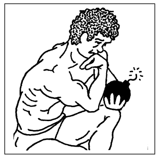
Think It Over



This section of* Resonance *is meant to raise thought-provoking, interesting, or just plain brain-teasing questions every month, and discuss answers a few months later. Readers are welcome to send in suggestions for such questions, solutions to questions already posed, comments on the solutions discussed in the journal, etc. to* Resonance*, Indian Academy of Sciences, Bangalore 560 080, with "Think It Over" written on the cover or card to help us sort the correspondence. Due to limitations of space, it may not be possible to use all the material received. However, the coordinators of this section (currently R Nityananda and C S Yogananda) will try and select items which best illustrate various ideas and concepts, for inclusion in this section.

Where is the Missing String?

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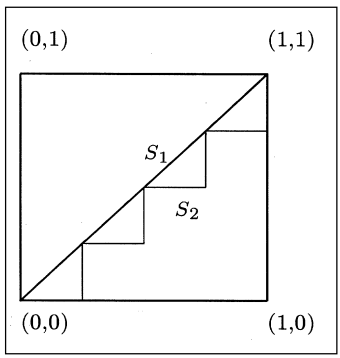
Consider the (unit) square with vertices at (0, 0), (1, 0), (1, 1) and (0, 1). Suppose you want to connect (0, 0) and (1, 1) with a string. If you place a string (S_1) along the diagonal from (0, 0) to (1, 1), it will have a length of $\sqrt{2}$. If you place a second string (S_2) such that it goes from (0, 0) to (1, 0) first and then to (1, 1) it will have a length of 2. Now take S_2 closer to S_1 by changing its course so that it goes from (0, 0) to (1/2, 0) (horizontally), then from (1/2, 0) to (1/2, 1/2) (vertically), from (1/2, 1/2) to (1, 1/2) (horizontally) and then from (1, 1/2) to (1, 1) (vertically). S_2 still has length 2. Bring it even closer to S_1 by changing its route such that from (1/4, 0) it goes vertically to (1/4, 1/4), from there to (1/2, 1/4) horizontally, then vertically to (1/2, 1/2), next to (3/4, 1/2), next to (3/4, 3/4), (1, 3/4), and finally to (1, 1). S_2 still has length 2. Continue this procedure. At the ith stage S_2 will look like a step function touching the diagonal at the points (0, 0), ($1/2^{i-1}$, $1/2^{i-1}$), ($1/2^{i-2}$, $1/2^{i-2}$), ... (1,1). Eventually S_2 will *merge* with S_1. But the length of S_2 is always 2. Where did more than a quarter of S_2 disappear in the limit?



While stating the problem submitted by Rangaswamy in the TIO of the September 97 issue the condition that zero should not appear in the array was left out by mistake. This condition makes the problem harder. We have received correct solutions to the problem as stated in the September 97 issue from V Subramanyam, N Hariharan and Anima Nagar but all the solutions given by them have zeroes occuring in the array. We thank them for their interest and encourage them (and also other readers) to solve the problem under the condition stated above.