Think It Over

This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

Answer to TIO, September 1997.

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Question:

Look at the following 3×3 square arrays of 'squares that read the same horizontally and vertically.

(i) 5 2 9 ($=23^2$)
2 5 6 ($=16^2$)
9 6 1 ($=31^2$)

(ii) 1 6 9 ($=13^2$)
6 7 6 ($=26^2$)
9 6 1 ($=31^2$)

(iii) 1 2 1 ($=11^2$)
2 8 9 ($=17^2$)
1 9 6 ($=14^2$)

Can you find a 4×4 square array of squares reading the same across and down? What about 5×5, 6×6, 7×7, ... ?

There are at least 13 arrays of 4×4 squares that read the same across and down if all the digits are allowed to appear, but if one is allowed only non-zero digits in the array then there is a unique solution. Compilation of this unique array is quite interesting and here we shall outline the procedure to find it.

Numbers ending with the digits, 2, 3, 7 and 8 are not squares and the two-digit endings of squares, neither digit being zero, can only be among: 16, 21, 24, 25, 29, 36, 41, 44, 49, 56, 61, 64, 69, 76, 81, 84, 89 and 96. Look for those four-digit squares which do not have the digits 0, 2, 3, 7 and 8. Observe that only these can occupy the last row (and last column) of the required array. There are only three such: 1156, 1444 and 6561. It can be easily checked with 1156 or 1444 in the last row we cannot complete the array while with 6561 we are led to the unique solution:

2	1	1	6	=	46^2
1	2	2	5	=	35^2
1	2	9	6	=	36^2
6	5	6	1	=	81^2

which we can denote by (46^2, 35^2, 36^2, 81^2). In similar notation here are examples of such square arrays of squares of size 5×5, 6×6, 7×7 and 8×8:

(239^2, 269^2, 118^2, 162^2, 107^2)
(389^2, 744^2, 365^2, 594^2, 482^2, 407^2)
(1311^2, 2708^2, 1172^2, 2887^2, 2694^2, 1639^2, 1204^2)
(3336^2, 4044^2, 3721^2, 5046^2, 9139^2, 9477^2, 9665^2, 8134^2).

Editorial Remark: We have had good response from our readers to this problem in both versions. The following readers sent in correct solutions: N Hariharan (Bangalore), J Srivatsava (Hyderabad), V Subramanyam (Secunderabad).

heading ...

Let us consider an acidic solution of pH, say, S. Since pH=–log[H^+], the [H^+] of the said solution will be 10^{-5}M. Now, if we dilute the solution 10 times with distilled water, the [H^+] will become 10^{-6} M and the pH will increase to 6. Further dilution should, then, theoretically lead to stepwise increase of pH from 6 to 7 to 8 to 9 and so on. This obviously does not mean that diluting an acidic solution with distilled water will at some point give us a basic/alkaline solution. But the calculations and the theoretical logic as shown above seems to indicate such a case. So wherein exactly does the anomaly lie?