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CP violating rate asymmetries in B decays

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Abstract. We briefly discuss measurements of angles β and α of the unitarity triangle. We then review rate asymmetries using $SU(3)$ relationships in the standard model (SM). Some methods to measure angle γ using $SU(3)$ are then discussed. We note that rate for $b \to s\gamma$ can be used to set limits on extra dimensions in which standard model particles propagate.

Keywords. Rate asymmetries; standard model; CP violation.

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Measurement of angles of the unitarity triangle is one of the prime goals of the B factories. Both Belle [1] and BaBar [2,3] collaborations have reported values of $\sin 2\beta$. The experimental world average is given by

$$\sin 2\beta = 0.79 \pm 0.10. \tag{1}$$

This value is in excellent agreement with the standard model. A recent theoretical analysis by Ciuchini *et al* [4] yields the value

$$\sin 2\beta = 0.698 \pm 0.066. \tag{2}$$

It is still important to measure the other angles by different techniques to make sure that CP violation arises only through the CKM matrix. The next measurement is likely to be $\sin 2\alpha$ through the study of time dependent asymmetries in $B_d \to \pi^+\pi^-$ decays. The coefficient of $\sin(\Delta mt)$ in this case yields $\sin(2\alpha_{\text{measured}})$ where α_{measured} is not the same as α because of large penguin contamination. It is possible to estimate the deviation $\delta\alpha = \alpha_{\text{measured}} - \alpha$ with some theory input. For example, using QCD improved factorization of Beneke *et al* [5], it is possible to deduce $\sin 2\alpha$ from $\sin(2\alpha_{\text{measured}})$ provided $|V_{ub}/V_{cb}|$ is known. This was done in ref. [6]. The following plot (figure 1) summarizes the result of the analysis.

Interesting relationships can be obtained between CP violating rate differences in the standard model if one uses flavor $SU(3)$ symmetry. The quark level effective Hamiltonian up to one loop level in electroweak interaction for hadronic charmless B decays, can be written as

$$H_{\text{eff}}^q = \frac{4G_F}{\sqrt{2}}[V_{ub}V_{uq}^*(c_1O_1 + c_2O_2) - \sum_{i=3}^{12}(V_{ub}V_{uq}^*c_i^{uc} + V_{tb}V_{tq}^*c_i^{tc})O_i]. \tag{3}$$

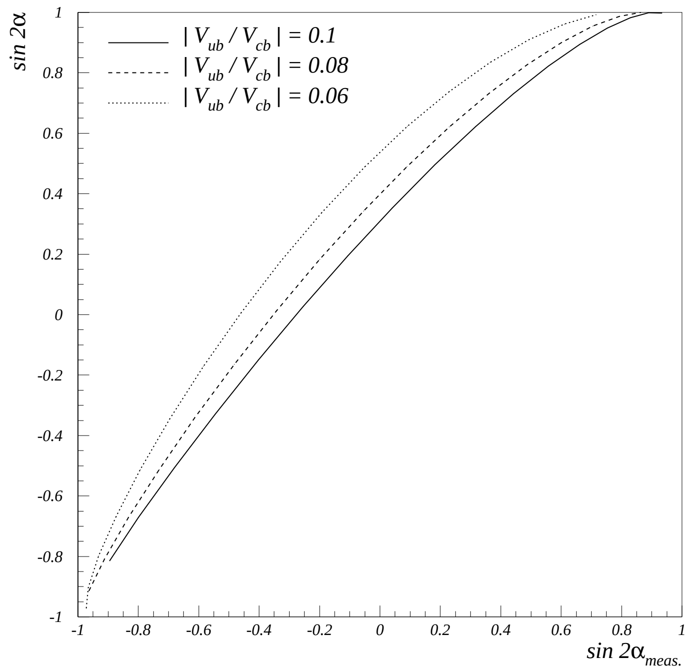

Figure 1. The 'true' value of $\sin 2\alpha$ as a function of the value of $\sin 2\alpha$ 'measured' in $B \to \pi^+\pi^-$ decays for $|V_{ub}/V_{cb}| = 0.1$ (solid curve), 0.08 (dashed curve) and 0.06 (dotted curve).

The operators are defined in ref. [7]. The coefficients $c_{1,2}$ and $c_i^{jk} = c_i^j - c_i^k$, with j indicates the internal quark, are the Wilson coefficients (WC). These WC's have been evaluated by several groups [7], with $|c_{1,2}| \gg |c_i^j|$. In the above the factor $V_{cb}V_{cq}^*$ has been eliminated using the unitarity property of the KM matrix.

The operators $O_{1,2}$, $O_{3-6,11,12}$, and O_{7-10} transform under $SU(3)$ symmetry as $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, $\bar{3}$, and $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, respectively. These properties enable us to write the decay amplitudes for $B \to PP$ in only a few $SU(3)$ invariant amplitudes.

For the $T(q)$ amplitude, for example, we have [8]

$$\begin{aligned} T(q) = {} & A_{\bar{3}}^T B_i H(\bar{3})^i (M_l^k M_k^l) + C_{\bar{3}}^T B_i M_k^i M_j^k H(\bar{3})^j \\ & + A_6^T B_i H(6)_k^{ij} M_j^l M_l^k + C_6^T B_i M_j^i H(6)_l^{jk} M_k^l \\ & + A_{\overline{15}}^T B_i H(\overline{15})_k^{ij} M_j^l M_l^k + C_{\overline{15}}^T B_i M_j^i H(\overline{15})_l^{jk} M_k^l , \end{aligned} \tag{4}$$

where $B_i = (B_u, B_d, B_s) = (B^-, \bar{B}^0, \bar{B}_s^0)$ is a $SU(3)$ triplet, M_i^j is the $SU(3)$ pseudoscalar octet, and the matrices $H(i)$ contain information about the transformation properties of the operators O_{1-12}.

For $q = d$, the non-zero entries of the matrices $H(i)$ are given by

$$H(\bar{3})^2 = 1, \quad H(6)^{12}_1 = H(6)^{23}_3 = 1, \quad H(6)^{21}_1 = H(6)^{32}_3 = -1,$$
$$H(\overline{15})^{12}_1 = H(\overline{15})^{21}_1 = 3, \quad H(\overline{15})^{22}_2 = -2, \quad H(\overline{15})^{32}_3 = H(\overline{15})^{23}_3 = -1. \tag{5}$$

And for $q = s$, the non-zero entries are

$$H(\bar{3})^3 = 1, \quad H(6)^{13}_1 = H(6)^{32}_2 = 1, \quad H(6)^{31}_1 = H(6)^{23}_2 = -1,$$
$$H(\overline{15})^{13}_1 = H(\overline{15})^{31}_1 = 3, \quad H(\overline{15})^{33}_3 = -2, \quad H(\overline{15})^{32}_2 = H(\overline{15})^{23}_2 = -1. \tag{6}$$

Due to the anti-symmetric property of $H(6)$ in exchanging the upper two indices, A_6 and C_6 are not independent. For individual decay amplitude, A_6 and C_6 always appear together in the form $C_6 - A_6$. We will absorb A_6 in the definition of C_6. In terms of the $SU(3)$ invariant amplitudes, the decay amplitudes for various B meson decays are given by [9]

$\Delta S = 0$

$$\begin{aligned}
&T^{B_u}_{\pi^-\pi^0}(d) = \frac{8}{\sqrt{2}} C^T_{\overline{15}},\\
&T^{B_u}_{\pi^-\eta_8}(d) = \frac{2}{\sqrt{6}}(C^T_{\bar{3}} - C^T_6 + 3A^T_{\overline{15}} + 3C_{\overline{15}}),\\
&T^{B_u}_{K^-K^0}(d) = C^T_{\bar{3}} - C^T_6 + 3A^T_{\overline{15}} - C^T_{\overline{15}},\\
&T^{B_d}_{\pi^+\pi^-}(d) = 2A^T_{\bar{3}} + C^T_{\bar{3}} + C^T_6 + A^T_{\overline{15}} + 3C^T_{\overline{15}},\\
&T^{B_d}_{\pi^0\pi^0}(d) = \frac{1}{\sqrt{2}}(2A^T_{\bar{3}} + C^T_{\bar{3}} + C^T_6 + A^T_{\overline{15}} - 5C^T_{\overline{15}}),\\
&T^{B_d}_{K^-K^+}(d) = 2(A^T_{\bar{3}} + A^T_{\overline{15}}),\\
&T^{B_d}_{\bar{K}^0K^0}(d) = 2A_{\bar{3}} + C^T_{\bar{3}} - C^T_6 - 3A^T_{\overline{15}} - C_{\overline{15}},\\
&T^{B_d}_{\pi^0\eta_8}(d) = \frac{1}{\sqrt{3}}(-C^T_{\bar{3}} + C^T_6 + 5A^T_{\overline{15}} + C_{\overline{15}}),\\
&T^{B_d}_{\eta_8\eta_8}(d) = \frac{1}{\sqrt{2}}(2A_{\bar{3}} + \frac{1}{3}C^T_{\bar{3}} - C^T_6 - A^T_{\overline{15}} + C_{\overline{15}}),\\
&T^{B_s}_{K^+\pi^-}(d) = C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} + 3C_{\overline{15}},\\
&T^{B_s}_{K^0\pi^0}(d) = -\frac{1}{\sqrt{2}}(C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} - 5C_{\overline{15}}),\\
&T^{B_s}_{K^0\eta_8}(d) = -\frac{1}{\sqrt{6}}(C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} - 5C_{\overline{15}}),
\end{aligned}$$

$\Delta S = -1$

$$\begin{aligned}
&T^{B_u}_{\pi^-\bar{K}^0}(s) = C^T_{\bar{3}} - C^T_6 + 3A^T_{\overline{15}} - C^T_{\overline{15}},\\
&T^{B_u}_{\pi^0K^-}(s) = \frac{1}{\sqrt{2}}(C^T_{\bar{3}} - C^T_6 + 3A^T_{\overline{15}} + 7C^T_{\overline{15}}) ,\\
&T^{B_u}_{\eta_8K^-}(s) = \frac{1}{\sqrt{6}}(-C^T_{\bar{3}} + C^T_6 - 3A^T_{\overline{15}} + 9C^T_{\overline{15}}),\\
&T^{B_d}_{\pi^+K^-}(s) = C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} + 3C^T_{\overline{15}},\\
&T^{B_d}_{\pi^0\bar{K}^0}(s) = -\frac{1}{\sqrt{2}}(C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} - 5C^T_{\overline{15}}),\\
&T^{B_d}_{\eta_8\bar{K}^0}(s) = -\frac{1}{\sqrt{6}}(C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} - 5C^T_{\overline{15}}),\\
&T^{B_s}_{\pi^+\pi^-}(s) = 2(A^T_{\bar{3}} + A^T_{\overline{15}}),\\
&T^{B_s}_{\pi^0\pi^0}(s) = \sqrt{2}(A^T_{\bar{3}} + A^T_{\overline{15}}),\\
&T^{B_s}_{K^+K^-}(s) = 2A^T_{\bar{3}} + C^T_{\bar{3}} + C^T_6 + A^T_{\overline{15}} + 3C^T_{\overline{15}},\\
&T^{B_s}_{K^0\bar{K}^0}(s) = 2A^T_{\bar{3}} + C^T_{\bar{3}} - C^T_6 - 3A^T_{\overline{15}} - C^T_{\overline{15}},\\
&T^{B_s}_{\pi^0\eta_8}(s) = \frac{2}{\sqrt{3}}(C^T_6 + 2A^T_{\overline{15}} - 2C^T_{\overline{15}}),\\
&T^{B_s}_{\eta_8\eta_8}(s) = \sqrt{2}(A^T_{\bar{3}} + \frac{2}{3}C^T_{\bar{3}} - A^T_{\overline{15}} - 2C^T_{\overline{15}}).
\end{aligned}$$

The amplitudes for $P(q)$ in terms of $SU(3)$ invariant amplitudes can be obtained in a similar way. We will indicate the corresponding amplitudes by A^P_i and C^P_i. Rate differences are defined as follows:

$$\Delta(B \to PP) = \Gamma(B \to PP) - \Gamma(\bar{B} \to \bar{P}\bar{P}). \tag{7}$$

$SU(3)$ symmetry relates $\Delta S = 0$ and $\Delta S = -1$ decays. One particularly interesting class of relations are the ones with $T(d) = T(s) = T$ and $P(d) = P(s) = P$. For this class of decays, we have [8]

$$A(d) = V_{ub}V^*_{ud}T + V_{tb}V^*_{td}P,$$
$$A(s) = V_{ub}V^*_{us}T + V_{tb}V^*_{ts}P. \tag{8}$$

Due to different KM matrix elements involved in $A(d)$ and $A(s)$, although the amplitudes have some similarities, the branching ratios are not simply related. However, when considering rate difference, $\Delta(B \to PP)$, the situation is dramatically different. Because a simple property of the KM matrix element, $\mathrm{Im}(V_{ub}V_{ud}^*V_{tb}^*V_{td}) = -\mathrm{Im}(V_{ub}V_{us}^*V_{tb}^*V_{ts})$, we find that in the $SU(3)$ limit,

$$\Delta(d) = -\Delta(s), \tag{9}$$

where $\Delta(i) = (|A(i)|^2 - |\bar{A}(i)|^2)\lambda_{ab}/(8\pi m_B)$ is the CP violating rate difference defined earlier and $\lambda_{ab} = \sqrt{1-2(m_a^2+m_b^2)/m_B^2 + (m_a^2-m_b^2)^2/m_B^4}$ with $m_{a,b}$ being the masses of the two particles in the final state.

In the $SU(3)$ limit we find the following equalities:

$$\begin{aligned}
&(1)\ \Delta(B^- \to K^-K^0) = -\Delta(B^- \to \pi^-\bar{K}^0)\ ,\\
&(2)\ \Delta(\bar{B}^0 \to \pi^-\pi^+) = -\Delta(B_s \to K^-K^+)\ ,\\
&(3)\ \Delta(\bar{B}^0 \to K^-K^+) = -\Delta(B_s \to \pi^-\pi^+) = -2\Delta(B_s \to \pi^0\pi^0)\ ,\\
&(4)\ \Delta(\bar{B}^0 \to \bar{K}^0K^0) = -\Delta(B_s \to K^0\bar{K}^0)\ ,\\
&(5)\ \Delta(\bar{B}^0 \to \pi^+K^-) = -\Delta(B_s \to K^+\pi^-),\\
&(6)\ \Delta(\bar{B}^0 \to \pi^0\bar{K}^0) = -\Delta(B_s \to K^0\pi^0) = 3\Delta(\bar{B}^0 \to \eta_8\bar{K}^0)\\
&\qquad\qquad\qquad\quad = -3\Delta(B_s \to K^0\eta_8).
\end{aligned} \tag{10}$$

If it turns out that the annihilation contributions are all small as can be tested in $B^- \to K^-K^0$, $B_s \to \pi^+\pi^-$ and $B_s \to \pi^0\pi^0$, there are additional relations for rate differences. We find

$$\begin{aligned}
&(1) \approx (4),\\
&(2) \approx -(5),\\
&(6) \approx \Delta(\bar{B}^0 \to \pi^0\pi^0).
\end{aligned} \tag{11}$$

In the limit that annihilation contributions are small, it is difficult to perform tests related to (1), (3) and (4) because the decay rates involved are all small. The equalities of (2) and (5) provide the best chances to test the SM.

We can use factorization assumption to estimate $SU(3)$ breaking. This should be a good approximation because corrections to factorization are $O(\alpha_s)$, thus neglected terms of order $SU(3)$ breaking times $O(\alpha_s)$

$$\begin{aligned}
&\Delta(\bar{B}^0 \to \pi^+\pi^-) \approx -\frac{f_\pi^2}{f_K^2}\Delta(\bar{B}^0 \to \pi^+K^-),\\
&\Delta(B_s \to K^+K^-) \approx -\frac{f_K^2}{f_\pi^2}\Delta(B_s \to \pi^-K^+).
\end{aligned} \tag{12}$$

Similar method can be applied to $B \to PV$ decays [10]. We find the following interesting relations:

$$\begin{aligned}
&\Delta(B_d \to \pi^-\rho^+) \approx -\frac{f_\pi^2}{f_K^2}\Delta(B_d \to K^-\rho^+),\\
&\Delta(B_d \to \pi^+\rho^-) \approx \Delta(B_d \to \pi^+K^*).
\end{aligned} \tag{13}$$

Table 1. The 6 (or 8) B decay modes used by each of the 4 cases to determine γ.

Case	Modes used $\Delta S=0$	Modes used $\Delta S=1$
1	$B^+\to\pi^+\pi^0$, $B_d\to\pi^+\pi^-,\pi^0\pi^0$	$B_d\to\pi^-K^+,\pi^0K^0$
	$\bar{B}_d\to\pi^+\pi^-,\pi^0\pi^0$	$B_s\to\pi^+\pi^-$ (or $\pi^0\pi^0$)
2	$B^+\to\pi^+\pi^0$, $B_d\to\pi^+\pi^-,\pi^0\pi^0$	$B_s\to K^+K^-$
	$\bar{B}_d\to\pi^+\pi^-,\pi^0\pi^0$	(CP-averaged)
3	$B^+\to\pi^+\pi^0$, $B_d\to\pi^+\pi^-,\pi^0\pi^0$	$B_d\to\pi^0K^0,\pi^-K^+$
	$B_d\to K^+K^-$	$\bar{B}_d\to\pi^0\bar{K}^0,\pi^+K^-$
4	$B^+\to\pi^+\pi^0$	$B_d\to\pi^0K^0,\pi^-K^+$
	$B_s\to\pi^+K^-$ (or $\pi^0\bar{K}^0$) (CP-averaged)	$\bar{B}_d\to\pi^0\bar{K}^0,\pi^+K^-$

Buras and Fleischer [11] gave a method to determine γ *without* neglecting rescattering using $B_d\to\pi^-K^+$, $B^+\to\pi^+\pi^0$ decays and time *dependent* measurements of the $B_d\to\pi^0K_S$ decay. For this method, they also require time dependent analysis of, for example, $B_d\to J/\psi K_S$ to measure β. Gronau and Pirjol [12] suggested a method using time independent measurements of *all* the $B_d\to\pi K$ and $B_s\to\pi K$ modes. In their method also rescattering effects are included. However, it might be difficult to measure the neutral modes of B_s decays since that will involve tagging at hadron machines.

We have discussed a technique to determine γ *including* rescattering effects (and the EWP operators) using B meson decays to π's and K's [13]. We will illustrate this technique for one of the cases discussed. This corresponds to case (3) in table 1.

We do *not* require any time dependent studies. The strategy is as follows: In cases 1 and 2, using five $\Delta S=0$ decay modes, we determine the strong phases and magnitudes of the tree level and penguin contributions as functions of γ (assuming flavor $SU(2)$ symmetry). Then, using flavor $SU(3)$ symmetry, we *predict* the rate for *one* $\Delta S=1$ mode in case 2. In case 1, two $\Delta S=1$ modes have to be measured to make a prediction for a third $\Delta S=1$ mode. The measurement of the decay for which we have a prediction (as a function of γ) then determines γ. A similar idea can be applied to predict a $\Delta S=0$ decay mode as a function of γ using measurements of $\Delta S=1$ (and some $\Delta S=0$) modes (cases 3 and 4).

The decay amplitudes for $B_d\to\pi K$ can be written as

$$\sqrt{2}\,\mathscr{A}(B_d\to\pi^0K^0)=-I_{1/2}+2I_{3/2}, \tag{14}$$

$$\mathscr{A}(B_d\to\pi^-K^+)=I_{1/2}+I_{3/2}, \tag{15}$$

where $I_{1/2}$ and $I_{3/2}$ are the amplitudes for B_d decay to $\pi\,K$ $(I=1/2)$ and $(I=3/2)$ respectively. Then,

$$3I_{3/2}=\sqrt{2}\mathscr{A}(B_d\to\pi^0K^0)+\mathscr{A}(B_d\to\pi^-K^+)$$

$$= -\lambda_u^{(s)} 8\tilde{C}_{15}^T + 8\lambda_c^{(s)} C_{15}^P = -8C_{15}^T \left(\lambda_u^{(s)} - \frac{3}{2}\kappa\lambda_c^{(s)}\right)$$
$$= -8\,|C_{15}^T||\lambda_u^{(s)}|\left(e^{i\gamma} + \delta_{EW}\right), \tag{16}$$

Here δ_{EW} is given by $-|\lambda_c^{(s)}|/|\lambda_u^{(s)}|\,3/2\,\kappa \sim -0.66$, and the EWP contribution *is* important for $B_d \to \pi K$ decays. $|C_{15}^T|$ can be obtained from the $B^+ \to \pi^+\pi^0$ decay rate.

$I_{1/2}$ is given by

$$I_{1/2} = -\lambda_u^{(s)}\left(\tilde{C}_3^T + \tilde{C}_6^T + \frac{1}{3}\tilde{C}_{15}^T\right) + \lambda_c^{(s)}\left(C_3^P + C_6^P + \frac{1}{3}C_{15}^P\right)$$
$$+\lambda_u^{(s)}\tilde{A}_{15}^T - \lambda_c^{(s)}A_{15}^P$$
$$\equiv e^{i\phi'_{\tilde{T}}}|\lambda_u^{(s)}|e^{i\gamma}\tilde{T}' - |\lambda_c^{(s)}|e^{i\phi'_P}P'. \tag{17}$$

The four quantities: $\tilde{T}'$, P', $\phi'_{\tilde{T}}$ and ϕ'_P can thus be determined as functions of γ from the measurements of the four decay rates: $B_d \to \pi^- K^+$, $B_d \to \pi^0 K^0$ and their CP-conjugates.

Due to the EWP contribution (see eq. (16)), the triangle construction is a bit different in this case as shown below.

We multiply the CP-conjugate amplitudes by $e^{i2\gamma}$ to get the 'barred' amplitudes. In this case there is an angle between $I_{3/2}$ and $\bar{I}_{3/2}$ denoted by $2\tilde{\gamma}$ and their magnitudes are functions of γ (see eq. (16)):

$$|I_{3/2}| = |\bar{I}_{3/2}| = \frac{8}{3}|C_{15}^T||\lambda_u^{(s)}|\sqrt{\left(1 + \delta_{EW}^2 + 2\delta_{EW}\cos\gamma\right)}, \tag{18}$$

$$\tan\tilde{\gamma} = \frac{\delta_{EW}\sin\gamma}{1 + \delta_{EW}\cos\gamma}. \tag{19}$$

Given γ, we can thus construct the triangles of eq. (16) and its CP-conjugate (see figure 2). Knowing the magnitudes and orientations of $I_{1/2}$ and $\bar{I}_{1/2}$ from figure 2, we can determine $\tilde{T}'$, P', $\phi'_{\tilde{T}}$ and ϕ'_P as functions of γ.

The $B_d \to K^+K^-$ amplitude is given by

$$\mathscr{A}\left(B_d \to K^+K^-\right) = -\lambda_u^{(d)}\left(2A_3^T + 2A_{15}^T\right) - \sum_q \lambda_q^{(d)}\left(2A_{3,q}^P + 2A_{15,q}^P\right)$$
$$\equiv a e^{i\phi_a}. \tag{20}$$

We can see that $\sqrt{2}\mathscr{A}\left(B_d \to \pi^0\pi^0\right) + \mathscr{A}\left(B_d \to K^-K^+\right)$ can be obtained from $\sqrt{2}\mathscr{A}\left(B_d \to K^0\pi^0\right)$ and $\mathscr{A}\left(B_d \to \pi^+\pi^-\right) - \mathscr{A}\left(B_d \to K^-K^+\right)$ can be obtained from $\mathscr{A}\left(B_d \to \pi^-K^+\right)$ by scaling the $\Delta S = 1$ amplitudes by appropriate CKM factors. Thus, we can determine γ, including *all* rescattering effects, by measuring the 8 decay modes: $B^+ \to \pi^+\pi^0$, B_d and $\bar{B}_d \to \pi K$ (all), $B_d \to K^+K^-$, $B_d \to \pi^0\pi^0$ and $B_d \to \pi^-\pi^+$ (or CP-conjugates of the last three modes).

If the annihilation amplitudes are small, we can determine γ by measuring any *one* $B_d \to \pi\pi$ decay mode, in addition to the $B^+ \to \pi^+\pi^0$, B_d (and $\bar{B}_d$) $\to \pi K$ decay modes. If we measure the CP-conjugate $B_d \to \pi\pi$ rates as well, then a CP-averaged measurement of the decay rate $B_d \to K^+K^-$ suffices.

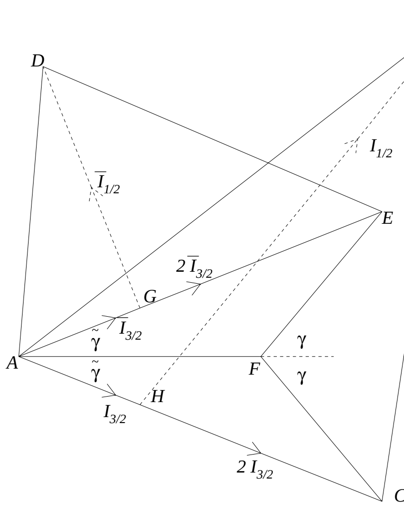

Figure 2. The triangles formed by the $B_d \to \pi K$ amplitudes: AB $= |\mathscr{A}(B_d \to K^+\pi^-)|$, BC $= |\sqrt{2}\mathscr{A}(B_d \to \pi^0 K^0)|$, AD $= |\mathscr{A}(\bar{B}_d \to K^-\pi^+)|$, DE $= |\sqrt{2}\mathscr{A}(\bar{B}_d \to \pi^0 \bar{K}^0)|$, AF $= |\sqrt{2}\,\mathscr{A}(B^+ \to \pi^+\pi^0)|\,|\lambda_u^{(s)}|/|\lambda_u^{(d)}|$ and FC = FE = AF δ_{EW} (see eq. (16)). In the phase convention where the strong phase of C_{15}^T is zero, the angle between AF and the real axis is $\pi + \gamma$.

We now show [14] how $b \to s\gamma$ decay can be used to set limits on the size of extra dimensions. The motivations for studying theories with *flat* extra dimensions of size (TeV) $^{-1}$ accessible to (at least some of) the SM fields are varied.

From the 4*D* point of view, these extra dimensions take the form of Kaluza–Klein (KK) excitations of SM fields with masses $\sim n/R$, where R is a typical size of an extra dimension. In models with *only* SM gauge fields in the bulk, there are contributions to muon decay, atomic parity violation (APV) etc. from tree-level exchange of KK states of gauge bosons [15,16]. Then, precision electroweak measurements result in a strong constraint on the size of extra dimensions and, in turn, imply that the effect on the process $b \to s\gamma$ is small.

To avoid these constraints, we will focus on models with *universal* extra dimensions, i.e., extra dimensions accessible to *all* the SM fields. In this case, due to conservation of extra dimensional momentum, there are *no* vertices with only one KK state, i.e., coupling of KK state of gauge boson to quarks and leptons always involves (at least one) *KK* mode of quark or lepton.

This, in turn, implies that there is no tree-level contribution to weak decays of quarks and leptons, APV $e^+e^- \to \mu^+\mu^-$ etc. from exchange of KK states of gauge bosons [17,18].

However, there is a constraint on R^{-1} from *one-loop* contribution of KK states of (mainly) the top quark to the T parameter. For $m_t \ll R^{-1}$, this constraint is roughly given by $\sum_n m_t^2/(m_t^2 + (n/R)^2) \lesssim 0.5 - 0.6$ (depending on the neutral Higgs mass) [18]. For the case of one extra dimension, this gives $R^{-1} \gtrsim 300$ GeV. The KK excitations of quarks appear as heavy stable quarks at hadron colliders and searches by the CDF collaboration also imply $R^{-1} \gtrsim 300$ GeV for one extra dimension [18]. We consider in this talk $b \to s\gamma$ for the case of only minimal SM with one Higgs doublet in extra dimensions.

The effective Hamiltonian for $\Delta S = 1$ B meson decays is

$$\mathscr{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{j=1}^{8} C_j(\mu) \mathscr{O}_j, \tag{21}$$

where the operator relevant for the transition $b \to s\gamma$ is

$$\mathscr{O}_7 = \frac{e}{16\pi^2} m_b \, \bar{s}_{L\alpha} \sigma^{\mu\nu} b_{R\alpha} F_{\mu\nu}. \tag{22}$$

The coefficient of this operator from $W - t$ exchange in the SM is

$$C_7^W(m_W) = -\frac{1}{2} A\left(\frac{m_t^2}{m_W^2}\right), \tag{23}$$

where the loop function A is given by

$$A(x) = x\left[\frac{\frac{2}{3}x^2 + \frac{5}{12}x - \frac{7}{12}}{(x-1)^3} - \frac{\left(\frac{3}{2}x^2 - x\right)\ln x}{(x-1)^4}\right]. \tag{24}$$

Of course, this includes the contribution from the charged would-be-Goldstone boson (WGB) (i.e., longitudinal W). With extra dimensions, there is a one-loop contribution from KK states of W (accompanied by KK states of top quark, $t^{(n)}$), but as we show now, this is smaller than that from KK states of charged WGB. In the limit $m_W \ll R^{-1}$, the KK states of W get a mass $\sim n/R$ by 'eating' the field corresponding to extra polarization of W in higher dimensions – this field is a scalar from the 4D point of view. Thus, the coupling of *all* components of $W^{(n)}$ to fermions is g, unlike the case of the zero-mode, where the coupling of *longitudinal* W to fermions is given by the Yukawa coupling of Higgs to fermions. Therefore, the contribution of $W^{(n)}$ to the coefficient of the dimension-5 operator $\bar{s}\sigma_{\mu\nu} b F^{\mu\nu}$ is $\sim e m_b g^2/(16\pi^2) m_t^2 \sum_n 1/(n/R)^4$, where the factor m_t^2 reflects GIM cancellation. In terms of the operator $\mathscr{O}_7$, the contribution of each KK state of W to C_7 is $\sim m_t^2 m_W^2/(n/R)^4$.

From the above discussion, it is clear that the KK states of charged would-be-Goldstone boson (denoted by WGB$^{(n)}$) are physical (unlike the *zero*-mode). The loop contribution of WGB$^{(n)}$ with mass n/R (and $t^{(n)}$ with mass $\sqrt{m_t^2 + (n/R)^2}$) is of the same form as that of physical charged Higgs in 2 Higgs doublet models [19] with the appropriate modification of masses and couplings of virtual particles in the loop integral

$$C_7^{\text{WGB}^{(n)}}\left(R^{-1}\right) \approx \frac{m_t^2}{m_t^2 + (n/R)^2} \times \left[B\left(\frac{m_t^2 + (n/R)^2}{(n/R)^2}\right) - \frac{1}{6} A\left(\frac{m_t^2 + (n/R)^2}{(n/R)^2}\right)\right]. \tag{25}$$

Here, the factor $m_t^2/(m_t^2 + (n/R)^2)$ accounts for (a) the coupling of WGB$^{(n)}$ to $t^{(n)}$ which is $\lambda_t \sim m_t/v$, i.e., the same as that of WGB$^{(0)}$ (longitudinal W), and (b) the fact that this contribution decouples in the limit of large KK mass – the functions A and B (see below) in the above expression approach a constant as n/R becomes large.

The loop function B is given by [19]

$$B(y) = \frac{y}{2}\left[\frac{\frac{5}{6}y - \frac{1}{2}}{(y-1)^2} - \frac{\left(y-\frac{2}{3}\right)\ln y}{(y-1)^3}\right]. \tag{26}$$

It is clear that the ratio of the contribution of $W^{(n)}$ and that of $\mathrm{WGB}^{(n)}$ is $\sim (m_W R/n)^2 \lesssim O(1/10)$ since $R^{-1} \gtrsim 300$ GeV (due to constraints from the T parameter and searches for heavy quarks). In what follows, we will neglect the $W^{(n)}$ contribution.

At NLO, the coefficient of the operator at the scale $\mu \sim m_b$ is given by [20]

$$C_7\left(m_b\right) \approx 0.698\, C_7\left(m_W\right) - 0.156\, C_2\left(m_W\right) + 0.086\, C_8\left(m_W\right). \tag{27}$$

Here, C_2 is the coefficient of the operator $\mathscr{O}_2 = (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{s}_{L\beta}\gamma_\mu c_{L\beta})$ and is approximately the same as in the SM (i.e., 1) since the KK states of W do not contribute to it at tree-level. C_8 is the coefficient of the chromomagnetic operator $\mathscr{O}_8 = g_s/(16\pi^2) m_b \bar{s}_{L\alpha}\sigma^{\mu\nu}T^a_{\alpha\beta} b_{R\beta} G^a_{\mu\nu}$.

In the SM, $C_8\left(m_W\right) \approx -0.097$ [20] due to the contribution of $W - t$ loop (using $m_t \approx 174$ GeV). The coefficient of this operator also gets a loop contribution from KK states which is of the same order as the contribution to C_7. Since the coefficient of C_8 in eq. (27) is small, we neglect the contribution of KK states to C_8.

The coefficient of $\mathscr{O}_7$ at the scale m_W is given by the sum of the contributions of $W^{(0)}$ (eq. (23)) and that of $\mathrm{WGB}^{(n)}$ (eq. (25)) summed over n.

Since $C_7^W(m_W) < 0$ and $C_7^{\mathrm{WGB}^{(n)}}(R^{-1}) > 0$, we see that contribution from $\mathrm{WGB}^{(n)}$ interferes destructively with the W contribution. The SM prediction for $\Gamma(b \to s\gamma)/\Gamma(b \to cl\nu)$ has an uncertainty of about 10% and the experimental error is about 15% (both are 1σ errors) [21]. The central values of theory and experiment agree to within $1/2\,\sigma$. The semileptonic decay is not affected by the KK states (at tree-level).

Combining theory and experiment 2σ errors in quadrature, this means that the 95% CL constraint on the contribution of KK states is that it should not modify the SM prediction for $\Gamma(b \to s\gamma)$ by more than 36%. Since $\Gamma(b \to s\gamma) \propto \left[C_7\left(m_b\right)\right]^2$, the constraint is $|[C_7^{\mathrm{total}}(m_b)]^2/[C_7^{\mathrm{SM}}(m_b)]^2 - 1| \lesssim 36\%$.

Using $m_t \approx 174$ GeV, we get $A \approx 0.39$ in eq. (23) and $C_7^{\mathrm{SM}}(m_b) \approx -0.3$ from eq. (27). Assuming $m_t \ll R^{-1}$, we get $B \approx 0.19$ and $A \approx 0.21$ in eq. (25). Then, using eq. (27) and the above criterion, we get the constraint

$$\sum_n m_t^2/(m_t^2 + (n/R)^2) \lesssim 0.5 \tag{28}$$

which is comparable to that from the T parameter. For one extra dimension, performing the sum over KK states with the exact expressions for A and B in eq. (25), the constraint is $R^{-1} \gtrsim 280$ GeV.

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