

AN Kolmogorov laid the foundations for an axiomatic and mathematically elegant theory of probability in his famous monograph *Grundbegriffe der Wahrscheinlichkeitrechnung* published in the year 1933. An English translation of the book appeared in 1950 under the title *Foundations of the Theory of Probability*. The book proved to be a monumental work of immense proportions (though couched in only 70 pages) since it established probability theory, for the first time, within the purview of rigorous mathematics and served as the mooring for all future sailing into the sea of probability theory and its applications.

Kolmogorov used abstract measure theory to define the notion of a space of elementary outcomes, the sigma field of observable events, probability measure, and random variables. These have become the basic ingredients in probability theory and provide a single framework to study random phenomena irrespective of the various physical descriptions of the problems themselves. Probability measure was defined on the sigma field as a countably additive, non-negative set function with unit total mass. Such a definition fits all idealised models of random processes.

The remarkable ingenuity of Kolmogorov is seen in his celebrated consistency theorem which establishes the existence of a stochastic process provided the finite-dimensional descriptions of the random phenomenon have no inconsistencies. In other words, given a consistent family of finite-dimensional distributions, one can construct a unique probability measure in an infinite-dimensional space. For this reason, the consistency theorem is also known as Kolmogorov's extension theorem. This single result showed the existence of a variety of processes such as Brownian motion, Poisson process, Markov processes with regular transition probabilities, etc. In contrast, Wiener's proof of the existence of Brownian motion (which appeared ten years earlier than Kolmogorov's monograph) is specific to the process on hand.

The basic notions of conditional probability and conditional expectation were defined rigorously by Kolmogorov using the Radon-Nikodym theorem which had just then appeared. The consummate teacher that he was, Kolmogorov illustrates the pitfalls of conditioning with respect to an isolated event of probability zero by explaining Borel's paradox.

The contributions of Kolmogorov to probability theory are basic, rich, varied and important as the following incomplete list would show: strong law of large numbers, zero-one law, the three series theorem, Kolmogorov's inequality, consistency theorem, law of the iterated logarithm, Chapman-Kolmogorov equation, Kolmogorov's forward and backward equations, Kolmogorov's criterion for continuity, Kolmogorov-Smirnov test, characterisation of infinitely divisible distributions with finite second moments, and interpolation and extrapolation of stationary processes. His research gave birth to important notions such as self-similarity, reversibility, and subordination of processes. It is noteworthy that each paper of Kolmogorov spawned an exciting area of research which flourishes to this day.

Kolmogorov's achievements discussed above pertain only to his work in the theory of probability. He has played a major role in other areas of mathematics such as Fourier series, mathematical logic, ergodic theory, descriptive set theory, topology, theory of turbulence, and of algorithms. He wrote a number of articles on the teaching of mathematics, and statistical analysis of Russian versification. It would doubtless be hard for the future to believe that one man could accomplish so much in a life time. A fuller account of the life and legacy of Kolmogorov can be found in the suggested reading.

Suggested Reading:[1] *J. Appl. Probability*, 25, No.2, 445–450, 1988 (see article by KR Parthasarathy in this issue, 56–65). [2] *Russian Mathematical Surveys*, 43, No.6, 1988. [3] *The Annals of Probability*, 17, No.3, 815–964, 1989. [4] *Theory of Probability and Applications*, 34, No.1, 1989.

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