

## Bridge between bound state and reaction effective nucleon–nucleon potentials

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**Abstract.** It has been a long standing problem to find the connection between the realistic nucleon–nucleon interaction and the interaction between two nucleons when they are embedded inside a nuclear medium, the so-called ‘effective interaction’. On the one hand various many-body approaches have been employed, while on the other hand, information is sought on the basis of static nuclear properties. Both provide partial information. An attempt is made to understand if there is an inherent link, a bridge, which connects all these informations, through the study of heavy ion fusion reactions.

**Keywords.** Effective nucleon–nucleon interactions; heavy-ion reactions; microscopic approach; fusion reactions.

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### 1. Introduction

There have been several attempts to use realistic nucleon–nucleon (N–N) interactions in understanding static nuclear properties such as, binding energies, r.m.s. radii, single particle spacings, etc. using Brueckner theory and its variations, as well as variational methods such as that of Shakin, Waghmare and collaborators [1]. Also, comparison of these informations is made by comparing with the ‘on-shell’ behaviour of the reaction mechanisms, involving the so-called ‘direct reactions’. However, the reaction processes considered do not involve analysis of the effective interactions themselves. Besides, studies related to heavy ion reactions (HIR) employ methods such as time dependent Hartree–Fock (TDHF) or their variations, which may not be quite justified. The reason is that the solutions of the TDHF equations are rather involved and non-unique even for delta function interactions. If one considers a finite range interaction, then the solutions are still more difficult and unless one uses the prescription of Pal [2] to select a particular solution which provides a unique solution which, at best is more theoretical than practical, the efforts are futile. Indeed, no practical, fully quantum mechanical theory of HIR exists. Recently, Satchler *et al* [3] have made an indepth study in this regard using other macro-microscopic interactions.

In this paper, we have considered systematic study of heavy ion fusion reactions starting from the other end – namely using classical equations of motion approach to semi-classical

or semi-quantum mechanical according to one's interpretation, to understand if such fusion cross sections indeed depend upon the nature of the effective N-N interaction.

## 2. Theoretical analysis

The theoretical frame work for the understanding of heavy ion fusion cross sections on the basis of effective N-N interactions, the so-called microscopic approach, was given in our earlier work [3]. It was observed that even though a good agreement was obtained between the observed and calculated fusion cross sections, by using pure classical (point particle) description for nucleons interacting via a two-body effective interaction, there was gross disagreement between and known 'depth' of the interaction as well as the 'incompressibility' parameter. In this case one has to solve coupled Newtonian equations of motion of the form

$$m \frac{d^2 \vec{r}_i}{dt^2} = - \sum_j \nabla_j V_{ij}, \quad (1)$$

where  $V_{ij}$  is the effective two-body potential, which was considered of the form [4]

$$V_{ij} = -v_0 \left( 1 - \frac{c}{r_{ij}} \right) \exp \left( - \frac{r_{ij}^2}{r_0^2} \right). \quad (2)$$

Clearly, in the case of  $^{16}\text{O} + ^{16}\text{O}$  reaction one has to solve 92 coupled equations to find the trajections of the particles. As mentioned above, this theory was only a partial success.

On the other hand, as is shown by Godre and Waghmare [3], if one has a *semi-classical* approach invoking wave packet description of the interacting particles, then not only one obtains a good agreement with the observed results but also correct depth of  $\sim 50$  MeV and the incompressibility parameter  $k \cong 230$  MeV are obtained. Indeed, because of the overlap of the wave packets, and including zero point motion of the nucleons, the essential ingredients of quantum mechanical aspects of the problem are automatically incorporated. It turns out [3] that the cross sections are sensitive not only to the width  $\sigma$  of the wave packets but also to the range  $r_0$  of the N-N interaction.

A summary of the calculated fusion cross sections for  $^{16}\text{O} + ^{16}\text{O}$  reaction as a function of the wave packet width  $\sigma$  and for various centres of mass energies is given in table 1. The parameters of the potential which reproduce the static properties of  $^{16}\text{O}$  nuclei are given in table 2. (The comparison of calculated and experimental results for  $\sigma = 0.8$  fm and 1.4 fm is taken from ref. [3].)

**Table 1.**  $^{16}\text{O} + ^{16}\text{O}$  fusion cross reaction (in mb).

Sr. No.	$E_{cm}$ (MeV)	$\sigma$ (fm)			
		0.8	1.0	1.2	1.4
I	16.7	838.6	876.9	991.1	1093.6
II	32.8	1320.5	1438.5	1546.7	1674.2
III	66.8	1106.0	1300.2	1517.5	1546.7

**Table 2.** Parameters of the potential.

$v_0$ (MeV)	590	432	448	495
$c$ (fm)	1.34	1.12	1.03	0.99
$r_0$ (fm)	1.2	1.2	1.2	1.2
$\sigma$ (fm)	0.8	1.0	1.2	1.4
BE (MeV)	–128.4	–127.6	–127.5	–126.8
$R$ (fm)	2.73	2.73	2.73	2.73

It must be mentioned here that in our earlier detailed analysis [5] it was shown that the results were indeed sensitive to the range  $r_0$  of the potential and the best agreement could be obtained for  $r_0 = 1.2$  fm.

### 3. Connection between various N–N potentials

In order to make a meaningful comparison of various N–N potentials we extract the zeroth multiple ( $I_0 = 0$ ) of the Talmi integral [6]

$$I_p = \left(\frac{1}{b^2}\right)^{p+(3/2)} \frac{2^{p+2}}{\sqrt{\pi}(2p+1)!!} \int_0^\infty r^{2p} \exp\left[\frac{-r^2}{b^2}\right] V(r)r^2 dr. \quad (3)$$

for various potentials  $V(r)$ . The potentials we have considered are from the following categories.

Sl. No	Potential	$I_0$ (MeV)
I	<i>Realistic potentials</i>	
	a) Reid potential [7]	82.9
	b) Kallio and Koltveit [8]	82.2
	c) Shakin and Waghmare [1]	82.0
II	<i>Effective Interactions</i>	
	d) Waghmare [9]	86.4
	e) Goldhammer [10]	48.2
III	<i>Fusion N–N potentials</i>	
	f) Godre and Waghmare [3] } g) Present calculations }	80.0

The parameters of the potentials are given in the references quoted against each potential.

### 4. Conclusions

In the calculations of realistic interactions we have included terms up to second order, including contributions due to tensor forces. It is interesting to note that while the value of  $I_0$  for the first two categories range between  $I_0 \cong -70$  MeV to  $I_0 \cong -80$  MeV, similar

value is obtained in the third case for the value of the extension of the wave packet of about  $\sigma \cong 1.6$  fm. This is also consistent with the following argument. For getting the  $^{16}\text{O}$  radius we have the expression:

$$R = \langle r^2 \rangle^{1/2} = \left[ 3\sigma^2 + \frac{1}{A^2} \sum_{i=1}^A \langle r_i \rangle^2 \right]^{1/2}. \quad (4)$$

For point particles  $\sigma = 0$  and we have the usual relation

$$R = \frac{1}{A} \left[ \sum_{i=1}^A \langle r_i \rangle^2 \right]^{1/2}. \quad (5)$$

If  $\sigma$  gives  $R$ , as in the wave packet description, then  $\langle r_i \rangle^2$  is zero and we obtain

$$R = \sqrt{3}\sigma \quad \text{or} \quad \sigma \cong 1.6 \text{ fm}. \quad (6)$$

Thus, this analysis shows that there is indeed a consistency between the varieties of N–N interactions derived from various scattering data as well as the ones derived from spectroscopic analysis.

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