Probability Theory Without Tears!

Not Too Austere, Not Too Chatty

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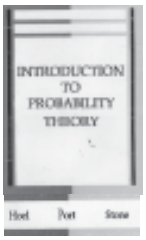

Introduction to Probability Theory
P G Hoel, S C Port, and C J Stone
Universal Book Stall, New Delhi. 1995
pp. 258. Rs. 75.

William Feller, in the preface to the third edition of his classic, observes that even in the 1940's few mathematicians recognised probability as a legitimate branch of mathematics and that applications were limited in scope. Today, however, as the opening paragraph of the book under review says, "probability theory is the branch of mathematics that is concerned with random (or chance) phenomena; it has attracted people to its study both because of its intrinsic interest and its successful applications to many areas within the physical, biological and social sciences, in engineering, and in the business world". A beginner to the subject is, therefore, likely to be interested not merely in the mathematical nuances but also in the applications (especially in statistics), and may want to acquire a reasonable working knowledge of the jargon, assumptions, limitations, etc.; in other words, he or she may be interested in understanding what probability theory is about and what it is not about. (It may interest the reader to note two of the extreme views about probability: the uninitiated very often consider probability theory to be a bag of tricks to solve combinatorial problems, and the professional mathematician at times dismisses it as a branch of measure theory. Needless to say these points of view reflect only two facets of the subject and are far from a complete picture.) Because of the relatively short history of probability theory as a "legitimate" branch of mathematics, a beginner may be misled by obscure, inelegant or even inaccurate presentations of important concepts. (One often wonders if the issue of "legitimacy" has anything to do with gambling dens where probability theory had its humble beginnings.) Also, she or he may be put off by the mathematical hairsplitting of advanced treatises. A good book intended for beginners should avoid these pitfalls. Too austere an approach will make one interested in a working knowledge of the subject uncomfortable, whereas too chatty a style will drive away a mathematically inclined student.

The book assumes only a knowledge of calculus (including multiple integration); no knowledge of measure theory or linear algebra is assumed.

In this context it is very satisfying to note that the highly successful text book by Hoel, Port and Stone is now available to Indian students at an affordable price. The book assumes only a knowledge of calculus (including multiple integration); no knowledge of measure theory or linear algebra is assumed. After quickly going through some very interesting combinatorial problems, the book deals with basic notions concerning discrete and (absolutely) continu-

ous random variables, expectations, moments, standard distributions, transformations of random variables, characteristic functions, central limit theorem and applications. A short chapter on random walks and Poisson processes is added at the end to serve as a brief encounter with stochastic processes. In other words, the book is ideally suited for a one-semester course on "non-measure theoretic" probability at the III B.Sc./I M.Sc. level.

A highlight of the book is that a proof of the central limit theorem is presented assuming inversion and continuity theorems for characteristic functions; this should whet the appetite of a mathematically inclined student. Another salient feature is that the sampling distributions of statistics are derived. Concepts are explained clearly and proofs are given whenever possible using only calculus. When proofs are not presented, what is being assumed and what needs to be proved are stated carefully. Examples and exercises have been chosen with care; exercises are designed to test the understanding and build the confidence of the student. Perhaps the greatest merit of the book is that it can be used even by a (diligent) student of average ability as self-study material.

The reviewer, however, would like to make the following comments. There could have been more non-routine exercises to provoke an above-average student. It could have been mentioned that characteristic functions are also called Fourier transforms (and Fourier series in the case of integer valued random variables). Some comments could have been made concerning the need to have the concept of σ-field; (that is, why it is not suitable to have the power set of the sample space as the set of events); this could have been done in the chapter on continuous random variables and references to advanced texts on measure theory cited. (Of course, to be fair to the authors it must be said that σ-field remains an enigma in most elementary texts.)

To sum up, the book will serve as an excellent preparation (without tears) for a course in statistics or further study in probability theory and stochastic processes. It will enable the reader to build up enough confidence to take on more advanced texts like the ones by Billingsley, Feller, Parthasarathy, and Rao.

Suggested Reading

P Billingsley. Probability and Measure . (Second Edition), John Wiley. 1994.

W Feller. An Introduction to Probability Theory and Its Applications . Vol. 1. (Third Edition). Wiley-Eastern, New Delhi. 1993.

W Feller. An Introduction to Probability Theory and Its Applications , Vol. 2. Wiley-Eastern, New Delhi. 1984.

K R Parthasarathy. Introduction to Probability and Measure. Macmillan Co. of India. New Delhi. 1977.

C R Rao. Linear Statistical Inference and its Applications . (Second Edition). Wiley-Eastern, New Delhi. 1984.

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