

Think It Over



This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

Shailesh A Shirali
Principal
Amber Valley Residential
School
K M Road Mugthihalli
Chickmagalur 577 101, India.
Email:
shailesh_shirali@rediffmail.com

On a Use of Normal Distribution

Problem

For positive integers n , consider the quantities A_n and B_n defined by

$$A_n = \int_{-1}^1 \cos^{2n} \frac{\pi x}{2} dx,$$

$$B_n = \frac{1}{A_n} \int_{-1/\sqrt{n}}^{1/\sqrt{n}} \cos^{2n} \frac{\pi x}{2} dx. \quad (1)$$

It is easy to anticipate that $A_n \rightarrow 0$ when $n \rightarrow \infty$; some sample values are displayed below, computed using *Mathematica*.

$$A_{100} \approx 0.1127, A_{200} \approx 0.0797,$$

$$A_{400} \approx 0.0564, A_{1000} \approx 0.0357. \quad (2)$$

Keywords
Stirling approximation.

Since the range of integration in the integral defining B_n steadily diminishes, we might also guess that $B_n \rightarrow 0$



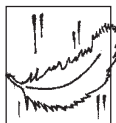
THINK IT OVER

as $n \rightarrow \infty$. But the figures tell us otherwise; indeed we find that

$$B_{100} \approx 0.9742, B_{200} \approx 0.9739,$$

$$B_{400} \approx 0.9738, B_{1000} \approx 0.9737. \quad (3)$$

The figures suggest that B_n converges to a non-zero value (approximately 0.974) as $n \rightarrow \infty$. How may this be explained? What is the limiting value, and can it be expressed in closed form?



Anagrams of William Burnside:

I WARN DUMB LILIES
SIMILAR, BLUE WIND
I WILL SUM IN BREAD
MEDUSA ILL; WIN RIB
I WILL DREAM IN BUS
BILL; I MUSE INWARD
INLAW IS MULE BIRD
I DRAW IN SEMI-BULL
I WILL DUB SEMINAR
BILL; I MUSE DARWIN
I AM ILL; WIN RED BUS
ALL MINIBUS WIRED
WELD RAIL, MINIBUS
I, SIR, BAN WILD MULE
DIAL; WE RUIN LIMBS
REDIAL US; WIN LIMB
I'M BRAINWISE DULL

... *Kanakku Puly*

