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Control Volume Analysis Related to Putt-Putt Boat

In fluid mechanics, control volume analysis has the same importance and usefulness as analysis based on free-body diagram in the rigid body mechanics. This note will enable one to develop a feel for the main principles. CV is an imaginary volume that has to be carefully selected. In general, fluid can flow in and out the surface (the control surface) of the CV.

CV analysis for the mass conservation gives:

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \text{ Rate of accumulation of mass in CV,}$$

where $\dot{m}_{\text{in}}$ is the mass flow rate (in for example kg/s) into and $\dot{m}_{\text{out}}$ is the mass flow rate out of the CV.

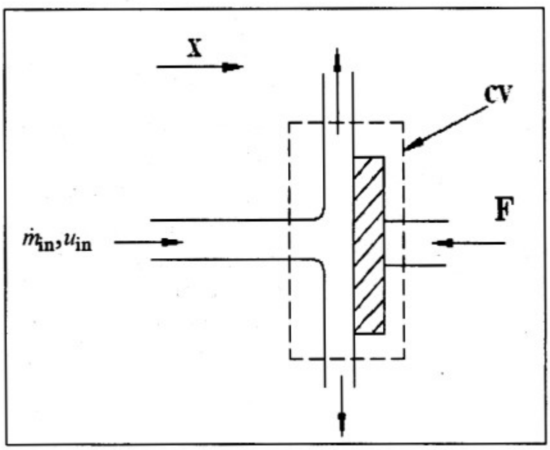
The CV analysis for momentum and forces in the x (say) direction gives:

Sum of the forces on CV in the X direction

= rate of x-momentum leaving C.V – rate of x-momentum entering CV + rate of accumulation of x-momentum in CV

Keywords
Integral analysis, thrust.

$$= \dot{m}_{\text{out}} u_{\text{out}} - \dot{m}_{\text{in}} u_{\text{in}} + \text{rate of accumulation of x-momentum in CV.}$$



u_{out} is the velocity in the X direction of the fluid flowing out and u_{in} is that of the fluid flowing in.

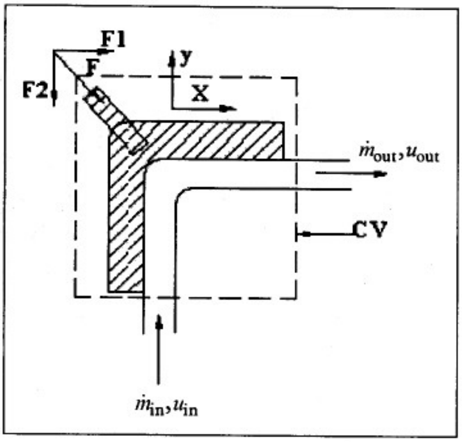
Below are some examples. Example 3 and 4 are relevant to the putt-putt boat analysis (see p.66).

1) Jet of fluid hitting a flat plate

Jet of fluid hitting a flat plate requires a force F to the left to kill the momentum of the jet.

The atmospheric pressure does not cause any force. CV analysis gives,

$$F = \dot{m}_{in} u_{in} \, .$$



2) Jet of fluid hitting a plate with right angle bend

There is no accumulation of mass in the CV, $\dot{m}_{out} = \dot{m}_{in}$.

A vertical downward force (F_2) is required to kill the upward momentum of the jet. A force (F_1) to the right is required to give the jet a momentum to the right.

$$F_1 = \dot{m}_{out} u_{out}$$

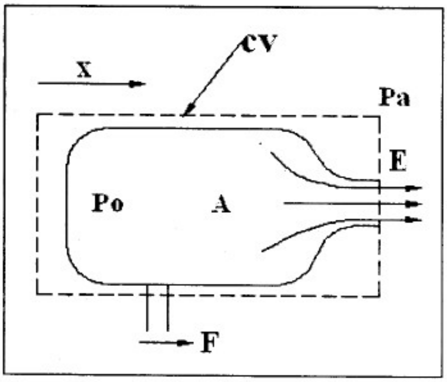
$$-F_2 = -\dot{m}_{in} u_{in} \quad \text{or} \quad F_2 = \dot{m}_{in} u_{in} \, .$$

3) Flow out of a pressurized vessel

This case corresponds to the ejection phase in the putt-putt boat. Consider the case of air gushing out of a small opening in a canister containing pressurized air, with pressure P_o; then

$$\Delta P = (P_o - P_a) > 0 \, .$$

We intuitively know that a 'thrust' is produced in a direction opposite to the jet of air issuing out. Let $-F$ be the force acting on the support; the support will exert an equal and opposite F force on the canister.



Besides F the only other force on the CV will be due to the pressure acting on the surface of the CV i.e. on the outer surface of canister and the exit. On the outer surface of the canister the pressure is the ambient pressure P_a; the exit pressure P_e is also P_a because the air comes out like a jet (see *Box* 1, p.67). Now, the exit velocity U_e can be found from Bernoulli's equation applied between some point A within the canister and E at the exit,

$$P_o + \frac{1}{2}\rho_o U_o^2 = P_e + \frac{1}{2}\rho_e U_e^2.$$

For pressure differences (P_o–P_e) small compared to P_o, changes in density are negligible, $\rho_e = \rho_o = \rho_a$[1]. If the canister dimension is large compared to the exit dimension then the velocity within the canister is negligible, $U_o = 0$. The exit velocity has the simple relation

[1] Subsript o refers to conditions within cannister, subscript e refers to exit and subscript a to ambient.

$$U_e = [2(P_o - P_a)/\rho_a]^{1/2} = (2\Delta P/\rho_a)^{1/2}.$$

Application of the integral momentum equation in the X direction

$$F - (P_e - P_a)A_e = \dot{m}_{out} U_e - \dot{m}_{in} U_{in}$$

gives, with $P_e = P_a$, $\dot{m}_{in} = 0$, $\dot{m}_{out} = \rho_a U_e A_e$

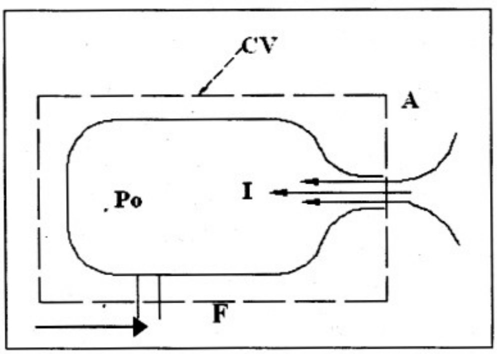
the thrust $F = \rho_a U_e^2 A_e = 2(P_o - P_a)A_e = 2\Delta P A_e$.

In other words , to create the exit momentum of $\dot{m}_{\text{out}} U_e$ requires a force F to the right. Such a canister (or balloon) can be used, for example, to push a toy boat forward.

4) Flow into a canister containing low pressure air

This case corresponds to the suction phase in the putt-putt boat. Now the pressure inside is less than the atmospheric pressure P_a; that is,

$$\Delta P = (P_o - P_a) < 0.$$



This case is more unusual and it is not obvious what the direction of the force would be due to the in-gushing air. Would the low pressure air inside push the canister to right, and produce a negative thrust, or would the momentum of the incoming air push it to the left and produce a positive thrust?

A jet is formed inside the canister as shown in the figure. Application of Bernoulli's equation between A and I, and again assuming density changes are negligible, gives the fluid velocity at the opening as

$$U_I = \left(2(P_a - P_o)/r_o\right)^{1/2}.$$

In the earlier case pressure at the opening was P_a; now it is below the ambient value and infact it is approximately equal to the canister pressure, $P_e \approx P_o$.

The momentum equation is

$$F - (P_a - P_o)A_e K \approx -\dot{m}_{\text{in}} u_{\text{in}}.$$

K is a factor greater than unity that takes into account the fact that a low pressure acts not only the exit velocity area A_e but on the surrounding lip area as well. We have

$$P_e = P_o,\ \dot{m}_{in} = rU_I A_e,\ u_{in} = -U_I$$

(u_{in} is in the negative X direction).

Substituting the above values gives the force

$$F = rU_I^2 A_e - (P_a - P_o)A_e K$$

$$= 2(P_a - P_o)A_e - (P_a - P_o)A_e K = (2 - K)(P_a - P_o)A_e.$$

The first term on the right hand side of the first line giving a positive thrust arises due to the incoming fluid losing its momentum within the canister; the second term producing a negative thrust is due to the region of low pressure existing near the exit. The value of K will depend on the details of the geometry at the exit. However, taking a large spherical CV surrounding the canister and assuming a sink type of flow into the canister gives zero force i.e., $K = 2$.

In summary, as expected, outflow creates a positive thrust; inflow is expected to create a small force. Thus, in the case of the putt-putt boat the ejection phase creates almost all of the propulsive thrust.