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Flavour-changing neutral currents in models with extra Z' boson

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Abstract. New neutral gauge bosons Z' are the features of many models addressing the physics beyond the standard model. Together with the existence of new neutral gauge bosons, models based on extended gauge groups (rank > 4) often predict new charged fermions also. A mixing of the known fermions with new states, with exotic weak-isospin assignments (left-handed singlets and right-handed doublets) will induce tree-level flavour-changing neutral interactions mediated by Z exchange, while if the mixing is only with new states with ordinary weak-isospin assignments, the flavour-changing neutral currents are mainly due to the exchange of the new neutral gauge boson Z'. We review flavour-changing neutral currents in models with extra Z' boson. Then we discuss some flavour-changing processes forbidden in the standard model and new contributions to standard model processes.

Keywords. Standard model; flavour-changing neutral currents; Z' gauge boson; E_6 model.

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1. Introduction

Z' bosons are known to naturally exist in well-motivated extensions of the standard model (SM) [1]. In particular, they often occur in grand unified theories (GUTs), superstring theories, and theories with large extra dimensions [2]. In traditional GUTs, the scale of the Z' mass is arbitrary. However, in perturbative heterotic string models with supergravity mediated supersymmetry breaking, the $U(1)'$ and electroweak breaking are both driven by a radiative mechanism, with their scales set by the soft supersymmetry breaking parameters, implying that the Z' mass should be less than 1 TeV [3]. There are stringent limits on the mass of an extra Z' from the non-observation of direct production followed by decays into e^+e^- or $\mu^+\mu^-$ by CDF [4], while indirect constraints from precision data also limit the Z' mass (weak neutral current processes and LEP II) and severely constrain the Z–Z' mixing angle θ [5,6]. These limits are model-dependent, but are typically in the range

$M_{Z'} > O(500)$ GeV and $|\theta| <$ few $\times 10^{-3}$ for standard GUT models. A Z' could be relevant to the NuTeV experiment [7] and, if the couplings are not family universal [5,8], to the anomalous value of the forward–backward asymmetry $\mathrm{A}_{\mathrm{FB}}^{\mathrm{b}}$ [6]. (Earlier hints of a discrepancy in atomic parity violation have largely disappeared due to improved calculations of radiative corrections [9].) There is thus both theoretical and experimental motivations for an additional Z', most likely in the range 500 GeV–1 TeV. Also, in this mass range, it should be possible to carry out significant diagnostic probes of Z' couplings at the LHC and at a future NLC [10], which would complement those from the precision experiments [6]. The existence of the Z' boson would also suggest a spectrum of sparticles considerably different from most versions of the MSSM [11].

In the SM, lepton flavour violating (LFV) currents are strictly forbidden. This is not the case in most of its extensions. For instance, if right-handed neutrinos are present, LFV currents are generated radiatively, proportional to very small GIM-like factors involving neutrino masses. Other extensions of the SM, which include new neutral fermions and/or new Higgses, have been discussed in ref. [12]. In model building, it is generally required that some natural mechanisms exist to suppress LFV currents at a level compatible with the present experimental constraints.

It has been stressed [13] that extended gauge models, characterised by additional $U(1)$ factors and by the presence of new charged fermions, predict flavour-changing neutral currents (FCNCs) mediated by the additional neutral gauge boson Z'. Since the flavour-changing Z' vertices are expected to be naturally large, these FCNCs must be suppressed by a large Z' mass. Depending on the precise value of Z' mass, the Z'-mediated FCNCs give sizable contributions to both B_s^0–$\overline{B_s^0}$ and B_d^0–$\overline{B_d^0}$ mixing [14]. Z'-Mediated FCNCs have large effects on CP-violating rate asymmetries in B decays [15]. Rare hadronic B decays can also be affected by Z'-mediated FCNCs. Recently, the effects of Z'-mediated FCNCs on B meson decays (e.g. $B_d \to \phi K_s$, $B_d \to \eta' K_s$, $B_s \to \mu^+\mu^-$) are discussed [16]. Although experiments on FCNC process have significantly constrained the Z' couplings of the first and second generation quarks to be almost the same and diagonal, the couplings to the third generation are not well-constrained. It has been shown in refs [17–19] that the third generation fermions have different Z' couplings from the other two generations. With FCNCs, the Z' contributes at tree level, and its contribution will interfere with the standard model contributions.

Anoka *et al* [20] recently analysed a special class of supersymmetric Z' models wherein the Z' properties get essentially fixed from constraints of SUSY breaking. They found $M_{Z'} =$ 2–4 TeV and the Z–Z' mixing angle $\theta \approx 0.001$. Constraints from the elctroweak precision observable are satisfied, with the Z' model giving a slightly better fit compared to the standard model. Nardi *et al* [21] analysed (i) the effects of new neutral gauge bosons and (ii) the effects of mixing of the known fermions with the new ones in E_6 and $SO(10)$ models. They derived the stringent bounds on Z–Z' mixing ($\leq$0.02), on the fermion mixing parameters (0.01 in most cases), and on the mass of new gauge boson ($m_{Z'} >$ 170–350 GeV, depending on model). They found that ordinary–ordinary fermion mixing could induce a LFV for the physical Z boson. However, this vertex is suppressed by Z–Z' mixing, which is severely constrained by the data $|\theta| \leq 0.02$, and they excepted that the FCNC process would be mainly induced by direct Z' exchange. Bernabéu *et al*

[22] presented the experimental limits on μ–e conversion in nuclei which give a nuclear-model-independent bound on the Z–e–μ vertex which is twice as strong as that obtained from $\mu \to eee$. In the case of E_6 models, these limits provide quite stringent constraints on the Z' mass and on the Z–Z' mixing angle. The proposed experiments to search for μ–e conversion in nuclei have good chances to find evidence of lepton flavour violation, either in the case that new exotic fermions are present at the electroweak scale, or if a new neutral gauge boson Z' of E_6 origin lighter than O(TeV).

There are theoretical and phenomenological motivations that there may exist additional heavy Z' bosons with family non-universal couplings. Flavour mixing in the quark and lepton sectors will then lead to flavour-changing couplings of the heavy Z', and also of the ordinary Z when Z–Z' mixing is included. In this paper, the general formalism of such effects is described and applications are made to a variety of flavour-changing and CP-violating tree processes. Results are described for a specific heterotic string model and by phenomenological considerations, the first two families, but not the third, have the same couplings. Even within a specific theory the results are model dependent because of unknown quark and lepton mixing matrices. However, assuming that typical mixings are comparable to the CKM matrix, processes such as coherent μ–e conversion in a muonic atom, K^0–$\bar{K}^0$ and B–$\bar{B}$ mixing, ϵ and ϵ'/ϵ lead to significant constraints on Z' bosons in the theoretically and phenomenologically motivated range $M_{Z'} \sim$1 TeV.

2. FCNCs in models with extra Z' boson

Here, we assume that the effective low energy gauge group is of the form $G = [SU(2)_{\rm L} \times U(1)_{\rm Y} \times SU(3)_{\rm C}] \times U_1(1)$, and that it originates from the breaking of a simple unification group, like E_6. The SM neutral gauge boson Z_0 can then mix with $U_1(1)$ gauge boson Z_1, resulting in the two mass eigenstates Z and Z'. The neutral current (NC) Lagrangian [21] in the physical Z and Z' basis can be written as

$$L_{\rm NC} = -eJ^{\lambda}_{\rm em}A_{\lambda} - g_0(J^{\lambda}Z_{\lambda} + J'^{\lambda}Z'_{\lambda}), \tag{1}$$

where $g_0 = (4\sqrt{2}G_{\rm F}M^2_{Z_0})^{1/2}$ is the SM gauge coupling of the Z_0, and J, J' are the fermionic currents coupled to the Z and Z' bosons. They are related to the gauge currents J_0 and J_1, coupled to Z_0 and Z_1 respectively by the relation

$$\begin{pmatrix} J^{\lambda}_Z \\ J^{\lambda}_{Z'} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} J^{\lambda}_0 \\ \sin\theta_{\rm w} J^{\lambda}_1 \end{pmatrix}, \tag{2}$$

where θ is the Z–Z' mixing angle and $\theta_{\rm w}$ is the mixing weak angle.

Besides predicting extra Z' bosons, extended gauge models like E_6 predict also the existence of 'new' fermions ψ^0_N. The new fermions will in general mix with the standard 'known' fermions ψ^0_K having the same electric and colour charges. Then for any specific value of the electric and colour charges, the component of chirality $\alpha = {\rm L,R}$ of the light mass eigenstates ψ_l will correspond to a general superposition of gauge eigenstates that can be written as [21]

$$\psi_{l\alpha} = A_\alpha^\dagger \psi_{K\alpha}^0 + F_\alpha^\dagger \psi_{N\alpha}^0. \quad (3)$$

The mixing matrices A_α and F_α describe respectively the mixing of the light states with the known and new fermions and satisfy the unitarity relation $A_\alpha^\dagger A_\alpha + F_\alpha^\dagger F_\alpha = I$. The presence of these mixings will affect the couplings of the gauge bosons to the light fermions ψ_l [13,21,23]. In particular, given a general current J_Q^λ, corresponding to a broken generator Q, its projection on the light fermions $\psi_{l\alpha}$ will read

$$J_{lQ}^\lambda = \sum_{\alpha=\mathrm{L,R}} \overline{\psi}_{l\alpha} \gamma^\lambda [q_\alpha^K I + (q_\alpha^N - q_\alpha^K) F_\alpha^\dagger F_\alpha] \psi_{l\alpha}, \quad (4)$$

where $q_\alpha^K (q_\alpha^N)$ is the Q-eigenvalue of the known (new) fermions $\psi_{K\alpha}^0 (\psi_{N\alpha}^0)$, and for simplicity we have assumed that all the new states have the same Q-charge. If the known fermions are mixed with new states having different assignments of weak-isospin ('exotic' fermions), then the coefficient $q_\alpha^N - q_\alpha^K = t_3(\psi_{N\alpha}^0) - t_3(\psi_{K\alpha}^0)$ multiplying the mixing matrix $F_\alpha^\dagger F_\alpha$ in eq. (4) is non-vanishing and the current J_{lO}^λ coupled to the Z_0 boson is affected. In this case, extremely stringent bounds on the off-diagonal terms can be obtained from the limits on FC processes. The diagonal elements of the matrix $F^\dagger F$ are also constrained mainly from LEP, NC, and charged current precision data [21,23] and the corresponding limits are in general $\leq 10^{-2}$. On the other hand, the mixing between the ordinary fermions and the new exotic ones are theoretically expected to be very small, since they arise in general from see-saw like formulas [13,24], so that the corresponding limits are not very effective in constraining the models under examination.

If, instead the mixing is with new states having the same $SU(2)$ assignments as the SM fermions ('ordinary' fermions), the coefficient of the mixing term in the J_0^λ current vanishes, and the couplings to the Z_0 boson are not affected. In this case no phenomenological bounds can be set on the elements of $F^\dagger F$, with the exception of the ordinary mixing of the left-handed quarks, that are constrained by the unitarity tests of the CKM matrix [23]. However, ordinary–ordinary fermion mixing does affect the J_1 current, since in general $q_{1\alpha}^N \neq q_{1\alpha}^K$. Clearly at low energy the possible effects of the ordinary–ordinary mixings is suppressed with respect to the effects of the ordinary–exotic mixings as the ratio of the gauge boson mass squared.

Now we will consider the case of E_6 models, in which new gauge bosons as well as new ordinary and new exotic fermions are present. Since E_6 rank is 6, as many as two additional new gauge bosons could appear in the low energy effective gauge group. We consider the embedding of the SM gauge group G_{SM} in E_6 through the pattern of subgroups such as $E_6 \to U(1)_\psi \times SO(10) \to U(1)_\chi \times SU(5) \to G_{\mathrm{SM}}$. Then the lightest additional gauge boson will in general correspond to an effective extra $U_1(1)$ resulting as a combination of the $U(1)_\psi$ and $U(1)_\chi$ factors. We will parametrize this combination in terms of an angle β. This will define an entire class of Z' models in which each fermion f is coupled to the new boson through the effective charge

$$q_1(f) = q_\psi(f) \sin\beta + q_\chi(f) \cos\beta. \quad (5)$$

Particular cases that are commonly studied in the literature [21,25,26] correspond to $\sin\beta = -\sqrt{5/8}, 0, 1$ and respectively denoted as Z_η, Z_χ, and Z_ψ models. The Z_ψ

boson is defined by $E_6 \to SO(10) \times U(1)_\psi$. It possesses only axial-vector couplings to the ordinary fermions. The Z_η boson is the linear combination $\sqrt{3/8}Z_\chi - \sqrt{5/8}Z_\psi$. It occurs in superstring models when E_6 directly breaks down to rank 5. Finally, the Z_χ boson is defined by $SO(10) \to SU(5) \times U(1)_\chi$. This boson couples to the known fermions in the same way as Z' present in $SO(10)$ GUTs.

However, since $SO(10)$ does not contain additional charged fermions, the FC effects we are studying here is absent. In contrast, new charged quarks and leptons are present in E_6. The 27 fundamental representations contain, beyond the standard 15 fermion degrees of freedom, 12 additional states for each generation, among which we have a vector doublet of new leptons $H = (NE^-)_{\rm L}^{\rm T}$, $H^{\rm C} = (E^+ N^{\rm C})_{\rm L}^{\rm T}$. The chiral couplings of the leptons to the Z_1 as well as the coefficient of the LFV term $F^\dagger F$ are determined by q_ψ and q_χ charges of the new and known states, which are

$$q_\psi(E_{\rm L}) = -q_\psi(E_{\rm R}) = -\frac{1}{3}\sqrt{\frac{5}{2}}, \quad q_\chi(E_{\rm L}) = q_\chi(E_{\rm R}) = -\frac{1}{3}\sqrt{\frac{3}{2}},$$
$$q_\psi(e_{\rm L}) = -q_\psi(e_{\rm R}) = \frac{1}{6}\sqrt{\frac{5}{2}}, \quad q_\chi(e_{\rm L}) = 3q_\chi(e_{\rm R}) = \frac{1}{2}\sqrt{\frac{3}{2}}. \tag{6}$$

With respect to the $SU(2)_{\rm L}$ transformation properties, the $E_{\rm L}^+$ new leptons are exotic and then the mixings of their CP conjugate states $E_{\rm R}^-$ with the standard R-handed leptons $e_{\rm R}$ violate weak-isospin by Y_2. In contrast, the $E_{\rm L}^-$ leptons are ordinary and their mixings with the light leptons are not expected to be suppressed by any small mass ratios since they do not violate weak-isospin. These mixings are generated by entries in the mass matrix corresponding to v.e.v.s. of the singlet Higgs fields $\langle\phi_{\rm S}\rangle_0$ which, since also contribute to the masses of the new (heavy) gauge bosons, are expected to be larger than the doublet v.e.v.s.

We note that in E_6 models the ordinary–ordinary lepton mixings occur between $SU(2)$ doublet. Then it is clear that for each entry in the charged lepton mass matrix of the form $E_{\rm R} e_{\rm L}\langle\phi_{\rm S}\rangle_0$, there must be a corresponding entry $N^{\rm C}\nu\langle\phi_{\rm S}\rangle_0$ in the mass matrix for the neutral states that would generate a Dirac mass for the light neutrinos. Even if in the 27 representations of the E_6 models several new neutral states (including two $SU(2)$ singlets) are present in the minimal E_6 models, it is not possible to generate naturally any small eigenvalue for the mass matrix if these Dirac mass entries are present, since the Higgs representation that could generate large Majorana masses and lead to see-saw mechanism is absent. Then, in the frames of these models, the limits on the neutrino masses automatically guarantee that any possible ordinary–ordinary mixing in the charged lepton sector should be unobservably small. However, as was discussed by Nandi and Sarkar [27], large Majorana masses for the singlet neutral fermions can be generated due to gravitational effects, leading to a rather complicated mass matrix for the neutral states for which a see-saw mechanism is effective, and produce naturally small masses for the light doublet neutrinos. In this scenario, in order not to conflict with the limits on the neutrino masses, there is no need to tune the Dirac mass entries to any unnaturally small value. Then the weak-isospin conserving mixings of the charged leptons are constrained, and in the limit in which the singlet v.e.v.s. are much larger than the doublet v.e.v.s. are theoretically expected to be $O(1)$ [13].

The LFV Lagrangian in E_6 models can be obtained from eqs (1) and (4). For the charged leptons of the first two generations it can be written as

$$-L_{\mathrm{LFV}}^{e\mu} = g_0 \lfloor k_0(\cos\theta Z_\lambda - \sin\theta Z'_\lambda)\bar{e}_{\mathrm{R}}\gamma^\lambda \mu_{\mathrm{R}} + k_1(\sin\theta Z_\lambda + \cos\theta Z'_\lambda)\bar{e}_{\mathrm{L}}\gamma^\lambda \mu_{\mathrm{L}} \rfloor, \quad (7)$$

where

$$k_0 = -\frac{1}{2}(F_{\mathrm{R}}^\dagger F_{\mathrm{R}})_{e\mu} \quad (8)$$

is induced by mixing with the exotic charged leptons E_{R}^-, while

$$k_1 = \sin\theta_{\mathrm{w}}[q_1(E_{\mathrm{L}}) - q_1(e_{\mathrm{L}})](F_{\mathrm{L}}^\dagger F_{\mathrm{L}})_{e\mu} \quad (9)$$

results from mixing with the new ordinary leptons E_{L}.

From the second term in eq. (7), we see that ordinary–ordinary fermion mixing can still induce a LFV vertex for the physical Z boson. However, this vertex is suppressed by Z_0–Z_1 mixing, which is severely constrained by the present data to $|\theta| \leq 0.02$ [21,25], and then we can expect that the FCNC process would be mainly induced by direct Z' exchange.

3. Formalism

In this section we discuss the general formalism and introduce our notations. The neutral current (NC) Lagrangian in the physical Z and Z' basis given by eq. (1) can be written as

$$L_{\mathrm{NC}} = -eJ_{\mathrm{em}}^\lambda A_\lambda - g_0(J^\lambda Z_\lambda + J'^\lambda Z'_\lambda),$$

where $g_0 = (4\sqrt{2}G_{\mathrm{F}}M_{Z_0}^2)^{1/2}$ is the SM gauge coupling of Z_0, and J, J' are the fermionic currents coupled to the Z and Z' bosons. The currents are

$$J_\lambda = \sum_i \bar{\psi}_i\gamma_\lambda[\epsilon_{\mathrm{L}}(i)P_{\mathrm{L}} + \epsilon_{\mathrm{R}}(i)P_{\mathrm{R}}]\psi_i, \quad (10)$$

$$J'_\lambda = \sum_{i,j} \bar{\psi}_i\gamma_\lambda[\epsilon'_{\psi\mathrm{L}ij}P_{\mathrm{L}} + \epsilon'_{\psi\mathrm{R}ij}P_{\mathrm{R}}]\psi_j, \quad (11)$$

where the sum extends over all quarks and leptons ψ_{ij} and $P_{\mathrm{R,L}} = (1 \pm \gamma_5)/2$. $\epsilon'_{\psi\mathrm{R,L}ij}$ denote the chiral couplings of the new gauge boson and the standard model chiral couplings are

$$\epsilon_{\mathrm{R}}(i) = -\sin^2\theta_{\mathrm{w}}Q_i, \quad \epsilon_{\mathrm{L}}(i) = t_3^i - \sin^2\theta_{\mathrm{w}}Q_i, \quad (12)$$

where t_3^i and Q_i are the third component of the weak-isospin and the electric charge of fermion i, respectively.

Flavour-changing effects immediately arise if ϵ' are non-diagonal matrices. If the Z' couplings are diagonal but non-universal, fermion mixing induces flavour-changing couplings. The fermion Yukawa matrices h_ψ in the weak eigenstate basis can be diagonalized by unitary matrices $V_{\mathrm{R,L}}^\psi$ as

$$h_{\psi,\text{diag}} = V_\text{R}^\psi h_\psi V_\text{L}^{\psi^+}, \tag{13}$$

where the CKM matrix is given by the combination

$$V_\text{CKM} = V_\text{L}^u V_\text{L}^{d^+}. \tag{14}$$

Hence, the chiral Z' couplings in the fermion mass eigenstate basis read

$$B_{ij}^{\psi\text{L}} \equiv (V_\text{L}^\psi \epsilon'_{\psi\text{L}} V_\text{L}^{\psi^+})_{ij} \quad \text{and} \quad B_{ij}^{\psi\text{R}} \equiv (V_\text{L}^\psi \epsilon'_{\psi\text{R}} V_\text{R}^{\psi^+})_{ij}. \tag{15}$$

Further, Z–Z' mixing is induced by electroweak symmetry breaking, implying that Z, Z' are related to mass eigenstates by an orthogonal transformation. Hence, the couplings of the massive gauge boson mass eigenstates Z^i are

$$L_\text{NC}^Z = -[g_1 \cos\theta J^\lambda + g_2 \sin\theta J'^\lambda] Z_\lambda - [g_2 \cos\theta J'^\lambda - g_1 \sin\theta J^\lambda] Z'_\lambda, \tag{16}$$

where θ is the Z–Z' mixing angle. g_1 and g_2 are the gauge couplings of the two $U(1)$ factors. The standard model weak neutral current J^λ is given in eq. (10) and J'^λ has a form analogous to eq. (11), with $\epsilon'_{\psi\text{R,L}}$ replaced by the couplings $B^{\psi\text{R,L}}$ from eq. (15). We have neglected kinetic mixing [28], since it only amounts to a redefinition of the unknown Z' couplings. At low energies, the effective four-fermion interactions are then given by

$$-L_\text{eff} = \frac{4G_\text{F}}{\sqrt{2}} \sum_{\psi,\chi} (\rho_\text{eff} J^2 + 2wJ \cdot J' + yJ'^2) \tag{17}$$

$$= \frac{4G_\text{F}}{\sqrt{2}} \sum_{\psi,\chi} \sum_{i,j,k,l} [C_{kl}^{i,j} Q_{k,l}^{i,j} + \tilde{C}_{k,l}^{i,j} \tilde{Q}_{k,l}^{i,j} + D_{k,l}^{i,j} O_{k,l}^{i,j} + \tilde{D}_{k,l}^{i,j} \tilde{O}_{k,l}^{i,j}], \tag{18}$$

with local operators (All these operators are not independent. For couplings of four fermions of the same type, $\psi = \chi$, e.g., four charged leptons, one has $Q_{kl}^{ij} = Q_{ij}^{kl}$, $\tilde{Q}_{kl}^{ij} = \tilde{Q}_{ij}^{kl}$ and $O_{kl}^{ij} = \tilde{O}_{ij}^{kl}$.)

$$\begin{aligned} Q_{kl}^{ij} &= (\bar{\psi}_i \gamma^\lambda P_\text{L} \psi_j)(\bar{\chi}_k \gamma_\lambda P_\text{L} \chi_l), \quad \tilde{Q}_{kl}^{ij} = (\bar{\psi}_i \gamma^\lambda P_\text{R} \psi_j)(\bar{\chi}_k \gamma_\lambda P_\text{R} \chi_l), \\ O_{kl}^{ij} &= (\bar{\psi}_i \gamma^\lambda P_\text{L} \psi_j)(\bar{\chi}_k \gamma_\lambda P_\text{R} \chi_l), \quad \tilde{O}_{kl}^{ij} = (\bar{\psi}_i \gamma^\lambda P_\text{R} \psi_j)(\bar{\chi}_k \gamma_\lambda P_\text{L} \chi_l). \end{aligned} \tag{19}$$

ψ and χ represent classes of fermions with the same SM quantum numbers, i.e., u, d, e^- and ν, while i, j, k, l are family indices. The coefficients are

$$\begin{aligned} C_{kl}^{ij} &= \rho_\text{eff} \delta_{ij} \delta_{kl} \epsilon_\text{L}(i) \epsilon_\text{L}(k) + w \delta_{ij} \epsilon_\text{L}(\psi_i) B_{kl}^{\chi\text{L}} \\ &\quad + w \delta_{kl} \epsilon_\text{L}(\chi_l) B_{ij}^{\psi\text{L}} + y B_{ij}^{\psi\text{L}} B_{kl}^{\chi\text{L}}, \end{aligned} \tag{20}$$

$$\begin{aligned} \tilde{C}_{kl}^{ij} &= \rho_\text{eff} \delta_{ij} \delta_{kl} \epsilon_\text{R}(i) \epsilon_\text{R}(k) + w \delta_{ij} \epsilon_\text{R}(\psi_i) B_{kl}^{\chi\text{R}} \\ &\quad + w \delta_{kl} \epsilon_\text{R}(\chi_l) B_{ij}^{\psi\text{R}} + y B_{ij}^{\psi\text{R}} B_{kl}^{\chi\text{R}}, \end{aligned} \tag{21}$$

$$\begin{aligned} D_{kl}^{ij} &= \rho_\text{eff} \delta_{ij} \delta_{kl} \epsilon_\text{L}(i) \epsilon_\text{R}(k) + w \delta_{ij} \epsilon_\text{L}(\psi_i) B_{kl}^{\chi\text{R}} \\ &\quad + w \delta_{kl} \epsilon_\text{R}(\chi_l) B_{ij}^{\psi\text{L}} + y B_{ij}^{\psi\text{L}} B_{kl}^{\chi\text{R}}, \end{aligned} \tag{22}$$

$$\tilde{D}^{ij}_{kl} = \rho_{\rm eff}\delta_{ij}\delta_{kl}\epsilon_{\rm R}(i)\epsilon_{\rm L}(k) + w\delta_{ij}\epsilon_{\rm R}(\psi_i)B^{\chi{\rm L}}_{kl} + w\delta_{kl}\epsilon_{\rm L}(\chi_l)B^{\psi{\rm R}}_{ij} + yB^{\psi{\rm R}}_{ij}B^{\chi{\rm L}}_{kl}. \tag{23}$$

The coefficients are given by

$$\rho_{\rm eff} = \rho_1\cos^2\theta + \rho_2\sin^2\theta, \quad \rho_i = \frac{M^2_{\rm w}}{M^2_i\cos^2\theta_{\rm w}}, \tag{24}$$

$$w = \sin\theta\cos\theta(\rho_1-\rho_2)(g_2/g_1), y = (\rho_1\sin^2\theta + \rho_2\cos^2\theta)(g_2/g_1)^2, \tag{25}$$

where M_i are the masses of the neutral gauge boson mass eigenstates and $\theta_{\rm w}$ is the electroweak mixing angle.

4. Flavour-changing processes

In this section we will discuss flavour-violating processes forbidden in the SM and new contributions to SM processes. Experimental bounds or results on these processes can then be used to constrain the Z' couplings.

4.1 *Z Decays*

Due to Z–Z' mixing, Z couples to neutral current J'. The decay width for a flavour-changing Z decay at tree level is given by

$$\Gamma(Z\to\psi_i\bar{\psi}_j) = \frac{CG_{\rm F}\rho_1M^3_1}{3\sqrt{2}\pi}\sin^2\theta(|B^{\psi{\rm L}}_{ij}|^2 + |B^{\psi{\rm R}}_{ij}|^2)(g_2/g_1)^2, \tag{26}$$

where g_1 and g_2 are the gauge couplings of the two $U(1)$ factors and $C=1$ ($C=3$) is the colour factor for leptons (quarks). Due to strong experimental constraints on the Z–Z' mixing angle θ (see refs [5,20]), the couplings $B^{\psi_{\rm R,L}}$ cannot be strongly constrained from flavour-violating Z decays.

4.2 *Lepton decays*

In the SM, each lepton generation has a separately conserved lepton number, if one neglects small effects from non-vanishing neutrino masses and non-perturbative effects. The effective Lagrangian (18) gives rise to lepton family number violating processes, although the total lepton number is still conserved.

Consider first the decay of a charged lepton l_j into three different charged leptons l_i, l_k and $\bar{l}_l$. At tree level, the decay width is

$$\Gamma(l_j\to l_il_k\bar{l}_l) = \frac{G^2_{\rm F}m^5_{l_j}}{48\pi^3}(|C^{l_il_j}_{l_kl_l} + C^{l_kl_j}_{l_il_l}|^2 + |\tilde{C}^{l_il_j}_{l_kl_l} + \tilde{C}^{l_kl_j}_{l_il_l}|^2 + |D^{l_il_j}_{l_kl_l}|^2 + |D^{l_kl_j}_{l_il_l}|^2 + |\tilde{D}^{l_il_j}_{l_kl_l}|^2 + |\tilde{D}^{l_kl_j}_{l_il_l}|^2), \tag{27}$$

where we have neglected the masses of the final state leptons. If two leptons in the final state are equal ($i = k$), taking permutations of the external fermion lines into account yields [29]

$$\Gamma(l_j \to l_i l_i \bar{l}_l) = \frac{G_F^2 m_{l_j}^5}{48\pi^3}(2|C_{l_i l_l}^{l_i l_j}|^2 + 2|\tilde{C}_{l_i l_l}^{l_i l_j}|^2 + |D_{l_i l_l}^{l_i l_j}|^2 + |\tilde{D}_{l_i l_l}^{l_i l_j}|^2). \quad (28)$$

Since such processes are free of hadronic uncertainties and well-constrained experimentally [29–31], they yield strong constraints on the leptonic couplings of Z'. (In ref. [32] these processes were considered in the case of vanishing Z–Z' mixing, Z' couplings of the V-A form, and assuming that the Z' has no diagonal couplings.)

The strongest constraint on the Z'–$\mu - e$ coupling however comes from coherent μ–e conversion in a muonic atom [33]. The branching fraction for this process, i.e., the ratio of the coherent μ–e conversion rate to the μ capture rate for a nucleus of atomic number Z and neutron number N is given by [22,29]

$$\begin{aligned} B(\mu^- N \to e^- N) = {} & \frac{G_F^2 \alpha^3 m_\mu^5}{2\pi^2 \Gamma_{\text{CAPT}}} \frac{Z_{\text{eff}}^4}{Z} |F_P|^2 (|B_{12}^{l_L}|^2 + |B_{12}^{l_R}|^2) \\ & \times |w[\tfrac{1}{2}(Z - N) - 2Z \sin^2\theta_w] \\ & + y[(2Z + N)(B_{11}^{uL} + B_{11}^{uR}) \\ & + (Z + 2N)(B_{11}^{d_L} + B_{11}^{d_R})]|^2, \end{aligned} \quad (29)$$

where Γ_{CAPT} is the μ capture rate, Z_{eff} is the effective atomic charge obtained by averaging the muon wave function over the nucleon, and F_P is a nuclear matrix element.

4.3 *Radiative decays*

Neutral current penguins give rise to radiative lepton decays. Neglecting the mass of the final state lepton, the decay width is

$$\Gamma(l_j \to l_i \gamma) = \frac{\alpha G_F^2 m_{l_j}^5}{8\pi^4}(|\xi_L^{l_i l_j}|^2 + |\xi_R^{l_i l_j}|^2), \quad (30)$$

where dipole moment couplings of an on-shell photon to the chiral μ–e currents are

$$\xi_L^{l_i l_j} = \frac{1}{m_{l_j}} \sum_k m_{l_k} D_{l_i l_k}^{l_k l_j} = \frac{y}{m_{l_j}}(B^{l_R} m_l B^{l_L})_{ij} + w\epsilon_L(l_j) B_{ij}^{l_R}, \quad (31)$$

$$\xi_R^{l_i l_j} = \frac{1}{m_{l_j}} \sum_k m_{l_k} \tilde{D}_{l_i l_k}^{l_k l_j} = \frac{y}{m_{l_j}}(B^{l_L} m_l B^{l_R})_{ij} + w\epsilon_R(l_j) B_{ij}^{l_L}, \quad (32)$$

where m_l is the charged lepton mass matrix.

Now consider the $b \to s\gamma$ decay. The branching ratio (BR) and the CP asymmetry (A_{CP}) of the process $B \to X_s \gamma$ can restrict the couplings ξ of the Z' [34]. Since the b-quark mass is much larger than the QCD-scale Λ, long-range strong interaction effects are not expected to be important in the inclusive decay $B \to X_s\gamma$ [35]. Hence, the rate for this process is usually approximated by considering the ratio

$$R \equiv \frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to X_c e\overline{\nu}_e)} \approx \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\overline{\nu}_e)}. \tag{33}$$

Neglecting SM contributions, the contributions to R from the one-loop neutral current penguin diagrams is

$$R = \frac{8\alpha}{3\pi}|V_{cb}|^{-2}f^{-1}\left(\frac{m_c^2}{m_b^2}\right)(|\xi_{\rm L}^{sb}|^2 + |\xi_{\rm R}^{sb}|^2), \tag{34}$$

where f is the phase-space factor in the semi-leptonic b-decay:

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x. \tag{35}$$

In analogy to eqs (31) and (32) the flavour-violating effective couplings $\xi_{\rm R,L}^{s,b}$ are given by

$$\begin{aligned} \xi_{\rm L}^{sb} &= \frac{1}{m_b}\sum_k m_{d_K} D_{sd_K}^{d_K b} = \frac{y}{m_b}(B^{d_{\rm R}} m_d B^{d_{\rm L}})_{23} + w\epsilon_{\rm L}(b) B_{23}^{d_{\rm R}}, \\ \xi_{\rm R}^{sb} &= \frac{1}{m_b}\sum_k m_{d_K} \tilde{D}_{sd_K}^{d_K b} = \frac{y}{m_b}(B^{d_{\rm L}} m_d B^{d_{\rm R}})_{23} + w\epsilon_{\rm R}(b) B_{23}^{d_{\rm L}}, \end{aligned} \tag{36}$$

where d_K stands for the Kth generation down-type quark and m_d is the diagonal mass matrix of down quarks.

Using the experimental constraints $0.000222 < {\rm BR}(b \to s\gamma) < 0.000408$ and $-0.027 < A_{\rm CP}(b \to s\gamma) < +0.10$, Sever and Aydemir [34] found the value of coupling constant as

$$0.132 \le \xi^{sb} \le 0.151. \tag{37}$$

4.4 *Mass splitting and CP violation*

The effective Lagrangian (18) also contributes to the mass splitting in a neutral pseudo-scalar meson system. Again denoting the flavour eigenstates P^0 and $\bar{P}^0$, the mass splitting Δm_P is given by

$$\Delta m_P = -2{\rm Re}\langle P^0|L_{\rm eff}|\bar{P}^0\rangle. \tag{38}$$

The relevant hadronic matrix elements of the operators (19) have been determined in the vacuum insertion approximation using PCAC [36]. Hence, for a meson with the quark content $P^0 = \bar{q}_j q_i$ we obtain the following contribution to the mass splitting:

$$\Delta m_P = 4\sqrt{2}G_{\rm F} m_P F_P^2 y \Bigg\{ \frac{1}{3}{\rm Re}[(B_{\rm ij}^{q{\rm L}})^2 + (B_{\rm ij}^{q{\rm R}})^2] \\ - \left[\frac{1}{2} + \frac{1}{3}\left(\frac{m_P}{m_{qi}+m_{qj}}\right)^2\right] {\rm Re}(B_{ij}^{q{\rm L}} + B_{ij}^{q{\rm R}}) \Bigg\}, \tag{39}$$

where m_P and F_P are the mass and decay constant respectively of the meson.

Further, the phases in the Z' couplings $B_{ij}^{\psi{\rm R,L}}$ will contribute to CP violating processes. Limits on the imaginary parts of the s–d–Z' couplings can be placed by considering indirect CP violation ϵ_K in the neutral kaon system:

$$|\epsilon_K| = \frac{1\,{\rm Im}\langle K^0|L_{\rm eff}|\bar{K}^0\rangle}{2\sqrt{2}\,{\rm Re}\langle K^0|L_{\rm eff}|\bar{K}^0\rangle} \tag{40}$$

$$= \frac{2G_{\rm F} m_K F_K^2 y}{\Delta m_K} \Bigg| \frac{1}{3}{\rm Im}[(B_{12}^{d_{\rm L}})^2 + (B_{12}^{d_{\rm R}})^2] \\ - \left[\frac{1}{2} + \frac{1}{3}\left(\frac{m_P}{m_d+m_s}\right)^2\right] {\rm Im}(B_{12}^{d_{\rm L}} B_{12}^{d_{\rm R}}) \Bigg|. \tag{41}$$

Direct CP violation ϵ' in the decays $K \to \pi\pi$ can be expressed in terms of the decay amplitudes $A_0 = A(K \to (\pi\pi)_0)$ and $A_2 = A(K \to (\pi\pi)_2)$, where the indices 0 and 2 denote the isospin of the final two-pion states:

$$\epsilon' = -\frac{1}{\sqrt{2}} \frac{\omega}{{\rm Re}A_0} \left({\rm Im}A_0 - \frac{1}{\omega}{\rm Im}A_2\right) {\rm e}^{i\tilde{\phi}}, \tag{42}$$

where

$$\omega = \frac{{\rm Re}\,A_2}{{\rm Re}A_0}, \quad \tilde{\phi} = \frac{\pi}{2} + \delta_2 - \delta_0. \tag{43}$$

The δ_I are the final state interaction phases. When using eq. (42) to constrain physics beyond the standard model, it is common practice to take ω, ${\rm Re}\,A_0$ and ϕ from the experiment,

$$\omega = 0.045, \quad {\rm Re}A_0 = 3.33 \cdot 10^{-7}\ {\rm GeV}, \quad \tilde{\phi} \approx \frac{\pi}{4}, \tag{44}$$

and consider new contributions to the imaginary parts of the amplitudes A_0 and A_2. This is due to the fact that the CP violating imaginary parts are dominated by short-distance effects and can be reliably determined by considering matrix elements of the effective Lagrangian (18). The hadronic matrix elements can be computed in the large $N_{\rm C}$ limit of chiral perturbation theory, and one finds the following neutral current contribution to the real part of the ratio ϵ'/ϵ_K:

$${\rm Re}\left(\frac{\epsilon'}{\epsilon_K}\right) = 2 \cdot 10^{-3} w \left({\rm Im}B_{21}^{d_{\rm L}} + \frac{3}{2}{\rm Im}B_{21}^{d_{\rm R}}\right) \\ +1.5 \cdot 10^3 y \lfloor (B_{11}^{u{\rm L}} - B_{11}^{d_{\rm L}})({\rm Im}B_{21}^{d_{\rm L}} + 2\,{\rm Im}B_{21}^{d_{\rm R}}) \\ -(B_{11}^{u{\rm R}} - B_{11}^{d_{\rm R}})(2\,{\rm Im}B_{21}^{d_{\rm L}} + {\rm Im}B_{21}^{d_{\rm R}}) \rfloor. \tag{45}$$

4.5 *Experimental constraints*

We use the experimental limits or results on these processes to constrain the flavour-violating Z' couplings. (Flavour diagonal Z' couplings can be constrained from fits to electroweak observable [5].) In the following we briefly discuss bounds coming from Z–Z' mixing contributions to these processes. The pure Z' contributions yield a multitude of bounds on products of Z' couplings, which are less illuminating. As already mentioned, the strongest bound on Z'–$\mu - e$ coupling comes from the non-observation of coherent μ–e conversion by Sindrum-II Collaboration [33]:

$$w^2(|B_{12}^{l_\mathrm{L}}|^2 + |B_{12}^{l_\mathrm{R}}|^2) < 4 \cdot 10^{-14}, \tag{46}$$

while the decays $\tau \to 3e$ and $\tau \to 3\mu$ yield the strongest bounds on flavour-violating τ couplings:

$$\begin{aligned} &w^2(|B_{13}^{l_\mathrm{L}}|^2 + |B_{13}^{l_\mathrm{R}}|^2) < 2 \cdot 10^{-5}, \\ &w^2(|B_{23}^{l_\mathrm{L}}|^2 + |B_{23}^{l_\mathrm{R}}|^2) < 10^{-5}. \end{aligned} \tag{47}$$

It is interesting to note that these contributions alone ensure that branching ratios for lepton flavour-violating meson decays are below the experimental bounds, provided that the parameters w and y, given in eq. (25), are of the same order. (This holds in the most interesting case of a TeV scale Z' with small mixing, $\theta \leq 10^{-3}$.) For example, upper limits on the branching ratios for the processes $K_\mathrm{L} \to \mu^\pm e^\mp$ from the BNL E871 Collaboration [37] and $K_\mathrm{L} \to \pi^0 \mu^\pm e^\mp$ from KTeV [38] yield

$$y^2(|B_{12}^{l_\mathrm{L}}|^2 + |B_{12}^{l_\mathrm{R}}|^2)|\mathrm{Re}B_{12}^{d_\mathrm{R}} - \mathrm{Re}B_{12}^{d_\mathrm{L}}|^2 < 10^{-14}, \tag{48}$$

$$y^2(|B_{12}^{l_\mathrm{L}}|^2 + |B_{12}^{l_\mathrm{R}}|^2)|\mathrm{Im}B_{12}^{d_\mathrm{R}} + \mathrm{Im}B_{12}^{d_\mathrm{L}}|^2 < 2 \cdot 10^{-10}. \tag{49}$$

Hence, the experimental bounds on these processes would have to be improved by several orders of magnitude to yield interesting constraints on the real and imaginary parts of $B_{12}^{d_{\mathrm{R,L}}}$. From eqs (46), (48), (49) it is clear that lepton flavour-violating meson decays cannot compete in constraining flavour-non-diagonal Z' couplings, except in the limit $|w| \ll y$. However, lepton flavours conserving meson decays can be used to constrain Z' couplings to quarks, e.g., limits on $K_\mathrm{L} \to \mu^+\mu^-$ [30] and $K_\mathrm{L} \to \pi^0\mu^+\mu^-$ [39] give

$$w^2|\mathrm{Re}B_{12}^{d_\mathrm{R}} - \mathrm{Re}B_{12}^{d_\mathrm{L}}| < 3 \cdot 10^{-11}, \quad w^2|\mathrm{Im}B_{12}^{d_\mathrm{R}} + \mathrm{Im}B_{12}^{d_\mathrm{L}}|^2 < 5 \cdot 10^{-11}. \tag{50}$$

The top-quark couplings to a Z' cannot be constrained from these tree-level processes. In future, studies of rare top decay [40] and associated top-charm production [41] at the Tevatron, LHC and a future e^+e^- linear collider will yield very useful constraints.

Further, experimental results on meson mass splittings (The $B_s - \bar{B}_s$ mass difference has not been measured. We required that new contributions be smaller than the lower limit on the mass splitting, $\Delta m_{B_s^0} > 14.3\ \mathrm{ps}^{-1}$ at 95% C.L. [42].) allow constraints on the real parts of the squared Z' coupling to quarks,

$$y|\mathrm{Re}[(B_{12}^{d_{\mathrm{R,L}}})^2]| < 10^{-8}, \quad y|\mathrm{Re}[(B_{13}^{d_{\mathrm{R,L}}})^2]| < 6\cdot 10^{-8} \tag{51}$$

$$y|\mathrm{Re}[(B_{23}^{d_{\mathrm{R,L}}})^2]| < 2\cdot 10^{-6}, \quad y|\mathrm{Re}[(B_{12}^{u_{\mathrm{R,L}}})^2]| < 10^{-7} \tag{52}$$

and CP violation in the kaon system yields constraints on the imaginary part of the Z'–d–s coupling:

$$y|\mathrm{Im}[(B_{12}^{d_{\mathrm{R,L}}})^2]| < 8\cdot 10^{-11}, \quad w|\mathrm{Im}B_{12}^{d_{\mathrm{R,L}}}| < 10^{-6}. \tag{53}$$

5. Models

In the following we study extended Abelian gauge structures with flavour non-universal couplings, in order to see where such effects are almost likely to be seen. Although we only discussed bounds coming from Z–Z' mixing contributions to FCNC processes in §4.5, we will also take pure Z' contributions into account here, since they are of the same order as mixing contributions in the models considered.

First we consider a perturbative heterotic string model, based on free fermionic construction [17]. Such models have been studied in detail in [18] and was shown that they generically contain extended Abelian gauge structures and additional matter at string scale. The running of a scalar mass-square due to large Yukawa couplings then triggers the radiative breaking of the $U(1)'$, naturally giving a Z' in the TeV mass range. The Z' couplings can be calculated and the fermion charges Q' can be found in table 1. In the quark sector, the first two generations have the same charges, i.e., in the fermion mass eigenstate basis only mixings of the third generation quarks induce flavour-changing quark couplings in eq. (15). Nevertheless, all the B_{ij}^q are non-zero in general. The same holds true for right-handed leptons, but all three left-handed lepton generations have different Q' charges, which could give rise to strong flavour-violating effects.

To study these FCNCs we have chosen a Z' with a mass of 1 TeV and a Z–Z' mixing angle θ as 10^{-3}. The Z' coupling strength, predicted from the string model, is $g_2 = 0.105$ [18]. Further, we have to specify the unknown fermion mixing matrices $V_{\mathrm{R,L}}^{\psi}$. As an example, we assume that they are equal to the CKM matrix. In the charged lepton sector, these couplings then predict the rates for flavour-violating processes which are six orders of magnitude above the experimental limits for coherent μ–e conversion, five orders of magnitude above limit for the $\mu \to 3e$ decay, and of the same order as the experimental bound from the MEGA Collaboration [43] for the radiative decay $\mu \to e\gamma$. On the other hand, predictions for flavour-violating τ decays are well below the experimental limits. This is due to the assumed CKM mixing, where the 13 and 23 elements are rather small. Assuming larger mixing of third generation leptons, as suggested by the atmospheric neutrino data [44], would give flavour-changing rates close to the experimental bounds, particularly for τ decays into three charged leptons.

For processes involving quarks, we obtain contributions of the same order as SM contributions for the $B - \bar{B}$ and $B_s - \bar{B}_s$ mass differences, and assuming maximal CP violation, a contribution to ϵ_K which is of the same order as the measured value. Predictions for lepton flavour violating meson decays are well below the experimental bounds. As we have seen, one obtains flavour-violating rates above the experimental limits in the lepton sector if the first two generations have different Q' charges. As an example of a model in which the first two quark families also have different charges, we again consider the string-motivated model; however, we set the charges to zero by hand for all the first generation fermions. Then the rates for coherent μ–e conversion and $\mu \to 3e$ are still too large by four and two orders of magnitude respectively, and we find the same contributions as before to the $B - \bar{B}$ and $B_s - \bar{B}_s$ mass differences. However, we obtain contributions to the mass splitting in the K-system, which is larger than the measured values by two orders of magnitude. Further, again assuming maximal CP violation, we have contributions to ϵ_K and $\mathrm{Re}(\epsilon'/\epsilon_K)$ which are too large by factors $6 \cdot 10^5$ and 20, respectively.

From these examples, we conclude that any TeV-scale Z' would almost certainly have equal couplings to the first two families. However, there is still the possibility of different couplings for the third family. As a final example we consider a flavour non-universal Z' that was recently shown to improve the fit to precision electroweak data (3rd paper in ref. [5]). Assuming that the first two fermion generations are flavour universal, one can determine the Z' couplings from the fit. The central values found in the third paper in ref. [5] are reproduced in table 1. Since the Z' coupling of right-handed top quarks was not determined, we have set $Q'_{t\mathrm{R}} = 1$ for definiteness, although this coupling has only very little influence on the processes we discussed. Since in this model the first two lepton generations have the same Z' couplings, the predicted rates for flavour-violating μ decays are well below the experimental limits. Only for coherent μ–e conversion do we find a predicted rate of the same order as the experimental limit. However, we obtain contributions to the $B - \bar{B}$ and $B_s - \bar{B}_s$ mass differences which are too large by factors 7 and 40 respectively. Further, the predicted value for ϵ_K is larger than the measured value by a factor 20 and contribution to ϵ' is of the same order as the measured one.

6. Conclusions

We conclude that additional Z' bosons with a TeV scale mass and family non-universal coupling are severely constrained by experimental results on flavour-changing processes. The most stringent bounds come from muon decays, coherent μ–e conversion in muonic atoms, and from lepton flavour-changing processes in the K-system, i.e., from processes involving the coupling of a Z' to first and second generation fermions. Couplings to third generation are less constrained, but future studies of rare top, bottom and τ decays will help to further constrain these models.

If the Z' couplings are diagonal but family non-universal in the gauge eigenstate basis, flavour-changing couplings arise due to fermion mixing. In the examples we assumed that these unknown mixing matrices are comparable to the CKM matrix. If all the three families have different couplings we find contributions to flavour-changing processes involving the first two generations, which are above the experimental bounds by several orders of magnitude. We obtain particularly large contributions to coherent μ–e conversion, $\mu \to 3e$ decay, meson mass splittings, K_{L} decays and CP violation in the neutral K system. Since couplings of third

Table 1. Fermion charges in the Z' models motivated from string theory and from precision electroweak data.

Multiplet	$100Q'$ (String model)	$100Q'$ (EW fit model)
$\begin{pmatrix} t \\ b \end{pmatrix}_{\rm L}$	-71	$+132$
$t_{\rm R}$	$+133$	$+100$
$b_{\rm R}$	-136	$+848$
$\begin{pmatrix} u \\ d \end{pmatrix}_{\rm L}, \begin{pmatrix} c \\ s \end{pmatrix}_{\rm L}$	$+68$	-52
$u_{\rm R}, c_{\rm R}$	-6	$+38$
$d_{\rm R}, s_{\rm R}$	$+3$	$+172$
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_{\rm L}$	$+74$	-24
$\tau_{\rm R}$	-130	$+3$
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_{\rm L}$	-65	-32
$\mu_{\rm R}$	$+9$	-31
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_{\rm L}$	-204	-32
$e_{\rm R}$	$+9$	-31

generation fermions are much less constrained and we assumed that the fermion mixing matrices have a structure similar to the CKM matrix, these problems can be alleviated by assuming that the first two families, but not the third, have the same $U(1)'$ charges. Mixing with the third family still induces flavour-changing effects involving the first two families, but they are suppressed since in the CKM matrix those mixings are small. In the model considered, the new contributions are too large for the $B-\bar{B}$ and $B_s-\bar{B}_s$ mass differences, and CP-violation in the neutral K system. The experimental bounds for all the other processes are respected.

All the constraints are model dependent. In addition to the Z' mass mixing with the Z, and charges, they are dependent on the mixing matrices for the left and right chiral quarks and leptons. In the standard model, the right chiral mixing matrices are unobservable, and only the combinations of the left chiral matrices in the CKM matrix and its leptonic analog are observable. However, all these matrices are in principle observable in the presence of non-universal Z' couplings. For example, the flavour-changing effects in the B and K systems could be eliminated if the CKM mixing were due entirely to the u quark sector, i.e., $V_{\rm CKM} = V_{\rm L}^u$, with $V_{\rm L}^d = V_{\rm R}^d = 1$. Similarly, μ–e conversion and $\mu \to 3e$ decay would be absent at tree level if all leptonic mixing observable in neutrino oscillations originated in neutrino (rather than charged lepton) mixing, i.e., $V_{\rm L}^e = V_{\rm R}^e = 1$. For models in which the first two families have the same couplings, these conditions could be relaxed so that $V_{\rm L,R}^{d,e}$ mixes the first two families only. Much stricter bounds on these and similar models, including models with alternative studied at Tevatron, LHC and a future

e^+e^- collider, and more stringent bounds on bottom and τ decays become available from existing b-factories and planned charm-τ-factories.

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References

[1] P Langacker, arXiv:hep-ph/0308033
[2] M Masip and A Pamarol, *Phys. Rev.* **D60**, 096005 (1999)
[3] M Cvetic and P Langacker, *Mod. Phys. Lett.* **A11**, 1247 (1996)
M Cvetic, D A Demir, J R Espinosa, L Everett and P Langacker, *Phys. Rev.* **D58**, 119905 (1998) (E)
[4] CDF Collaboration: F Abe *et al*, *Phys. Rev. Lett.* **79**, 2192 (1997)
[5] J Erler and P Langacker, *Phys. Lett.* **B456**, 68 (1999)
G C Cho *et al*, *Nucl. Phys.* **B531**, 65 (1998); *Nucl. Phys.* **B555**, 651 (1999) (E)
J Erler and P Langacker, *Phys. Rev. Lett.* **84,** 212 (2000)
[6] The LEP Electroweak Working Group and SLD Heavy Flavour Group, arXiv:hep-ex/0212036
[7] NuTeV Collaboration: G P Zeller *et al*, *Phys. Rev. Lett.* **88**, 091802 (2002)
[8] M Plumacher, *Phys. Rev.* **D62**, 013006 (2000); hep-ph/0001204
[9] For a recent review, see J S Ginges and V V Flambaum, arXiv:physics/0309054
[10] A Leike, *Phys. Rep.* **C317**, 143 (1999)
[11] L Everett, P Langacker, M Plümacher and J Wang, hep-ph/0001073
[12] W J Marciano and A I Sanda, *Phys. Rev. Lett.* **38**, 1512 (1977)
J D Vergados, *Phys. Rep.* **133**, 1 (1986)
J Bernabéu, *An. Fís.* **5**, 36 (1989)
[13] E Nardi, Preprint UM-TH 92-19 (1992), to appear in *Phys. Rev. D*
[14] K Leroux and D London, *Phys. Lett.* **B526**, 97 (2002)
[15] M Gronau and D London, *Phys. Rev.* **D55**, 2845 (1997)
Y Nir and D Silverman, *Phys. Rev.* **D42**, 1477 (1990)
[16] V Barger, C-W Chiang, P Langacker and H-S Lee, arXiv:hep-ph/0310073 v2, 14 Oct. 2003
[17] S Chaudhuri, S W Chung, G Hockney and J Lykken, *Nucl. Phys.* **B456**, 89 (1995); arXiv:hep-ph/9501361
[18] G Cleaver, M Cvetic, J R Espinosa, L L Everett, P Langacker and J Wang, *Phys. Rev.* **D59**, 055005 (1999); arXiv:hep-ph/9807479
[19] M Cvetic, P Langacker and G Shiu, *Phys. Rev.* **D66**, 066004 (2002)
M Cvetic, G Shiu and A M Uranga, *Phys. Rev. Lett.* **87**, 201801 (2001); arXiv:hep-th/0107143; *Nucl. Phys.* **B615**, 3 (2001); arXiv:hep-th/0107166
[20] O C Anoka, K S Babu and I Gogoladze, hep-ph/0401133 v1 19 Jan 2004

[21] E Nardi, E Roulet and D Tommasini, *Phys. Rev.* **D46**, 3040 (1992)
[22] J Bernabéu, E Nardi and D Tommasini, *Nucl. Phys.* **B409**, 69 (1993)
[23] P Langacker and D London, *Phys. Rev.* **D38**, 886 (1988)
E Nardi, E Roulet and D Tommasini, *Nucl. Phys.* **B386**, 239 (1992)
[24] P Langacker and D London, in ref. [23]
[25] J Layssac, F M Renard and C Verzegnassi, *Z. Phys.* **C53**, 97 (1992)
G Altarelli *et al*, *Phys. Lett.* **B263**, 459 (1991)
P Langacker and M Luo, *Phys. Rev.* **D45**, 278 (1992)
F del Aguila, J M Moreno and M Quirós, *Nucl. Phys.* **B361**, 45 (1991)
[26] J L Hewett and T G Rizzo, *Phys. Rep.* **183,** 195 (1989) and references therein
[27] S Nandi and U Sarkar, *Phys. Rev. Lett.* **56**, 564 (1986)
M Cvetic and P Langacker, *Phys. Rev.* **D46**, R2759 (1992)
[28] K S Babu, C Kolda and J March-Russell, *Phys. Rev.* **D54**, 4635 (1996)
[29] Y Kuno and Y Okada, hep-ph/9909265
[30] C Caso *et al*, *Euro. Phys. J.* **C3**, 1 (1998)
[31] S Gentile and M Pohl, *Phys. Rep.* **274**, 287 (1996)
[32] K K Gan, *Phys. Lett.* **B209**, 95 (1988)
[33] P Wintz, in *Proceedings of the First Int. Symp. on Lepton and Baryon Number Violation*, Trento, 1998 (IOP Publishing, Bristol and Philadelphia, 1999) edited by H V Klapodor, Kleingrothaus and I V Krivosheina, p. 534
[34] R Sever and A Aydemir, hep-ph/0302270 v1 28 Feb. 2003
[35] G Buchalla, A J Buras and M E Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996)
[36] F Gabbiani, E Gabrielli, A Masiero and L Silvestrini, *Nucl. Phys.* **B477**, 321 (1996)
[37] BNL E871 Collaboration: D Ambrose *et al*, *Phys. Rev. Lett.* **81**, 5734 (1998)
[38] G Buchalla, Presented at *International Europhysics Conference on High-Energy Physics* (EPS-HEP 99), Tampere, Finland, 15–21 July 1999, hep-ph/9912369
[39] J Whitmore, Preprint FERMI-CONF-99-266-E, To be published in the *Proceedings of 1999 Chicago Conference on Kaon Physics (K99)*, Chicago, IL, 21–26 June 1999
[40] T Han, R D Peccei and X Zhang, *Nucl. Phys.* **B454**, 527 (1995)
T Han, K Whisnant, B-L Young and X Zhang, *Phys. Rev.* **D55**, 7241 (1997)
J L Diaz-Cruz, M A Perez, G Tavares-Velasco and J J Toscano, *Phys. Rev.* **D60**, 115014 (1999)
[41] T Han, M Hosh, K Whisnant, B-L Young and X Zhang, *Phys. Rev.* **D58**, 073008 (1998)
F del Aguila, J A Aguilar-Saavedra and R Miquel, *Phys. Rev. Lett.* **82**, 1628 (1999)
F del Aguila and J A Aguilar-Saavedra, hep-ph/9909222
[42] S Willocq, presented at *2000 Aspen Winter Conference on Particle Physics*, 16–22 January 2000, Aspen, CO
[43] MEGA Collaboration: M L Brooks *et al*, *Phys. Rev. Lett.* **83**, 1521 (1999)
[44] G L Fogli, E Lisi and A Marrone, *Phys. Rev.* **D63**, 053008 (2001)
Super-Kamiokande Collaboration: Y Fukuda *et al*, *Phys. Rev. Lett.* **81**, 1562 (1998)