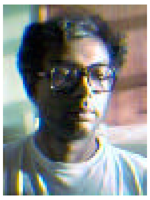
Symmetries of Particle Physics: Space-time and Local Gauge Symmetries

Sourendu Gupta



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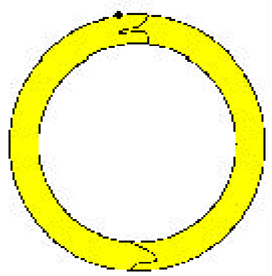
Introduction

Richard Feynman wrote that if all scientific knowledge in the world were to be destroyed and he could choose only one piece of understanding to be passed on to the future, then he would choose as his message: "matter is composed of atoms, ceaselessly moving and bouncing against each other". This is indeed the starting point of all modern physics. During the last two centuries, experiments have led us from these atoms to ever smaller particles, in a search for a theory of the material universe.

Underlying the modern extension of the atomic theory is the theme of symmetry. In retrospect, the physics of the last century can be seen as successive waves of experiment and theory, each feeding on the other. Experiments have slowly revealed the symmetries of the universe, and these have been incorporated into an unified quantum field theory. The final achievement of this century following from this line of investigation is called the standard model of particle physics (see [1]).

Figure 1. Experiment and theory feed on each other.

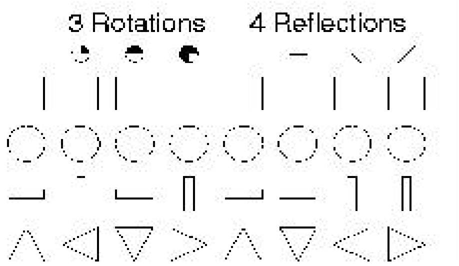


Why are symmetries so important in quantum mechanics? The answer is simple – quantum states are distinguished only by how they transform under symmetries (see *Box* 1). The symmetries of what? That of their energy written out as a function of space and time. This function is called the Hamiltonian. Under the symmetries of the Hamiltonian, the quantum state could be scalar, or have more complicated transformations.

All states with equivalent transformations are completely

Box 1. Symmetries of a Square

All the symmetry transformations of a square are shown in the *figure* below. These consist of 3 rotations about the center (by 90, 180 and 270 degrees) and reflections about 4 lines (the vertical and horizontal bisectors and two diagonals). Including also the trivial operation 'do nothing', there are 8 possible transformations. Every operation can be undone by another transformation. Doing two transformations one after the other is exactly like doing another of the same 8 operations. Any set of transformations which satisfy these properties (and another slightly technical property called associativity) are said to form a group. So these 8 operations form a group – the symmetry group of a square.



The square is said to be invariant under its symmetry group, since you cannot by looking tell whether or not it has been acted upon by any of these transformations. A circle is also invariant under this group. We can also say that these two shapes are scalars of this group. The circle and the square are not distinguishable by these transformations. However, if these operations are carried out on a rectangle or a triangle, then some of the transformed shapes can be distinguished from the originals. We can say that a rectangle transforms non-trivially under the symmetries of a square, and so does a triangle. However, the transformation properties of a triangle are not the same as those of a rectangle – these are two classes of objects under the symmetries of a square.

identical – there is no way to tell them apart. Thus two molecules of any chemical compound are exactly the same if their energies are equal. There is no way to tell them apart. The molecules could be hydrogen, water, chalk, tri-chloro ethane, or even as complicated as the amino acid leucine.

2. Symmetries of Space and Time

Some symmetries of the universe seem obvious. We can see by experience that the length of a scale remains the same everywhere in the universe; it does not depend upon whether the scale faces north or east. These are the translational and rotational symmetries that we are all familiar with. They leave the distance between two fixed points in space invariant.

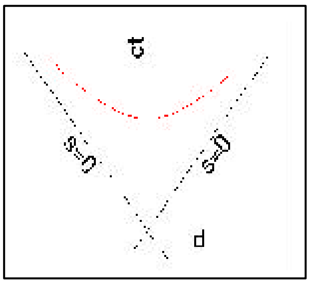


Figure 2. In a space-time diagram, lines of constant s (in red) appear as hyperbolae. Light travels along the lines s=0, all of which lie on the surface of a cone called the light cone.

There is more to space-time symmetries than this, as Albert Einstein found. If you sit in a moving train, time passes a little slower, and distances become a little smaller than if you sat in your house[1]. What is really invariant in this universe is the 4-dimensional distance between two events –

$$s = \sqrt{d^2 - c^2t^2},$$

where c is the speed of light, d the spatial distance and t the time between the two events. This symmetry includes rotational symmetry, but also something more. It is called Lorentz symmetry. Lorentz symmetry and translations (in space and time) together is called Poincaré symmetry. The transformation of wave-functions of particles under Poincaré symmetries are described by 4-momentum and spin angular momentum [2].

[1] This is, of course, very well known by all those who have used the Indian Railways.

[2] Spin takes values 0, 1/2, 1, 3/2, 2, and so on in units of Planck's constant h=1.0546 x 10^{-34} Joule-seconds.

When two things are exactly the same, we can ask what happens when we interchange them. For example take the wave-function of a pair of leucine molecules. There is a joint wave-function $\Psi(x_1, x_2)$ that one molecule of leucine is at position x_1 and the other is at position x_2. Since the molecules are identical, the probability, $|\Psi(x_1, x_2)|^2$, cannot depend on which one of the molecules is at x_1 and which is at x_2.

This means that

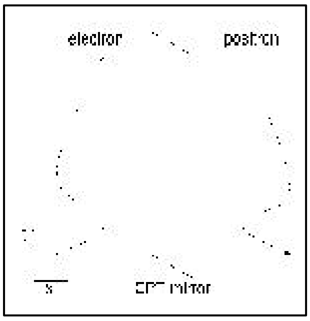
$$\Psi(x_1, x_2) = \pm\Psi(x_2, x_1).$$

This is another of the symmetries of the universe. All particles[3] with integer values of spin obey this law with the positive sign. They are called bosons. All particles with half-integer values of spin obey the law with a negative sign. They are called fermions. The Indian physicist Satyen Bose, working in Dhaka University was the first to work out the consequences of quantum indistinguishability of particles. Enrico Fermi, an Italian physicist, and the first person to build a nuclear reactor, worked out consequences that Bose had missed.

[3] Particles with spin 0, 1, and so on are called bosons. Those with spin 1/2, 3/2, etc. are called fermions.

There is still another symmetry of space-time called CPT symmetry. This relates the action of changing a particle into its anti-particle(C), observing it in a mirror (parity: P) and observing a movie running backwards (time reversal: T). For a long time it was believed that they are individually symmetries. Then in the mid-50's Chen Ning Yang and Tsung-Dao Lee hypothesized that parity is not conserved in certain interactions. After this was verified by Madame C S Wu and her collaborators, Yang and Lee were awarded a Nobel Prize.

Figure 3. CPT symmetry.



Parity breaking was incorporated into particle physics by two separate groups – Richard Feynman and Murray Gell-Mann working together, and also by Marshak and his graduate student George Sudarshan. In another Nobel Prize-winning experiment conducted in 1964, James Cronin and Val Fitch demonstrated that the combined symmetry CP is violated. We now believe that CPT together is a symmetry, although each is separately broken. Tests of CPT violation have been carried out with negative results.

There is one last class of space-time symmetries. These change fermions into bosons, and goes under the name of supersymmetry (SUSY). At present there is no observational support for this, although it is a favourite symmetry of theorists.

3. Gauge Symmetries

The real action in the last thirty years has been in the field of internal symmetries. What does that mean? To understand this, compare a twisted ribbon with a twisted rod. In both cases, elastic energy is stored in the twisted system. It is easy to see whether a ribbon is twisted. But it is not so simple to say whether the rod is twisted. The round cross-section of the rod has full rotational symmetry about the axis of the rod (this is called an $U(1)$ symmetry group). This 'internal' symmetry makes it hard to see whether or not energy is stored in the rod without allowing it to untwist itself.

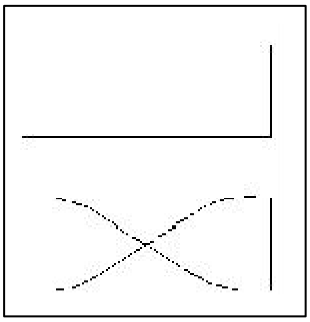


Figure 4. Which ribbon is twisted?

To help you, let me draw guidelines along the rod to make the twist visible. Which rod is twisted? The one with red lines along it, or the one with green? Again, you will be in a quandary. If the red lines were drawn for the untwisted rod, then the green lines show the twist. On the other hand, I could perversely have drawn the green lines on an untwisted rod. Then the red lines would show an opposite twist. This freedom of drawing the guidelines in any way one chooses is called a local gauge freedom. You would not know which is the twisted rod until I have told you my choice of gauge. But once you know this, you can compute the elastic energy stored in the rod.

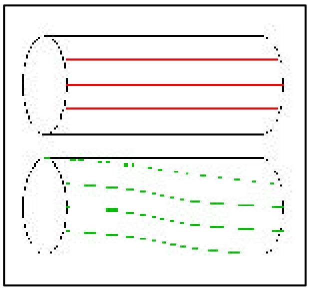


Figure 5. Which rod is twisted?

Let us do this. First imagine that the rod is made of a whole lot of thin cylinders stacked together. The torsional energy of each cylinder is proportional to the square of its angle of twist, and the torsional energy, E, of the rod is just the sum of such terms for each cylinder. This is expressed by the formula

$$\mathcal{E} = \frac{1}{2}\int dx \left(\frac{\partial\theta}{\partial x}\right)^2 .$$

We have chosen units such that the constant of proportionality is $\frac{1}{2}$. Now we have to figure out how to measure the local twist $\theta(x)$ whose derivative gives the energy.

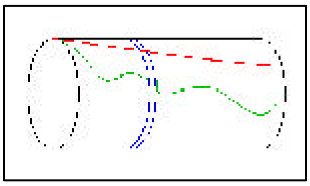


Figure 6.

We can measure the twist $\Psi_0(x)$ by first drawing a gauge line on the untwisted rod (shown in red). After twisting, this line is deformed to the line $\Psi(x)$ shown in green. The twist is $\theta(x) = \Psi(x) - \Psi_0(x)$. Local gauge freedom means that we could have changed the gauge line $\Psi_0(x)$ to any other $\Psi_0'(x) = \Psi_0(x) + \phi(x)$. This would of course have changed $\Psi(x)$ by the same amount, but the energy E is independent of this change.

We can push this model only so far. It turns out that a rod, or any such one-dimensional object is too simple to support a real gauge theory. To really construct such a theory, one needs a connected mass of loops. We shall not pursue this analogy further (though it would lead us to lattice gauge theory and eventually to Kenneth Wilson's approach to renormalisation).

Maxwell electrodynamics is a typical gauge theory. The field energy is –

$$\mathcal{E} = \frac{1}{2}\int d^3x \Big(\mathbf{E}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{B}\Big).$$

The electric and magnetic fields are given in terms of scalar and vector potentials –

$$\mathbf{E} = -\mathrm{grad}\phi - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \mathrm{curl}\mathbf{A}.$$

We could make a change called a gauge transformation,

$$\phi \rightarrow \phi - \frac{\partial \psi}{\partial t}, \qquad \mathbf{A} \rightarrow \mathbf{A} + \mathrm{grad}\psi$$

to the scalar and vector potentials and still get the same physics. (You can verify from these three equations that a gauge transformation does not change $\mathbf{E}$, $\mathbf{B}$ and $\mathcal{E}$.) The freedom of adding derivatives of ψ to the gauge potential, without changing the energy, is precisely the same freedom as being able to choose, locally, the zero of the phase of the gauge potentials. The quantum theory of a $U(1)$ field at every point of space-time is a genuine, if somewhat trivial, quantum field theory. It describes free photons, and is the theory that Max Planck started off quantum mechanics with, exactly 100 years ago.

4. The Laws of Nature

The theory of charged particles interacting with electromagnetic fields is called quantum electro-dynamics (QED). It is an $U(1)$ gauge theory interacting with fermions. It's solution was found by three people working independently – Richard Feynman and Julian Schwinger in post-war USA and Sin-Itiro Tomonaga working in wartime Japan. This solution, called renormalisation, got the three a Nobel Prize. QED is the most successful physical theory to date, and works to a precision of better than 1 part per million.

The $U(1)$ symmetry group is said to be Abelian. In an Abelian group you can apply transformations in any order without changing the result. Rotations about a fixed axis are specified by an angle. Two successive rotations are the same as a single rotation with an angle equal to the sum of the angles of the two separate rotations. Because the angles just add up, the order in which the two separate rotations are performed does not matter.

More interesting are the non-Abelian groups, where the same two transformations done in two different orders give different results. The symmetry group of a square is non-Abelian (as shown in the margin, the combined result of a rotation and a reflection depends on the order in which they are performed). Another example of a non-Abelian group is the group of rotations in three dimensions. Rotations about different axes do not commute.

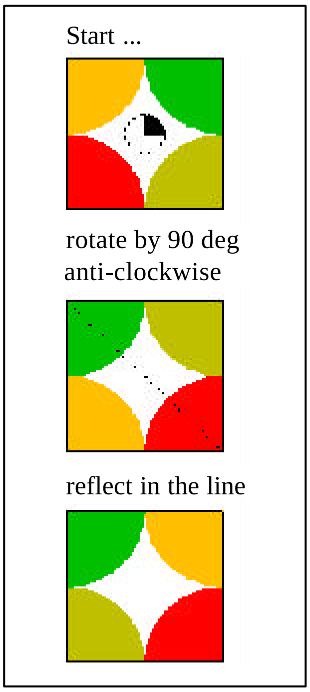
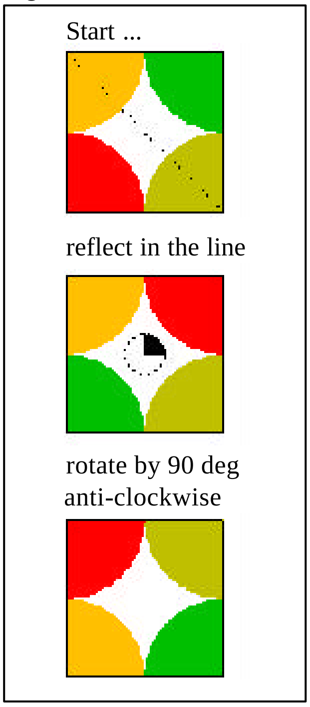


Figure 7.

Figure 8.



In the 1950's Chen Ning Yang and Robert L Mills showed independently how to formulate quantum field theories with non-Abelian gauge groups. There were enormous technical and mathematical difficulties in dealing with

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these theories, which were gradually solved over the years. Martin Veltman and his then student Gerard t'Hooft found how to renormalise a non-Abelian gauge theory. They were given the Nobel Prize in 1999 for this (see [2]).

These non-Abelian local gauge symmetries turned out to be important internal symmetries of the universe. A gauge group called $SU(2) \times U(1)$ unifies QED with the weak interaction. The weak interaction is responsible for β-decay of nuclei. We have already met the Abelian group $U(1)$ – the symmetries of a circle. The non-Abelian group $SU(2)$ is a close cousin of the group of rotations in three dimensions, i.e., the symmetries of a sphere. The solution of this gauge theory (called the electro-weak theory) and its application to the structure of matter brought the Nobel Prize to Sheldon Glashow, Abdus Salam and Steven Weinberg, who independently worked on this.

It turns out that the strong interactions are also described by a gauge theory. The relevant gauge group is $SU(3)$, the symmetry group of a certain 8-dimensional shape. This theory is called quantum chromo-dynamics (QCD). It was put together slowly by many different people, and has turned out to be extremely hard to solve. Although there has been enormous progress in our understanding of this theory, there are many calculations which we would like to perform but are unable to do. At present it seems that a combined attack using mathematics and computers is the best chance we have of solving this theory.

Much of the late-twentieth century theoretical physics, including the early evolution of string theories, has come from attempts to solve QCD. In turn, much of the modern theoretical physics, including the string theory in its new avatars, is often brought to bear on QCD, and sometimes succeeds in illuminating how it works.

Box 2. The Standard Model of Particle Physics

The standard model of particle physics is the twentieth century's most fundamental theory of matter. It describes all present day experiments on

1. the interactions of charged particles through the usual electro-magnetic forces (a theory called quantum electro-dynamics, QED)
2. the weak decays of nuclei and particles (this theory is unified with QED and is called the electro-weak theory)
3. the strong interactions which give rise to bound states called mesons and baryons (this theory is named quantum chromo-dynamics).

Normal matter consists of two sets of fermions – leptons (electrons, muons and tau are the charged leptons and three types of neutrinos are uncharged leptons) and quarks (up, down, strange, charm, bottom and top are all charged). These interact by exchange of bosons called gauge particles – photons mediate the forces of QED, $W^{\pm}$ and Z mediate the remaining forces in the electro-weak theory, gluons mediate the strong interactions between quarks. The interactions between quarks are so strong that they are never found free in nature, but are always bound up inside mesons or baryons such as the proton and neutron. Nevertheless, all the particles named above have been observed in experiments. The standard model also postulates another boson called the Higgs particle which has not been observed yet. The Higgs boson is responsible for giving masses to some of the gauge bosons and all the fermions.

One of the highest priorities of experimental particle physics is to search for the Higgs particle. Another is to test the prediction that at the extremely high temperatures and pressures which existed milli-seconds after the big bang, quarks were found free in nature.

The standard model of particle physics (see *Box* 2) is now written as a quantum field theory with local gauge invariance given by the gauge group $SU(3) \times SU(2) \times U(1)$. All the space-time symmetries except supersymmetry are included in its structure. The theory has been tested critically in experiments, and passed all tests. (There are still a few important ones left to do). The theoretical consequences are in the process of being worked out.

Several experiments have already been planned for the next decade. These will test the standard model in many new ways (see *Box* 2), because no theory is for ever. The future always brings surprises. Each surprise is an invitation to new generations of physicists to modify the achievements of the old. Even the standard model will have to change. But how?

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Suggested Reading

[1] Ashoke Sen, *Resonance*, Vol.5, No. 1, p.4, 2000.
[2] Rohini Godbole, *Resonance*, Vol. 5, No. 2, p.16, 2000.
[3] The Particle Adventure, http://particleadventure.org/.
[4] The Nobel e-museum, http://www.nobel.se/.

"The search for meaning is not limited to science: it is constant and continuous – all of us engage in it during all our waking hours; the search continues even in our dreams. There are many ways of finding meaning, and there are no absolute boundaries separating them. One can find meaning in poetry as well as in science; in the contemplations of a flower as well as in the grasp of an equation. We can be filled with wonder as we stand under the majestic dome of the night sky and see the myriad lights that twinkle and shine in its seemingly infinite depths. We can also be filled with awe as we behold the meaning of the formulae that define the propagation of light in space, the formation of galaxies, the synthesis of chemical elements, and the relation of energy, mass and velocity in the physical universe. The mystical perception of oneness and the: religious intuition of a Divine intelligence are as much a construction of meaning as the postulation of the universal law of gravitation."

Ervin Laszlo