Functional Analysis – Spectral Theory[1]

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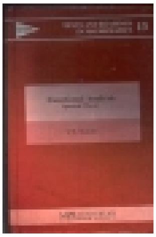

Functional Analysis – Spectral Theory
Texts and Readings in Mathematics - 13
by V S Sunder
Hindustan Book Agency
1997, pp. 306.

The surfeit of books on functional analysis over the years might lead one to believe that the subject has been presented from every possible perspective and in every imaginable style. The appearance of this book by V S Sunder will quickly dispel that attitude, for in it, the point of view taken, and the choice of ideas described, are significantly different from many typical expositions of the subject.

"The only real way to understand the spectral theorem is as a statement concerning representations of commutative C^* algebras." Thus speaks the author in the preface and these words summarize the outlook that he espouses. The book, as the subtitle indicates, is directed largely towards a study of spectral theory, using ideas from the theory of operator algebras (the latter being a subject in which the author has made notable research and expository contributions).

[1] Reproduced from *Current Science*, Vol.75, No.2, 25th July, 1998.

Beginning with some preliminaries about normed spaces (including the Hahn–Banach theorem, Open Mapping and Closed Graph theorems, Uniform Boundedness Principle and so on), the author goes on to discuss Hilbert spaces. Here the section on orthonormal bases describes in great detail many facts which are often passed off as 'intuitively obvious'.

This is followed by a chapter on C^* algebras (especially commutative ones) which includes the Gelfand–Naimark theory as well as representation theory. It culminates in the Hahn–Hellinger theorem whose proof is laid out very meticulously. The author's enthusiasm for von Neumann algebras finds expression in the Double Commutant Theorem (as in the use of these algebras elsewhere, for example, in the section on unbounded operators).

The above ideas are then used in a derivation of the spectral theorem and polar decomposition for bounded operators. A couple of sections providing a fairly detailed treatment of compact and Fredholm operators, and their interplay, wrap up the chapter.

A final chapter on unbounded operators (including the spectral theorem, using the Cayley transform) introduces the reader to the various subtleties one encounters in going beyond the confines of bounded operators.

The Appendix is both voluminous and broad enough in its content to merit being called a book within a book. It opens with some linear algebra and ends with the Stone–

Weierstrass and Riesz Representation theorems. In between are sandwiched a number of standard theorems... . Urysohn's lemma, Tychonoff's theorem, Tietze Extension theorem, Monotone and Dominated Convergence theorems and a host of others. The section on 'Transfinite Considerations' introduces some formal aspects of set theory, including Zorn's lemma.

From definitions of elementary concepts (even matrix multiplication is defined!) to a comprehensive appendix, every attempt is made to ensure that prior knowledge assumed is kept to a minimum.

Many of the constructs used (for example, Hilbert spaces, locally convex topologies) are well motivated in the text. The proofs provided are thorough and painstakingly detailed. A liberal sprinkling of well thought out exercises, which are essential to the main thread of the discussion, ensures that this book is not directed at the casual reader.

The use of an elaborate notational style and the repeated emphasis of assumptions and attributes, by parenthetical phrases, are hallmarks of the author. Though this device occasionally makes sentences long winded, it dispels any ambiguity.

The necessity of the assumptions for various theorems is clarified by means of (counter) examples. Instances are, the fact that the Closed Graph Theorem can be false in the absence of completeness, and the fact that a non-normal operator can have spectral values which are not approximate eigenvalues.

The use of more distinctive symbols for linear operators ($L(X,Y)$) and bounded linear operators ($\mathcal{L}(X,Y)$) and a more detailed index may have been useful. The mild mismatch of symbols, that has crept into the Hahn–Banach theorem, is obvious enough to be rectified even by an unsophisticated reader.

The book is pitched at the level of a serious master's student or a beginning research student. In the case of a master's student, some sections may require a particularly dedicated effort.

The Texts and Readings in Mathematics (TRIM) series has, undoubtedly, made noteworthy contributions to mathematical literature. Sunder's book adds to the stature of the series in significant measure.

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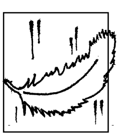

The ideals that have lighted my way and time after time have given me new courage to face life cheerfully, have been Kindness, Beauty and Truth.

Albert Einstein