Comments

The basic purpose of stratification is to achieve homogeneous strata. However, in practice, a given stratified set-up could be far from ideal. If some of the strata are indeed heterogeneous and further the cost of surveying is rather low in those strata then the sample size allocations to those strata obtained by the mathematical model would be naturally high. At times the sample size allocated to a particular stratum may even exceed the stratum size. A simple explanation for such an anomaly is that the mathematical model used to derive optimal allocation does not incorporate the condition $0 < n_h \leq N_h$, $1 \leq h \leq k$.

This can happen in any optimization problem. A mathematical model may not incorporate a constraint that the optimal solution should to satisfy. If the optimal solution so obtained does not satisfy the desired constraint then it is necessary to take certain correcting steps. For our problem, or the one discussed in Cochran's book (1977), whenever the allocated sample size exceeds the corresponding stratum size, the entire stratum is enumerated. For the remaining strata a modified optimization problem is formulated. Another more recent book *Model Assisted Survey Sampling* by Sarndal, Svenson and Wretman (Springer-Verlag. New York, 1992) also discusses this very same problem in its Remark 12.7.1.

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Increase of Entropy and the Arithmetic–Geometric Mean

It is instructive to work out the change in entropy when two identical blocks of the same material but at different temperatures are brought together into thermal contact. The system as a whole is thermally isolated.

Let T_1 and T_2 be the absolute temperatures of the two bodies. Assume $T_1 > T_2$. When they attain thermal equilibrium both will reach a common temperature T given by:

$$T_1 - T = T - T_2$$

Now the change in the entropy of each body is given by:

$$\delta S = ms \int_i^f \frac{dT}{T} = ms \ln \frac{T_f}{T_i} \cdot$$

Here m = mass and s = specific heat, T_i and T_f are the initial and final temperatures of the body. Hence, the net change in entropy is

$$\Delta S = \delta S_1 + \delta S_2 = ms \ln\left(\frac{T^2}{T_1 T_2}\right) .$$

Now $T = (T_1 + T_2)/2$ is the arithmetic mean between T_1 and T_2 and it is greater then the geometric mean $\sqrt{T_1 T_2}$.

Therefore the entropy increases and this can be looked upon as a physical manifestation of the arithmetic mean being greater than the geometric mean.

Suggested Reading

- **G Almvist, B Berndt, Arithmetic - Geometric Mean. *American Mathematical Monthly*. pp. 585-607, January 1987.**

Arithmetic-Geometric Mean

Recall the identity $(T_1 + T_2)^2 = (T_1 - T_2)^2 + 4T_1 T_2$
so that

$$\frac{(T_1+T_2)^2}{4\, T_1\, T_2} = 1 + \frac{(T_1-T_2)^2}{4\, T_1\, T_2}$$

> 1 as T_1 , T_2 are positive

Taking square roots, $\frac{T_1+T_2}{2} > (T_1\, T_2)^{1/2}$

A.M > G.M.

It is interesting to note that the increase in entropy is related to the fact AM > GM. In this context we may recall an interesting result due to Gauss (see Suggested Reading) .

Let a,b be positive numbers with a > b. Consider the arithmetic and geometric mean,

$$a_1 = (a + b)/2, \quad b_1 = (ab)^{1/2}$$
$$a_2 = (a_1 + b_1)/2, \quad b_2 = (a_1 b_1)^{1/2}$$
$$a_{n+1} = (a_n + b_n)/2, \quad b_{n+1} = (a_n b_n)^{1/2} .$$

Gauss showed that a_n, b_n converge to the same limit.