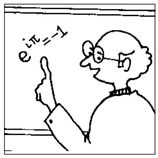
Classroom



In this section of* Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

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The Magnetohydrodynamic Generator
A Physics Olympiad Problem (2001)

The XXXII International Physics Olympiad was held in Antalaya, Turkey from June 28 - July 6, 2001. It marked India's fourth foray into this exciting event where sixty-five nations participated. As the delegation leader of the Indian team at Antalaya, one of us (VAS) was privileged to be in the thick of action. Our performance was a resounding success. With three gold and two silver medals we were placed second after the People's Republic of China (four gold medals and one silver medal) and at par with Russia and USA. This article describes an interesting problem on the magnetohydronamic generator that was posed as part of the five hour theoretical examination.

Preamble

The International Physics Olympiad (IPhO) is an annual event initiated by erstwhile East European nations three decades ago. India has been a late-comer to this event which is part festival, part competition – in short a celebration of pre-college physics. We participated for the first time in 1998. This year was our fourth foray

Keywords
Magnetohydrodynamics, generator, power, efficiency, Faraday's law, Physics Olympiad.

into this exciting event. One of us (Vijay A Singh) was the delegation leader of the Indian team consisting of ¯ve pre-college (Standard XII) students which participated in the XXXII International Physics Olympiad at Antalaya, Turkey from June 28 to July 6, 2001. We participated in the training and selection of our team, gave tips and advice to them in Turkey, and shared the agony and the eventual ecstasy during the prize-distribution ceremony. In short, we were privileged to be in the thick of events.

The road to the Physics Olympiad is indicated in Table 1. Our intense involvement begins in May when we run the International Physics Olympiad Training Camp (IPhOTC) in the Homi Bhabha Center for Science Education (HBCSE), Mumbai. One of us (Vijay A Singh) was the Academic Co-ordinator of the Theory Part of the training camp. It is perhaps fair to say that at least a dozen of us have to make do with four hours of sleep a day for the next two months. We had a di±cult time selecting the chosen ¯ve students from the thirty-two who were present at the camp. This done, we conducted a pre-departure training (PDT) for the ¯ve students before °ying to Antalya for the IPhO. More details of the procedure, preparation and strategy can be found elsewhere [1-3].

The examinations consist of two components: experiment and theory. Each component is of ¯ve hours duration. The theory component consists of three questions

Table 1. The road to the Physics Olympiad.
The acronyms (NSEP, IAPT, etc.) are explained separately in the text.

#	Exam	Duration	Date	Participants	Assoc.
1.	NSEP	2.5 hrs	Nov./Dec.	> 25,000	IAPT
2.	INPhO	8 hrs.	Jan. end	200	IAPT + HBCSE
3.	IPhOTC	3 weeks	May-June	35	HBCSE
4.	IPhO-PDT	1 week	June	5	HBCSE
5.	IPhO	10 hrs. 10 days	July	> 300	Intnl. location!

#	Name	Place	Points	Medal
1.	NANDAN DIXIT	Mumbai	45.8/50	Gold
2.	PARAG AGRAWAL	Mumbai	43.5/50	Gold
3.	ARVIND T	Chennai	42.2/50	Gold
4.	NARESH SATYAN	Bangalore	41.85/50	Silver
5.	VIJAY KUMAR S	Bangalore	40.3/50	Silver

Table 2. Results of the XXXII International Physics Olympiad.

Prof. Vijay A Singh, IIT-Kanpur and Prof. D A Desai, Ruparel College, Mumbai were the two leaders of the Indian team. Note that Naresh Satyan missed the gold by only 0.15 points!

each of ten points. The experimental component carries a weightage of 20 points. As mentioned above, our efforts were met with a good degree of success. With three gold medals and two silver medals we were placed second, just behind the People's Republic of China (four gold medals and one silver medal) and at par with Russia and USA. However, there was some heartbreak too (see Table 2). Naresh Satyan missed the gold by only 0.15 points! If only we could have bridged this narrowest of gaps we would have qualified for the top ranking alongside China. Sixty-five nations participated in this event.

The current article describes one of the challenging theoretical problems posed in the XXXII International Physics Olympiad and its solution. The problem is presented in a modified form to make it suitable for presentation in article form.

The Magnetohydrodynamic Generator: the Problem

A horizontal rectangular insulating pipe of width w and height h, which closes upon itself, is filled with liquid metal such as mercury or liquid sodium of resistivity ρ. An overpressure P is produced by a turbine which drives this fluid with constant speed v_0. The two opposite vertical walls of a section of the pipe with length L are made of copper. The copper walls are electrically shorted externally and a uniform magnetic field of magnitude B is applied vertically upward only to this section. The

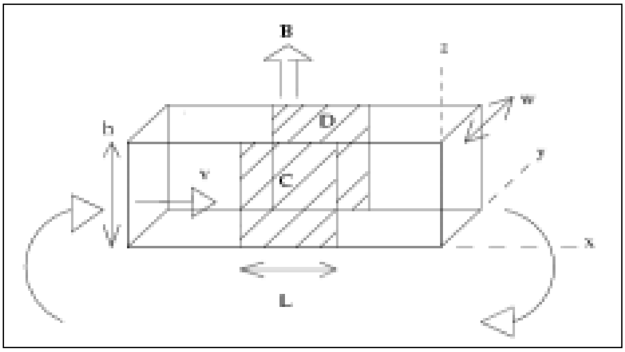

Figure 1. The MHD generator consists in its idealized form of a horizontal rectangular pipe. A liquid metal (say mercury or sodium) is driven along its length by a turbine (not shown). The broad arrows at the two ends indicate that the pipe closes upon itself. A uniform magnetic field B extends over a section L, and the two copper plates placed on vertically facing walls are electrically shorted across CD. In later parts of the problem a load resistance R_L is inserted between CD.

set-up is illustrated in Figure 1, with the axes $x; y;$ and z as indicated.

The motion of a real fluid is very complicated. To simplify the situation we assume the following:

- Although the fluid is viscous, its speed is uniform over the entire cross-section.
- The speed of the fluid is always proportional to the net external force acting upon it.
- The fluid is incompressible.

1. Find the force acting on the fluid due to the magnetic field (in terms of $L; B; h; w; \rho$ and the new speed v).
2. Derive an expression for the new speed v of the fluid (in terms of $v_0; P; L; B$ and ρ) after the magnetic field is applied.
3. Derive an expression for the additional power that must be supplied by the turbine to increase the speed to its orignal value v_0.
4. Instead of shorting the copper walls, we connect them with a load resistance R_L. Such a set-up represents a magnetohydrodynamic (MHD) generator in idealized form. Derive the modified expressions for the parts (1), (2), and (3).
5. Derive an expression for the efficiency of the MHD generator. For what value of the load R_L is it a maximum?

The Magnetohydrodynamic Generator: the Solution

1. Let the speed of a fluid particle passing through the section be v. From Faraday's law of induction the induced electromotive force (emf) across the copper walls will be $\varepsilon = Bvw$ (see Figure 1). The section of the fluid across L will impart resistance to the electrically shorted circuit. Using Ohm's Law the induced current will become $BvhL/\rho$. This current will produce a Lorentz force on the fluid in the presence of the magnetic field. The force acting on the fluid is thus

$$F = \left(\frac{BvhL}{\rho}\right)(-w\hat{y}) \times (B\hat{z})$$
$$= -\frac{B^2 vhwL}{\rho}\hat{x}. \qquad (1)$$

As expected, this magnetic force opposes the motion of the liquid metal produced by the turbine. Note that $\hat{x}, \hat{y}$ and $\hat{z}$ are unit vectors in the x, y, and z directions, respectively.

2. The forward force on the fluid due to turbine is Phw. The speed of the fluid in the absence of the magnetic field can be written as

$$v_0 = \alpha Phw; \qquad (2)$$

where α is the proportionality constant. On the application of the magnetic field, the backward Lorentz force will also act on the fluid. Now, the new speed of the fluid will be proportional to the net external force on the fluid,

$$v = \alpha(Phw - F).$$

Here F is the magnitude of the force in (1). Substituting the expression for α and F from (2) and (1), respectively, we obtain after some algebra

$$v = v_0\left[1 + \frac{v_0 B^2 L}{P\rho}\right]^{-1}. \qquad (3)$$

3. The additional power supplied by the turbine to restore the speed of the fluid to its original value v_0 should be same as the power dissipated across the fluid section L. This is given by the Joule heating formula. Hence the additional power is

$$p = \left(\frac{BvhL}{\rho}\right)^2 \left(\frac{\rho w}{hL}\right)$$

$$= \frac{B^2 v^2 hwL}{\rho}. \qquad (4)$$

Note that we may also derive (4) by the well-known expression that the instantaneous power is the scalar product of the force (1) and the instantaneous velocity.

4. With load, the circuit will offer total resistance $(R_L + \rho w/hL)$ and the induced current will reduce accordingly to $BvwhL/(R_L hL + w\rho)$. The force acting on the fluid due to magnetic field under this modified situation will become

$$F = -\frac{B^2 w^2 vhL}{R_L hL + w\rho}\hat{x}. \qquad (5)$$

The net external force will change and hence the new speed of the fluid is

$$v = v_0 \left[1 + \frac{B^2 w v_0 L}{P(R_L hL + w\rho)}\right]^{-1}. \qquad (6)$$

The total power dissipated will also get modified,

$$p = \frac{B^2 v^2 w^2 hL}{R_L hL + w\rho} \qquad (7)$$

Note that the above modified expressions (5, 6, 7), will reduce to (1, 3, 4) respectively, if we put the value of R_L to zero.

5. Efficiency η of the device can be expressed as

$$\eta = \frac{\text{Power across load}}{\text{Total power delivered}}$$

$$= \frac{R_L}{R_L + \rho w/hL}. \qquad (8)$$

When R_L is equal to $\rho w/hL$, the power delivered across the load will be maximum. Under this condition the efficiency will assume its maximum value, i.e. 50%.

Postscript: Calculations for Power and Efficiency

In the discussion so far we have neglected Hall effect, an aspect that was explicitly excluded in the Olympiad problem since it was out of reach of even a bright high school student. The Hall effect arises since the moving charged fluid particle is deflected in the presence of the transverse magnetic field. This results in an increase of the resistivity by a factor $(1+\omega^2\tau^2)$. Here ω is the cyclotron frequency of the charged fluid particle such as an electron and τ is the mean lifetime between collisions. We shall not derive this result here but refer the reader to an authoritative text [4].

Table 3 depicts numerical estimates of $F; v; p$ and η for a set of input parameters [4] $L = 0.5$ m; $h = 0.2$ m; $w = 0.2$ m; $B = 2.0$ Wb/m^2; $v_0 = 3000$ m/s; $P = 15$ kPa; $\rho = 0.1\ \Omega$-m for our idealized MHD generator. These estimates are obtained from (1, 3 to 8). We have neglected Hall effect in other words the Hall number $\omega\tau$ taken to be zero. For nonzero Hall number, one should replace ρ in all the above mentioned equations with $\rho(1+\omega^2\tau^2)$. Note that this will lead to an appreciable decrease in the efficiency. An inspection of Table 3 makes this clear.

Apart from the efficiency there are a number of other thermodynamic, hydraulic and electrical factors which

Table 3. The table encapsulates some performance characterstics of an elementary MHD generator. The load resistance R_L has a value of 0 Ω in columns two and three and 0.2 Ω in columns 4 and 5.

	$\omega\tau = 0$	$\omega\tau = 10$	$\omega\tau = 0$	$\omega\tau = 10$
F (N)	480	22.9	400	22.6
v (m/s)	600	2886	1000	2887
p (KW)	288	66	400	65.4
η	0	0	0.5	0.01

Abbreviations and Acronyms

HBCSE: The Homi Bhabha Center for Science Education [National Center set up by TIFR in Mumbai, India]
IAPT: The Indian Association of Physics Teachers [A voluntary, grassroots organization of physics teachers active for over 15 years]
IIT: The Indian Institute of Technology
INPhO: The Indian National Physics Olympiad (Exam) [An annual national level examination conducted jointly by HBCSE and IAPT]
IPhO: The International Physics Olympiad (Exam) [An annual examination in which sixty-five nations participated this year at Antalaya, Turkey.]
IPhOTC: The International Physics Olympiad Training Camp [This is run for three weeks in May-June at HBCSE with in-house and IAPT faculty's help.]
JEE: The Joint Entrance Examination [An annual entry level national examination conducted by the IITs]
NSEP: National Standard Examination of Physics [An annual nationwide examination conducted by the voluntary body, IAPT. It is the entry point for INPhO and IPhO]
TIFR: The Tata Institute of Fundamental Research [A premier research body situated in Mumbai, India]

affect the performance of MHD generator. However, in its elementary form these are ignored and in fact that complex analysis is out of the scope of high school students.

Acknowledgements

We wish to thank Arvind Kumar (HBCSE), H C Pradhan (HBCSE), D A Desai (Ruparel College) and R M Dharkar (IAPT) with whom we have had many useful and enjoyable discussions. We would be guilty of a great injustice if we did not acknowledge the staff at HBCSE, particularly Shirish, Sumana and Gajanan. This work was supported in part by the Department of Atomic Energy through the Board of Research in Nuclear Sciences, India (No. 2001/37/16/BRNS).

Suggested Reading

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