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# Theory of sheath in a collisional multi-component plasma

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**Abstract.** The aim of this brief report is to study the behaviour of sheath structure in a multi-component plasma with dust-neutral collisions. The plasma consists of electrons, ions, micron size negatively charged dust particles and neutrals. The sheath-edge potential and sheath width are calculated for collisionally dominated sheath. Comparison of collisionless and collisionally dominated sheath are made.

**Keywords.** Multi-component; collisional sheath-width; collisional sheath-edge potential.

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For many decades the behaviour of unmagnetized two component plasma near a solid wall has been qualitatively studied [1,2]. In particular, it is known that the electrostatic potential at the wall remain negative with respect to the plasma and that the potential drop occurs mainly in a very narrow region adjacent to the wall. And near the wall a potential barrier (the Debye sheath) appears, electrostatically reflecting the electrons. Furthermore, the average speed of the plasma ions entering the sheath is at least of acoustic speed (the Bohm criterion).

There is more interest recently in low-temperature plasmas containing negatively charged micron-size dust particles [3–7]. Such plasmas are often found near solid objects or walls, such as the planetary rings and comets, artificial satellites, the workpiece in plasma assisted material and chemical processing. It is thus of practical interest to investigate the interaction between a dusty plasma and a solid boundary.

It has been mentioned that dust strongly influences all parameters of the sheath, in particular, the electric field distributions and the ion flow velocities. The electric field of dust particles contributes to the distribution of the electric potential in the sheath and influence the ion flow, at the same time, the dust grain fields themselves strongly depending on the flow.

In the absence of dust, there are two regions in the near wall plasma; the plasma sheath itself, where main drop of electric field potential occurs, and the presheath, where the potential drop is small. In the presence of dust, there are three distinguished layers: the plasma boundary presheath (containing no dust), the dust cloud (there is main drop of the electric field potential) and the wall plasma layer (containing no dust) [8,9].

Mikikian *et al* [10] have reported the basic characteristics of dust particles trapped in the sheath of a plate embedded in a plasma produced by emissive hot filaments. For different plate biases they measured the sheath potential profiles and then determined the dust charge. When several dust particles levitate, a sheath steepening is observed.

In two-component plasmas several authors have recently considered the effect of ions collisionality on the sheath. Jurgensen and Shaqfeh [11] developed a kinetic model for ions suffering charge exchange collision. Godyak and Sternberg [12] presented a fluid model where the ions experience a collisional drag. Sheridan and Goree [13] studied the amount of collisionality needed to cause the transition from the collisionless to the collisionally dominated regime.

However to the best of our knowledge, the effect of collisionality on the sheath formation in case of dusty plasma has not been studied until now.

The aim of this brief report is to study the effect of collisionality in a dusty plasma. We have considered here the near wall region of an unmagnetized dusty plasma which consists of electrons, ions, micron size dust particles and neutral particles. Since the dust particles are much heavier than both the electrons and ions, the latter can be assumed to in thermal equilibrium with dusts as a cold fluid. The neutrals are taken as immobile. Here, since we have used fluid theory for dusts, we can ignore the variations in shape, size, and charge separations among the individual dust particles. This is so because these variations are sufficiently small and that they cannot be distinguished on the scale of the fluid element. Moreover, though the dusts are massive with respect to the ions and electrons, due to their inertia and positive ion-dust interactions, the dusts will possess a drift velocity.

Since electrons and ions are Boltzmannian they will move faster than the dusts towards the wall and will recombine at the wall. Electrons being more mobile, they are lost faster at the wall leaving the plasma at positive potential with respect to the wall. As a result, this negative potential will repel the negatively charged dusts which are moving towards the wall. Due to this repulsion, the dust density will become more at the sheath plasma boundary; because, inertial dusts moving from the bulk plasma towards the wall and the dusts repelled by the wall potential will be accumulated in that region. Due to the higher density of dust particles, the inter-particle distances between the dust and the neutrals become less than their mean free path. As a result, dust will collide with the immobile neutrals at the sheath region. This dust-neutral collisions change the sheath behaviour of the plasma widely. These studies are the main task of this brief report. For convenience, a steady state (i.e. $\partial/\partial t = 0$) plasma we assume here as one dimensional.

The electron and ion distributions are given by

$$n_e = n_{e0} \exp\left(\frac{e\phi}{KT_e}\right) \tag{1}$$

and

$$n_i = n_{i0} \exp\left(\frac{-e\phi}{KT_i}\right), \tag{2}$$

where $n_{e0}, n_{i0}$ are the densities of the electrons and ions respectively at the sheath edge, i.e. at $x = 0$, where $\phi$, the electrostatic potential is taken to be zero. Here, $e$ is the electronic charge and $T_e, T_i$ are the electron and ion temperatures respectively.

The cold dust fluid obey the source free, steady state of continuity equation

$$n_d v_d = n_{d0} v_{d0} \tag{3}$$

and momentum transfer equation

$$m_d(v_d\nabla)v_d = Z_d e\nabla\phi - F_c, \tag{4}$$

where $m_d$, $n_d$, $v_d$ and $Z_d$ are respectively the dust particle mass, density, velocity and charge number and $n_{d0}$, $v_{d0}$ are dust density and velocity at the sheath edge respectively.

As the dust fluid travels towards the sheath region (due to its inertial motion and $+$ve ion-dust interactions), the dust velocity increases gradually and then, as soon as the dusts reach the sheath-plasma boundary, they will be repelled by the negative potential of the wall. This causes the dust density to increase at the sheath-plasma boundary and as a result, the inter-particle distances between the dusts and neutrals becomes less than their mean free path. The collisional effects between the dust and the neutrals are introduced. We use the collisional force term $F_c$ in eq. (4), which is given by [14]

$$F_c = m_d n_n {v_d}^2\sigma, \tag{5}$$

where $\sigma = \sigma_s g(v_d)$, $\sigma_s = \pi {r_d}^2$ is the collisional cross-section for collisions between the dusts and neutrals , $n_n$ is the neutral gas density and $v_d$ is the relative dust velocity. The function $g(v_d)$ is suitably selected.

Elastic and charge-exchange collisions contribute to this cross-section $\sigma$, which depends on the dust's relative velocity $v_d$. Poisson equation now becomes [15]

$$\nabla^2\phi = -4\pi e(n_i - n_e - Z_d n_d). \tag{6}$$

To complete the model we must specify the dependence of the cross-section on dust energy. We assume that it has a power law dependence on the dust speed of the form

$$\sigma(v_d) = \sigma_s\left[\frac{v_d}{c_d}\right]^\gamma = \sigma_s g(v_d), \tag{7}$$

where $c_d = (KT_e/m_d)^{1/2}$ is the dust acoustic speed and $\gamma$ is a dimensionless parameter ranging from 0 to $-1$. This power law scaling contains the two special cases:
(1) mobility limited case (constant dust mobility) [16], $\gamma = -1$ and
(2) constant dust mean free path (constant cross-section) [17], $\gamma = 0$.

For strong dust-neutral collisions the movements of dusts are mobility limited. Therefore, here we are interested only about the constant dust mobility case (i.e. $\gamma = -1$). Hence, we do not consider the case of constant dust mean free path (i.e. $\gamma = 0$).

Combining eqs (1) to (7) we find two coupled, differential equations describing the sheath structure as

$$v_d\frac{\mathrm{d}v_d}{\mathrm{d}x} = \frac{Z_d e}{m_d}\frac{\mathrm{d}\phi}{\mathrm{d}x} - n_n\sigma_s\frac{v_d^{2+\gamma}}{c_d^\gamma} \tag{8}$$

and

$$\nabla^2\phi = -4\pi e n_{e0}\left[\delta\exp\left(\frac{-e\phi}{KT_e}\theta\right) - \exp\left(\frac{e\phi}{KT_e}\right) - (\delta-1)\frac{v_{d0}}{v_d}\right], \tag{9}$$

where $\theta = \frac{T_e}{T_i}$, $\delta = \frac{n_{i0}}{n_{e0}}$, $Z_d\frac{n_{d0}}{n_{e0}} = (\delta-1)$.

Now, normalizing the governing equations by an appropriate choice of variables like, the electric potential $\phi$ is scaled by the electron temperature, $x$ is scaled by Debye length $\lambda_{\mathrm{D}}$ and the dust velocity $v_d$ by dust acoustic speed i.e. $\eta = \frac{-e\phi}{KT_e}$, $\xi = \frac{x}{\lambda_{\mathrm{D}}}$, $u = \frac{v_d}{c_d}$.

Let the dimensionless sheath width be $d = \frac{D}{\lambda_{\mathrm{D}}}$, which means that the wall is at $x = D$ and let the dimensionless entry velocity (i.e. the Mach number) be $M = u_0 = \frac{v_{d0}}{c_d}$.

The degree of collisionality in the sheath is parameterized by $\alpha$, which is given by the number of collisions in a Debye length where $\alpha = \frac{\lambda_{\mathrm{D}}}{\lambda_i} = \lambda_{\mathrm{D}} n_n \sigma_s$. Here $\lambda_i$ is the mean free path of the dust particles. The collisionless case (i.e. $\alpha = 0$), is the limit of zero gas density. If the gas density is high enough, or Debye length is short enough, so that the dust mean free path is one Debye length $\lambda_{\mathrm{D}}$, then $\alpha = 1$.

The average number of collisions in the sheath is given by $\frac{D}{\lambda_i} = \frac{D}{\lambda_{\mathrm{D}}}\frac{\lambda_{\mathrm{D}}}{\lambda_i} = \mathrm{d}\alpha$. Now, substituting the dimensionless variables in eqs (8) and (9) we get

$$uu' = -Z_d \eta' - \alpha u^{2+\gamma} \tag{10}$$

and

$$\eta'' = \delta \exp(\theta\eta) - \exp(-\eta) + (1-\delta)\frac{u_0}{u}, \tag{11}$$

where the prime denotes differentiation with respect to the spatial co-ordinate $\xi$ and hence $\eta'$ is the dimensionless electric field. Here, eq. (10) represents the conservation of dust momentum and (11) is the Poisson equation. These two equations, together with appropriate boundary conditions, provide the description of the collisional sheath which is the main concern of this paper.

To solve these equations boundary conditions must be specified. At the wall $\xi = d$, $\eta(d) = \eta_w$. At the sheath plasma boundary $\xi = 0$, the boundary conditions are $\eta(0) = 0$, $\eta'(0) = 0$ and $u(0) = u_0$. In fact, these conditions are only an approximation to the conditions that actually hold at the sheath plasma interface. (In fact, the location of the sheath-plasma boundary is not well-defined). To find the correct boundary conditions, it would be necessary to include source terms and solve the entire discharge problem self-consistently.

In this section we derive expressions that give the potential profile and the thickness for the collisionally dominated sheath. In the limit of strong dust neutral collisions (i.e. the case of mobility-limited dust motions) the collision parameter $\alpha$ is large . The equation of motion (10) is simplified by neglecting the convective term on the left hand side. The equation thus becomes

$$\eta' = -\left(\frac{\alpha}{Z_d}\right) u^{2+\gamma} \tag{12}$$

which relates dust velocity to the electric field.

Now, by using eq. (12) in the Poisson equation (11) and neglecting the electron and ion terms, i.e. $\exp(-\eta)$ and $\delta\exp(\eta\theta)$ respectively we arrive at a power law solution for $\eta$ as follows:

$$\eta = \frac{3+\gamma}{5+2\gamma}\left[\frac{3+\gamma}{2+\gamma}u_0\,(1-\delta)\right]^{\frac{2+\gamma}{3+\gamma}} \left(\frac{-\alpha}{Z_d}\right)^{\frac{1}{3+\gamma}} \xi^{\frac{5+2\gamma}{3+\gamma}}. \tag{13}$$

The sheath thickness is obtained by putting the boundary condition $\eta(d) = \eta_w$ as

$$d = \left(\frac{5+2\gamma}{3+\gamma}\right)^{\frac{3+\gamma}{5+2\gamma}} \left[\frac{3+\gamma}{2+\gamma}u_0(1-\delta)\right]^{-\left(\frac{2+\gamma}{5+2\gamma}\right)} \left(\frac{-\alpha}{Z_d}\right)^{\frac{-1}{5+2\gamma}} \eta_w^{\frac{3+\gamma}{5+2\gamma}}. \tag{14}$$

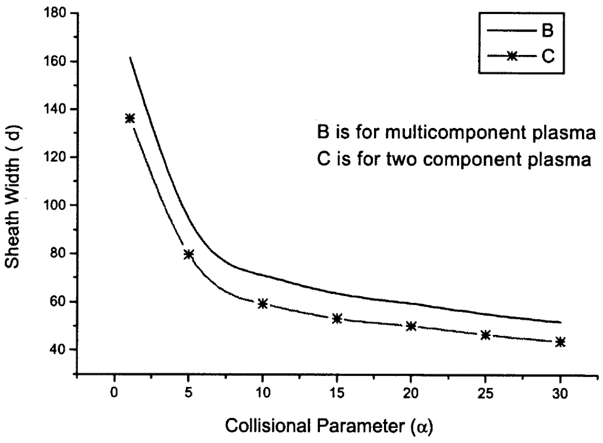

**Figure 1.** Graphical representation of the sheath-width with collisional parameter.

The electric potential $\eta$ varies not only with $\xi$ and $\alpha$, but also with the energy dependence of the cross-section, characterized by $\gamma$.

In the mobility limited case (i.e. $\gamma = -1$) of dusts, eqs (13) and (14) simplify to give

$$\eta = 3^{-1}2^{3/2}u_0^{1/2}\alpha^{1/2}\left(\frac{\delta-1}{Z_d}\right)^{1/2}\xi^{3/2} \tag{15}$$

and

$$d = 3^{2/3}2^{-1}u_0^{-1/3}\alpha^{-1/3}{\eta_w}^{2/3}\left(\frac{Z_d}{\delta-1}\right)^{1/3}. \tag{16}$$

From eq. (16) we find that, sheath thickness $d$ decreases with increasing collisionality $\alpha$.

In absence of dust i.e. in case of a two component plasma the sheath thickness for constant ion-mobility case is obtained [12] as

$$d = 3^{2/3}2^{-1}u_0^{-1/3}\alpha^{-1/3}{\eta_w}^{2/3}. \tag{17}$$

Now comparison of (16) and (17) shows that the collisional sheath thickness is more in case of a dusty plasma with respect to the two component plasma because of the factor $(Z_d/(\delta-1))^{1/3}$. But in both the cases the sheath thickness decreases asymptotically (figure 1).

In this brief report, we have presented a fluid model to study the sheath structure in a collisional dusty plasma which includes a power law dependence of the dust collision cross-section on energy. Here, we have considered the case of strong dust-neutral collisions. Hence, the special case of this power law dependence include constant dust mobility (i.e. $\gamma = -1$). Approximate solutions of this model appropriate for the collisionally dominated sheath are derived. Here, a comparison of sheath width is made for a two component plasma and for a dusty (or multicomponent) plasma. It is found that the sheath width for a two component plasma is less than the collisional sheath width in a multicomponent plasma with dust-neutral collisions. In particular, we derived expressions for potential profile and sheath width.

Our model can be applied to study the potential profile in various material plasma processing techniques where negatively charged dust particles are usually found to be present.

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