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What can we learn from high precision measurements of neutrino mixing angles?

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Abstract. Many experiments are being planned to measure the neutrino mixing angles more precisely. In this note, the theoretical significance of a high precision measurement of these parameters is discussed. It is emphasized that they can provide crucial information about different ways to understand the origin of large atmospheric neutrino mixing and move us closer towards determining the neutrino mass matrix. They may also be able to throw light on the question of lepton–quark unification as well as the existence of any leptonic symmetries. For instance if exact $\mu \leftrightarrow \tau$ symmetry in the neutrino mass matrix is assumed to be the reason for maximal ν_μ–ν_τ mixing, one gets $\theta_{13} = 0$ and $\theta_{13} \simeq \sqrt{\Delta m^2_\odot/\Delta m^2_{\rm A}}$ or $\theta_{13} \simeq \Delta m^2_\odot/\Delta m^2_{\rm A}$ can provide information about the way the $\mu \leftrightarrow \tau$ symmetry breaking manifests in the case of normal hierarchy.

Keywords. θ_{13}; μ–τ symmetry.

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1. Introduction

Neutrino physics is poised on the brink of an exciting set of experiments that could elevate our knowledge of neutrino masses and mixings to the same level as that of quarks and charged leptons. At the same time, they are also likely to provide important information about physics beyond the standard model. The most crucial experiments in this regard are: (i) searches for neutrinoless double beta decay which will confirm whether neutrinos are Dirac or Majorana fermions; (ii) sign of the atmospheric mass difference which will determine whether the mass hierarchy is normal or inverted and (iii) more precise determination of the various mixing angles including θ_{13}, which will complete our knowledge of mixings.

In this article, I discuss the impact of a high precision search for θ_{13} as well as a more precise determination of the other angles assuming that neutrinos are Majorana fermions. There are several experimental proposals for such searches e.g. refs [1,2]. Some of these experiments are also likely to yield a more precise value of the atmospheric neutrino mixing angle. There are also proposals to have a more accurate determination of the solar mixing angle θ_{12}.

Determination of these angles in addition to providing a complete picture of neutrino mixings, could be a signal of the underlying physics responsible for lepton mixings and as such could be an important clue to physics beyond the standard model [3]. As is argued recently [4], the value of θ_{13} in conjunction with a high precision measurement of the maximality of the atmospheric mixing angle $\theta_{\rm A} \equiv \theta_{23}$ could indeed be a very useful way to determine whether there is a leptonic permutation symmetry between μ and τ as is indicated by the near maximal atmospheric neutrino mixing angle.

Similarly, there is an empirical relation between the angles θ_{12} for the lepton sector and the corresponding angle for the quark sector (the Cabibbo angle), i.e. $\theta^q_{12} + \theta^\nu_{12} = \pi/4$. (Note that the present data indicate $\theta^\nu_{12} = 32.3^\circ \pm 2.4^\circ$ whereas $\theta^q_{12} = 12.8^\circ \pm 0.15^\circ$.) It has been speculated that this could be a signal of an underlying quark–lepton unification [5] in nature.

To begin the discussion, let us note that the Pontecorvo, Maki, Nakagawa and Sakata (PMNS) mixings arise from the lepton mass Lagrangian as follows:

$$\mathcal{L}_m = \nu^T_\alpha C^{-1}\mathcal{M}_{\nu,\alpha\beta}\nu + \bar{e}_{\alpha,\rm L} M^e_{\alpha\beta} e_{\rm R} + \text{h.c.} \tag{1}$$

Diagonalizing the mass matrices by the transformations $U^T_\nu \mathcal{M}_\nu U_\nu = \mathcal{M}^\nu_{\rm diag}$ and $U_\ell M^e V^\dagger = M^e_{\rm diag}$, one defines $U_{\rm PMNS} = U^\dagger_\ell U_\nu$. Clearly, any symmetry in the lepton mass matrices is likely to manifest itself in the $U_{\rm PMNS}$ elements. We will parametrize $U_{\rm PMNS}$ as follows:

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}{\rm e}^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}{\rm e}^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}{\rm e}^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}{\rm e}^{i\delta} & c_{12}s_{23} - c_{12}c_{23}s_{13}{\rm e}^{i\delta} & c_{23}c_{13} \end{pmatrix} K, \tag{2}$$

where $K = {\rm diag}(1, {\rm e}^{i\phi_1}, {\rm e}^{i\phi_2})$. For the sake of simplicity, we will first ignore the CP phases and work with the mixings alone to probe the new symmetries.

The paper is organized as follows: in §2, the implications of a broken $\mu \leftrightarrow \tau$ symmetry for the case of normal hierarchy is discussed; in §3, the corresponding discussion for the inverted hierarchy is taken up. In §4, possible gauge theoretic origin of such models is explored; in §5 the idea of quark–lepton complementarity and its possible origin in gauge theories is addressed in brief. In §6, the conclusions are presented.

2. θ_{13} as a probe of leptonic $\mu \leftrightarrow \tau$ symmetry

To see how a $\mu \leftrightarrow \tau$ symmetry of the neutrino mass matrix appears in the mixing matrix, let us consider the case of only two neutrino generations, i.e. that of μ and τ. Experiments indicate that the atmospheric mixing angle is very nearly maximal, i.e. $\theta_{\rm A} = \pi/4$. Working on the basis that the charged lepton mass matrix is diagonal, it is obvious that the neutrino Majorana mass matrix that gives maximal mixing is

$$\mathcal{M}^{(2)}_\nu = \begin{pmatrix} a & b \\ b & a \end{pmatrix}. \tag{3}$$

Furthermore, the fact that solar neutrino mass difference square $\Delta m^2_\odot \ll \Delta m^2_{\rm A}$ and allowing for small departures from the maximal atmospheric angle, we can write

$$\mathcal{M}^{(2)}_\nu = \frac{\sqrt{\Delta m^2_{\rm A}}}{2}\begin{pmatrix} 1+a\epsilon & 1 \\ 1 & 1+\epsilon \end{pmatrix}, \tag{4}$$

where a is a parameter of order one and $\epsilon \ll 1$. For the case of normal hierarchy we have $\sqrt{\Delta m^2_\odot/\Delta m^2_{\rm A}} \simeq \frac{1}{4}(1+a)\epsilon$. The atmospheric mixing angle is given by $\theta_{\rm A} \simeq \frac{\pi}{4} - \frac{\epsilon(1-a)}{4}$. It is clear that if $a=1$, the neutrino mass matrix has symmetry $\nu_\mu \leftrightarrow \nu_\tau$ and $\theta_{\rm A} = \pi/4$. Thus departures from these symmetries remain imprinted in the values of the mixing angles.

2.1 *Exact $\nu_\mu \leftrightarrow \nu_\tau$ symmetry and $\theta_{13} = 0$*

Let us now extend the above considerations to the case of three generations. First point to note is that in the zeroth order, clearly unrealistic, approximation, maximal atmospheric mixing can arise from two kinds of neutrino mass matrices:

Case (*i*):

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m^2_{\rm A}}}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \tag{5}$$

This is the case of normal hierarchy.

Case (*ii*):

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m^2_{\rm A}}}{2}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{6}$$

This is the case of inverted hierarchy. Both these mass matrices are of course invariant under $\nu_\mu \leftrightarrow \nu_\tau$ symmetry. Furthermore, the second case has the additional symmetry: $L_e - L_\mu - L_\tau$ [6]. In both cases of course one has $\Delta m^2_\odot = 0$ and $\theta_{12} = \pi/4$ and $\theta_{13} = 0$.

In order to depart from this unrealistic symmetry limit to the more realistic case and to see how the various mixing angles are affected, let us first ask the question whether one can have mass matrices invariant under $\nu_\mu \leftrightarrow \nu_\tau$ symmetry while giving $\Delta m^2_\odot \neq 0$ and $\theta_{12} < \pi/4$. The answer to this question is 'yes'. An example of such a mass matrix is

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m^2_{\rm A}}}{2}\begin{pmatrix} c\epsilon & d\epsilon & d\epsilon \\ d\epsilon & 1+\epsilon & -1 \\ d\epsilon & -1 & 1+\epsilon \end{pmatrix}. \tag{7}$$

Mass matrices of this type have been considered in [7]. A mass matrix with $\Delta m^2_\odot \neq 0$ but $\theta_{12} = \pi/4$ was discussed early on from considerations of $\nu_\mu \leftrightarrow \nu_\tau$ symmetry [8]. Both these [8,9] $\nu_\mu \leftrightarrow \nu_\tau$ symmetric neutrino mass matrices lead to $\theta_{13} = 0$.

For this mass matrix, we have

$$\epsilon = 2\frac{\Delta m_\odot^2}{\Delta m_{\rm A}^2},$$
$$\tan 2\theta_\odot \simeq \frac{2\sqrt{2}d}{1-c},$$
$$\theta_{23} = \frac{\pi}{4}; \quad \theta_{13} = 0. \tag{8}$$

Thus two of the three parameters of this matrix are determined by already existing data and if θ_{13} is found to be smaller than the limit expected in many forthcoming experiments and it is found that $\Delta m_{31}^2 > 0$, then there would be a strong case for this as the mass matrix for the neutrinos (in the basis where the charged leptons are mass eigenstates) and an underlying $\nu_\mu \leftrightarrow \nu_\tau$ symmetry. A test for this mass matrix would be a value of $\theta_{23} = \pi/4$. Since neutrinoless double beta decay can in principle determine the parameter c, one can determine all the parameters of this model. This would clearly be extremely useful for probing physics beyond the standard model.

2.2 *Departures from $\nu_\mu \leftrightarrow \nu_\tau$ symmetry and expectations for θ_{13}*

We now consider perturbations around the symmetric limit for the normal hierarchy case and discuss its consequences. Many discussions of such cases exist in the literature [10] (though not necessarily in the context of $\nu_\mu \leftrightarrow \nu_\tau$ symmetry).

In two recent papers [4], this question of how θ_{13} provides a measure of departures from $\nu_\mu \leftrightarrow \nu_\tau$ symmetry has been discussed.

We motivate our discussion from the angle of this symmetry. We mostly discuss the case without CP violation and in the end of this section, comment on a case with CP violation.

The most general CP-conserving perturbation of the neutrino mass matrix around the $\nu_\mu \leftrightarrow \nu_\tau$ symmetric limit that maintains the hierarchy $\Delta m_\odot^2 \ll \Delta m_{\rm A}^2$ and near maximal atmospheric mixing is

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m_{\rm A}^2}}{2}\begin{pmatrix} c\epsilon & d\epsilon & b\epsilon \\ d\epsilon & 1+a\epsilon & -1 \\ b\epsilon & -1 & 1+\epsilon \end{pmatrix}. \tag{9}$$

The parameters characterizing the departures from symmetry limit are: $b \neq d$ and $a \neq 1$. Two characteristic predictions appear depending on the way the symmetry breaking appears.

Case (i): $a = 1, b \neq d$

In this case, one can diagonalize the mass matrix and conclude that $(c, d \ll 1)$.

$$\epsilon \simeq 2\sqrt{\frac{\Delta m_\odot^2}{\Delta m_{\rm A}^2}},$$

$$\theta_{13} \simeq (b-d)\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}},$$
$$\tan 2\theta_\odot \simeq \frac{\sqrt{2}(b+d)}{1-c},$$
$$m_{\beta\beta} \simeq c\sqrt{\Delta m^2_\odot}. \tag{10}$$

Using the present data, in this case one would expect $\theta_{13} \simeq 0.16$. The predictions in models where atmospheric neutrino mixing arises from some dynamical mechanism [11] will also lead to predictions of this type. The difference between this approximate $\mu \leftrightarrow \tau$ symmetry case and the 'dynamical' case is that the atmospheric mixing angle in the symmetry case being discussed here is very close to maximal with departure from maximality being of order $\Delta m^2_\odot/\Delta m^2_{\rm A}$ which is a few per cent (of order $\leq 4^\circ$) whereas in the dynamical case, this departure can be larger (of order $\sim 8^\circ$ or so). The prediction for neutrinoless double beta decay in this case is beyond the range of accessibility of the next round of searches for double beta decay [12].

The physical meaning of this case is that while $\nu_\mu \leftrightarrow \nu_\tau$ symmetry is exact in the $\nu_\mu - \nu_\tau$ sector, it is broken in their mixing with ν_e. Unless this breaking is constrained by extra symmetries, one would expect a large θ_{13} in this case, as noted.

Case (ii): $a \neq 1, b = d$

In this case, we get

$$\epsilon \simeq \frac{4}{1+a}\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}},$$
$$\theta_{13} \simeq \frac{\epsilon^2 d(1-a)}{4\sqrt{2}}. \tag{11}$$

In this case there is a departure from maximality in the atmospheric mixing angle given by the following equation:

$$\theta_{\rm A} \simeq \frac{\pi}{4} - \frac{a-1}{a+1}\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}}. \tag{12}$$

Thus, the expectation for θ_{13} for this way of symmetry breaking is around $\theta_{13} \simeq 0.03$. The smallness of θ_{13} here compared to the previous case can be understood as follows: the $\nu_\mu \leftrightarrow \nu_\tau$ symmetry is broken only in the ν_μ–ν_τ sector of the mass matrix and not in the mixing with ν_e. As a result, to leading order in $\epsilon \simeq \frac{4}{1+c}\sqrt{\Delta m^2_\odot/\Delta m^2_{\rm A}}$, there is no contribution to θ_{13} and it arises only to order ϵ^2. Also as noted above, the departure from maximality of the atmospheric mixing angle in this case can be significant (~8–10%).

Case (iii): $a = 1, |b| = |d|$

An interesting way to break $\nu_\mu \leftrightarrow \nu_\tau$ is to maintain $a = 1$ so that symmetry breaking is in the mixing with ν_e; but choose $b = d^*$ [13]. In this case, one has $\theta_{13} = 2{\rm Im}b\sqrt{\Delta m^2_\odot/\Delta m^2_{\rm A}}$ and one has the Dirac phase at its maximal value of $\pi/2$.

Table 1. The predictions for θ_{13} and $\theta_{\rm A}$ for different ways of $\mu \leftrightarrow \tau$ symmetry breaking.

Symmetry breaking	θ_{13}	$\theta_{23} - \pi/4$
None	0	$\pi/4$
μ–τ sector only	$\sim \Delta m^2_\odot/\Delta m^2_{\rm A}$	$\leq 8^\circ$
e-Sector only	$\sim \sqrt{\Delta m^2_\odot/\Delta m^2_{\rm A}}$	$\leq 4^\circ$
Dynamical	$\sim \sqrt{\Delta m^2_\odot/\Delta m^2_{\rm A}}$	Large

In table 1, we summarize our results.

3. Departures from $\nu_\mu \leftrightarrow \nu_\tau$ symmetry: The inverted hierarchy case

The case of inverted hierarchy has been discussed in great detail in [14] (although connection to $\mu \leftrightarrow \tau$ symmetry was not discussed). Here I summarize the discussion in the language of $\mu \leftrightarrow \tau$ symmetry.

The most general mass matrix in this case is

$$\mathcal{M}_\nu = \sqrt{\Delta m^2_{\rm A}} \begin{pmatrix} z & c & s \\ c & y & d \\ s & d & x \end{pmatrix}, \tag{13}$$

where c and s stand for 'cos' and 'sin' of θ_{23} and $x, y, z, d \ll 1$. In the perturbative approximation, we find the following sum rules involving the neutrino observables and the elements of the neutrino mass matrix. It follows from this matrix that

$$\begin{aligned} &\sin^2 2\theta_\odot = 1 - \left(\frac{\Delta m^2_\odot}{4\Delta m^2_{\rm A}} - z\right)^2 + O(\delta^3), \\ &\frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}} = 2(z + \vec{v}\cdot\vec{x}) + O(\delta^2), \\ &U_{e3} = \vec{A}\cdot(\vec{v}\times\vec{x}) \ + \ O(\delta^3), \end{aligned} \tag{14}$$

where $\vec{v} = (\cos^2\theta, \sin^2\theta, \sqrt{2}\sin\theta\cos\theta)$, $\vec{x} = (x, y, \sqrt{2}d)$ and $\vec{A} = \frac{1}{\sqrt{2}}(1,1,0)$. Now we can discuss the exact $\nu_\mu \leftrightarrow \nu_\tau$ limit and departures from it. The exact symmetry limit occurs when we have $c = s = \frac{1}{\sqrt{2}}$ (maximal atmospheric mixing angle) and $x = y$. It is clear from above that $\theta_{13} = 0$ in this limit. Therefore, a non-vanishing θ_{13} is related to breakdown of this symmetry as in the case of normal hierarchy.

It is clear from this way of parametrizing the mass matrix that the current best fits for the large mixing angle solution to the solar neutrino observations [15] require $z \geq 0.3$ or so. This translates into a value for about $m_{\beta\beta} \geq 15$ MeV [16]. Similar to the case of normal hierarchy case, there are two broken symmetry situations.

Case (i): $c = s = \frac{1}{\sqrt{2}}$, $x \neq y$: In this case, we have

$$\theta_{13} = \frac{x-y}{2},$$
$$\frac{\Delta m^2_{\odot}}{\Delta m^2_{\mathrm{A}}} = 2(x+y+z+d). \tag{15}$$

In this case, θ_{13} could be quite large. It is worth noting that in this case even though smallness of $\Delta m^2_{\odot}/\Delta m^2_{\mathrm{A}}$ implies that there must be cancellations among the parameters x, y, z and d, it does not put any constraint on how large θ_{13} can be.

Case (ii): $c \neq s$, $x = y$: In this case we find

$$\theta_{13} \simeq -d\cos 2\theta_{\mathrm{A}}, \tag{16}$$

In this case, there is a close connection between the value of θ_{13} and departure from maximality of θ_{A}.

It is clear that the expectations for θ_{13} for the inverted hierarchy are very different from the normal hierarchy case. Specially missing in this case is the close connection between θ_{13} and the ratio $\Delta m^2_{\odot}/\Delta m^2_{\mathrm{A}}$. The reason for this is that the value of $\sin^2 2\theta_{\odot}$ required by the present solar and KamLand data requires the m_{ee} term in the neutrino mass matrix to be large in the case of inverted hierarchy. This therefore enters as a new parameter in the $\Delta m^2_{\odot}$ unlike the case of normal hierarchy.

4. Possible gauge theory of broken $\nu_\mu \leftrightarrow \nu_\tau$

So far the discussion has focussed on the testability of $\nu_\mu \leftrightarrow \nu_\tau$ symmetry in the neutrino Majorana mass matrix. In this section we would like to address its implications for physics beyond the standard model. We would like to seek plausible gauge models that lead to this symmetry. We will focus only on the normal hierarchy case since the case of inverted hierarchy has been studied extensively in the literature.

The first clear obstacle one must overcome is that the neutrinos are part of the $SU(2)_{\mathrm{L}}$ doublet that contains (e, μ, τ) and there is no apparent $\mu \leftrightarrow \tau$ symmetry in the charged lepton masses. However, in the limit of $m_\mu = m_\tau$, one can have such a symmetry implying that in the charged lepton sector, there must clearly be a mechanism to break the symmetry by a large amount without affecting the neutrino. We explore below how such a symmetry can emerge in gauge theories and in particular, how it can be broken in a consistent manner. The goal is a modest one of simply trying to give an existence proof. The main point is that if one cannot even construct a consistent model within the loose framework of arbitrary fine tuning, the symmetry has a less chance of being meaningful in reality. In our case it turns out that in addition to assuming a spontaneously broken $\mu \leftrightarrow \tau$ symmetry, if one assumes a Z_4 symmetry, then there are several models that one can construct that realize the mass matrix in eq. (7) without conflicting with charged lepton spectrum. We only discuss the symmetric limit. One can easily extend them to include small breaking effects, e.g. by adding higher dimensional terms to the Lagrangian.

We present two models, one with the right-handed neutrinos and a second one without them. In the first case we will use the conventional see-saw mechanism and in the second one, we will use a triplet dominated type II see-saw [17].

Model I with right-handed neutrinos

We use the standard model gauge group with supersymmetry and standard assignment of matter superfields [18] but with three pairs of Higgs doublets $(H_{\rm u}, H_{\rm d})$. We impose on the model an $S_2 \times Z_4$ symmetry. The multiplets (L_μ, L_τ), (μ^c, τ^c), (N^c_μ, N^c_τ), $(H_{\rm d,1}, H_{\rm d,2})$ transform into each other under S_2 symmetry and $H_{{\rm u},i}$ $(i = 1, 2, 3)$ and the rest of the fields transform as singlets. Under Z_4, we assign $(\mu^c, H_{\rm d,2})$ to transform as $i(\mu^c, H_{\rm d,2})$ whereas $(\tau^c, H_{\rm d,1})$ go to $-i(\tau^c, H_{\rm d,1})$. Rest of the fields are invariant. First point to note is that, the right-handed neutrino mass matrix invariant under this has the form

$$M_{\rm R} = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & M_{22} & M_{23} \\ M_{12} & M_{23} & M_{22} \end{pmatrix}. \tag{17}$$

The Dirac mass matrix for the neutrinos also has similar form:

$$m_{\rm D} = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{23} \\ m_{12} & m_{23} & m_{22} \end{pmatrix}. \tag{18}$$

It is clear that the neutrino mass matrix obtained from the above two equations after type I see-saw has the form which is as in eq. (7) which is $\mu \leftrightarrow \tau$ invariant.

To complete the discussion of model I, note that the charged lepton mass matrices arise from the superpotential:

$$W = h_1(L_\mu H_{\rm d,1}\mu^c + L_\tau H_{\rm d.2}\tau^c) + h_e L_e H_{\rm d,3} e^c \tag{19}$$

whereas we have set all other allowed Yukawas involving e^c, μ^c, τ^c allowed by the symmetry of the theory to zero. Now suppose we break the $\mu \leftrightarrow \tau$ symmetry by the soft $H_{\rm d,1,2}$ mass terms, then $H_{\rm d,1,2}$ will have different and arbitrary VEVs. As a result, we can get correct values for all the charged lepton masses.

Model II without right-handed neutrinos

This model is very similar to the model above except that there are no right-handed neutrinos, instead there are Higgs triplets $\Delta_{\rm L}$ with standard model hypercharge +2 so that couplings of type $LL\Delta_{\rm L}$ are allowed. The $\Delta_{\rm L}$ is given a mass term M which is of the order of 10^{14} GeV, so that the VEV of $\Delta_{\rm L}$ is suppressed due to the term $\Delta_{\rm L} H_{\rm d} H_{\rm d}$ to be $v^2_{wk}/M \simeq 10^{-1}$ eV, which can give neutrino masses of the right order. As in the first case, we require the model to be invariant under $S_2 \times Z_4$ symmetry with assignments as in the previous case. The fields in $\Delta_{\rm L} \oplus \bar{\Delta}_{\rm L}$ pair are invariant under it. The neutrino masses come from the superpotential

$$f_1(L_\mu + L_\tau)((L_\mu + L_\tau))\Delta_\mathrm{L} + f_2(L_\mu - L_\tau)((L_\mu - L_\tau))\Delta_\mathrm{L} + (L_\mu + L_\tau)L_e\Delta_\mathrm{L} + L_eL_e\Delta_\mathrm{L}. \tag{20}$$

Again this leads to a neutrino mass matrix invariant under $\mu \leftrightarrow \tau$ symmetry. The charged lepton masses arise in exactly the same way as in model I.

There are also other models in the literature with similar properties (e.g. see ref. [19]) also. It would therefore seem that considering $\mu \leftrightarrow \tau$ symmetry for leptons, despite its strong breaking in the charged lepton sector it is quite a meaningful and useful way to obtain information about physics beyond the standard model from neutrinos.

Let us make a few comments on the models described above. Note that we have not incorporated any breaking of $\mu \leftrightarrow \tau$ into the model. There could be many sources for such breakings: for example, there could be higher dimensional operators that involve $H_{\mathrm{d},1,2}$ that can break this symmetry. There could also be other effects such as radiative corrections from charged lepton Yukawa couplings that give mass to tau lepton and the muon etc.

5. Quark–lepton complementarity and quark–lepton unification

To see how quark–lepton unification can possibly lead to such a complementarity relation, first note that $\theta_\odot$ is the 12 entry of the PMNS matrix defined as $U_\mathrm{PMNS} = U_\ell^\dagger U_\nu$ (where U_ℓ is the unitary matrix operating on the left-handed charged leptons that diagonalizes the (e, μ, τ) mass matrix and U_ν is the corresponding one for the neutrino Majorana mass matrix) and θ_c is the corresponding entry in the CKM matrix defined as $V_\mathrm{CKM} = U_\mathrm{u}^\dagger U_\mathrm{d}$ (where $U_\mathrm{u,d}$ diagonalize the up and down quark mass matrices). Secondly suppose that the structure of neutrino and quark mass matrices at the high scale are such that to a leading order, the PMNS matrix is exact bi-maximal [20] whereas the CKM matrix is an identity matrix and to next leading order we have the down quark and charged lepton mass matrices equal in the basis where the up quark and neutrino Dirac masses remain unchanged, then the above complementarity relation can emerge and will be an indication of quark–lepton unification at high scale. Recently, we made an attempt to [21] provide one using the gauge group $SU(2)_\mathrm{L} \times SU(2)_\mathrm{R} \times SU(4)_\mathrm{c}$ symmetry that unifies quarks and leptons, which includes the $SU(4)$-color group of Pati and Salam. We found that if we use the double see-saw formula for understanding small neutrino masses, then the conditions for the complementarity relations can be satisfied.

We first outline a set of conditions that are sufficient for obtaining the QLC relation. The full quark and lepton mass matrices should be written in a form such that, we have

$$M_\mathrm{u,d} = M^0_\mathrm{u,d} + \delta M_\mathrm{u,d}, \tag{21}$$

$$M_\ell = M^0_\ell + \delta M_\mathrm{d}, \tag{22}$$

$$\mathcal{M}_\nu = U^*_\mathrm{bm}\mathcal{M}^d_\nu U^\dagger_\mathrm{bm}, \tag{23}$$

where U_bm is the bi-maximal PMNS matrix given by

$$U_{\rm bm} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \tag{24}$$

We require that $M^0_{\rm u} = \kappa M^0_{\rm d}$ so that to leading order $V^0_{\rm CKM} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix and $M^0_\ell = M^0_{\rm d}$ due to quark–lepton symmetry. We also choose $\delta M_{\rm u} = 0$. Note also that we have chosen $\delta M_\ell = \delta M_{\rm d}$, i.e. quark–lepton symmetry is maintained up to the first-order terms that generate the 12 mixing angles. The observed pattern in the $V_{\rm CKM}$ then arises from the matrix $\delta M_{\rm d}$ which is also the change in the leading order lepton mass matrix. It is then clear that one obtains the QLC relation by diagonalizing the full mass matrices for quarks and leptons. We do not provide an explanation why Cabibbo angle is small but our main point is that regardless of whatever mechanism is responsible for the magnitude of the Cabibbo angle, QLC relation can be derived.

6. Summary and conclusion

In this article, it is pointed out that the measurement of the neutrino mixing angle θ_{13} in conjunction with a measurement of the departure from maximality of the atmospheric mixing angle can be a very powerful way to probe any possible $\nu_\mu \leftrightarrow \nu_\tau$ symmetry present in the neutrino mass matrix. In table 1, the expectations for θ_{13} and different cases (with and without approximate $\nu_\mu \leftrightarrow \nu_\tau$ symmetry) are presented for the case of normal hierarchy and can be used as a way to specify the mass matrix. We also have discussed the case of inverted mass hierarchy and pointed out the implications of broken $\mu \leftrightarrow \tau$ symmetry.

Evidence for any approximate $\nu_\mu \leftrightarrow \nu_\tau$ symmetry will clearly be a significant indicator of which way to proceed as we probe physics beyond the standard model. For instance, such a symmetry is highly non-trivial to obtain within the framework of grand unification and point to alternative directions, which will be a useful information.

I have also discussed the possible implications of a quark–lepton complementarity relation indicated by the present solar neutrino data. I have given a set of criteria for understanding this relation as a consequence of quark–lepton unification group $SU(2)_{\rm L} \times SU(2)_{\rm R} \times SU(4)_{\rm c}$.

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