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Workshop IV – Cosmology-theoretical models/alternative scenarios: A report

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Abstract. Due to its subject matter, this workshop included a number of rather disjointed contributions in a number of areas, including exact solutions, mathematical cosmology and alternative theories. We shall therefore give a brief summary of each talk in this section in alphabetical order.

K S Govinder presented a talk entitled 'Causal solutions for radiating stellar collapse'. In a recent joint publication with Maharaj and Maartens [1], the speaker had put forward a model of radiating stellar collapse with a shear-free and non-accelerating interior matched to a Vaidya exterior. This simplified model allows some insight into the temperature profile of the star during its evolution. The only exact causal solution for the temperature presented in this work was in the case of constant mean collision time ($\sigma = 0$) (with perturbative results for small σ). The speaker reported extensions to these previous results by presenting two new exact causal solutions for the temperature equation and discussed the resulting temperature profiles of the collapsing star as well as providing similar results for the case of non-zero acceleration. In addition he presented non-constant mean collision time temperature solutions for the most general shearing line element with spherical symmetry. The speaker argued that this provides an indication that the temperature evolution of a collapsing star can *always* be obtained for both constant and non-constant mean collision time, regardless of the complexity/generality of the metric.

Reference

[1] M Govinder, S D Maharaj and R Maartens, *Class. Quantum Gravit.* **15**, (1998)

The talk entitled 'Kalb-Ramond field as source of cosmic polarization anisotropy' by Sayan Kar, Parthasarathi Majumdar and Soumitra Sengupta was based on a follow up of an earlier proposal of two of the said authors (PM and SS) on coupling of torsion to Maxwell electromagnetism within the Einstein cartan framework. Here the torsion is generated by massless Kalb–Ramond field with its field strength augmented by U(1) Chern–Simons terms in accordance with requirements of supersymmetry and anomaly cancellation in perturbative heterotic string theory compactified to 4 dimensions. Here the pseudoscalar axion field, whose derivative is dual to the Kalb–Ramond field strength, is argued to lead to the possibility of circular birefringence in synchrotron radiation from cosmologically distant radio sources for flat as well as an isotropic background space time with flat three space. The Bianchi identity of the Kalb–Ramond field is shown to lead to exactly computable phase difference between the two states of circular polarization of the observed radiation, when the contribution due to Chern–Simons terms are ignored, and the axion field is assumed to be spatially uniform. The authors show that the space time with torsion exhibit optical activity. This is in qualitative agreement with observational data from a large number of radio sources displaying such polarization asymmetry (after eliminating effects of Faraday rotation due to galactic magnetized plasma), recently examined by Jain and Ralston, following up an earlier analysis by Nodland and Ralston. The presence of Kalb–Ramond field with a Chern–Simons extension signals the possibility of connecting this theory with heterotic string theory.

S D Maharaj presented a talk entitled 'Some Einstein–Maxwell models with spherical symmetry', in which he reported on a study of the gravitational behaviour of charged spherically symmetric fluids in general relativity [1]. This involved the analysis of the coupled Einstein–Maxwell system and was organized in two parts. In the first part the simpler shear-free line element

$$\mathrm{d}s^2 = -\mathrm{e}^{2\nu(t,r)}\mathrm{d}t^2 + \mathrm{e}^{2\lambda(t,r)}\left[\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\Phi^2\right)\right]$$

was considered in spherically symmetric models. In this case the solution of the Einstein–Maxwell system reduces to the differential equation

$$y'' = f(x)y^2 + g(x)y^3,$$

for a particular choice of the electromagnetic gauge potential. An analysis was given of this equation as well as a first integral, which reduces to that found by Stephani [3] and Srivastava [2] in the uncharged limit.

In the second part, the general shearing line element in spherically symmetric models was considered. In this case the Einstein–Maxwell system is simplified by introducing new variables, the particular forms of which are motivated from the Lie analysis of differential equations. Also presented were two classes of shearing charged solutions. In the first class the evolution of the gravitational field is reduced to the solution of a Riccati equation. In the second class the behaviour of the gravitational field is governed by a second order nonlinear equation, the general solution of which was given in terms of elementary functions by raising the order [4].

References

[1] S D Maharaj, P G L Leach and R Maartens, *Gen. Relativ. Gravit.* **28**, 35 (1996)
[2] D C Srivastava, *Class. Quantum Gravit.* **4**, 1093 (1987)
[3] H Stephani, *J. Phys.* **A16**, 3529 (1983)
[4] S D Maharaj, in preparation (2000)

F Nasseri presented a talk entitled 'A model Universe with variable space dimension: its dynamics and wave function'. The central theme of the talk was the possibility that physical 'constants' may in fact turn out to be variables.

The speaker started from a recent proposal by herself and Mansouri [1] that the number of spatial dimensions in the universe may be a variable and considered a cosmological model with variable spatial dimensions (and a single time dimension). According to this scenario, the spatial dimension of the universe varies with the expansion of the universe and is supposed to be reduced from a large value (bounded from above by observational data to be 25) at the Planck epoch to its value of 3 (or a possible fractal dimension of about 2.5) at the present time. The authors consider this scenario by employing a Lagrangian formulation of a toy universe model. They obtain the corresponding field equations and demonstrate that such a generalization of the FRW cosmology is not unique. They consider the deviations of such a model from the standard FRW model as the spatial dimension is varied. They do this within the framework of quantum cosmology, by considering the corresponding Wheeler–De Witt equation for these models. Using the appropriate boundary conditions and the semiclassical approximation, they calculate the wave function of this model universe in the limit of $C \to +\infty$, corresponding to the case of constant space dimensions, and show that their wave function does not have a unique behaviour. It can lead to either of the following possibilities: the Hartle–Hawking wave function, a modified Linde wave function or to a more general one, but not to that of Vilenkin. They also calculate the probability density in their model universe, which they find is always more than the probability density of the de Sitter minisuperspace in 3-space as suggested by Vilenkin, Linde and others. They show that in the limit of constant space dimensions, the probability density of this model universe approaches that of the Vilenkin and Linde probability density, being $\exp(-2|S_E|)$, where S_E is the Euclidean action. They argue that this is an indication that the Vilenkin wave function is not stable with respect to the variation in space dimension.

Reference

[1] R Mansouri and F Nasseri, *Phys. Rev.* **D60**, 123512 (1999)

S M Wagh presented a paper entitled 'Inhomogeneous low density Universe with heat flow'. In this work he considered an exact solution to the Einstein field equations representing a spherically symmetric, inhomogeneous shear free Universe admitting heat flow. The imperfect fluid has acceleration nonvanishing and satisfies a barotropic equation of state. The author argues that any viable cosmological model should incorporate the density inhomogeneity and the heat flow simultaneously. He uses the FLRW scale factor, but the fluid considered is imperfect because it admits heat flow. The energy momentum tensor used is of the form

$$T_{ij} = (\rho + p)v_i v_j + b g_{ij} + q_i v_j + q_j v_i$$

and the heat flow is in the radial direction. The existence of the heat flow leads to the resolution of the entropy problem with the increase of the entropy density monotonically with time. The present Universe has large entropy because it is old. The solution presented has an arbitrary radial density distribution. The cosmological constant problem according to the author can also be resolved within the context of inhomogeneous cosmology.

It is now believed that the distribution of galaxies all around may contain voids of low density region. In the paper 'Collapse of a model of the void in Robertson Walker Universe with nonzero spatial curvature' S Banerji along with his coworkers M K Ray and S Chaudhuri generalizes their previous work [1] using FRW Universe with zero spatial curvature. The present model is of spherical void (or its precursor) with a heat conducting fluid having density less than the average density in the region I. The metric in this region was previously given by Maiti [2]. This is surrounded by a spherical shell filled with radiation (region II) satisfying Vaidya metric. The combination of the above two regions is embedded in the Robertson Walker Universe (region III) with nonzero spatial curvature. This region is filled up with a perfect fluid having a barotropic equation of state. Matching the first and the second fundamental forms at the boundaries of the regions and allowing time to be future directed in all regions, we find that the void appears to collapse as viewed by the observer in region III situated a little away from the boundary for all values of spatial curvature k. The rate of collapse is the fastest for $k = +1$, moderate for $k = 0$ and the slowest for $k = -1$.

References

[1] D R Mondal and S Banerji, *Pramana – J. Phys.* **51**, 673 (1998)
[2] S R Maiti, *Phys. Rev.* **D25**, 2518 (1982)

B C Paul presented a talk entitled 'Creation of an open inflationary universe in quantum cosmology' based on joint work with S Mukherjee.

The starting point of this work is an instanton solution in the framework of quantum cosmology which describes the creation of an open inflationary universe, recently obtained by Hawking and Turok. This instanton solution is singular with a finite corresponding action. Given the interest that this work has provoked, the authors ask whether this singular solution is generic, as implied by Hawking and Turok. They do this by considering its occurrence in the setting of quadratic Lagrangian theories with a cosmological constant, described by $L = R + \alpha R^2 - 2\Lambda$. They obtain a Hawking–Turok instanton solution in these theories by writing these theories as a scalar field minimally coupled to Einstein gravity by a suitable conformal transformation, using the 'equivalence theorem'. They obtain both singular and non-singular instantons for these theories and evaluate the probability of creation of an open inflationary universe in the context of quadratic theories. They show that the presence of instanton solutions require constraints on the allowed ranges of parameters. For example, they find that for the quadratic theories, the Hawking–Turok instantons only exist in the parameter ranges given by $\alpha < 0, \Lambda = f(\alpha) > 0$ and $8|\alpha| f(\alpha) < 1$. The authors conclude that in this sense the Hawking–Turok instanton solutions are not always generic in all parameter regimes in quadratic theories.

M V Safnova in the paper 'Gravity balls in induced gravity model – Gravitational lense effects' (coauthor, D Lohiya) presented a nontopological solitonic solution in nonminimally coupled effective gravity theories. A typical solution represents a spherical region with $G(\text{eff}) = 0$ outside and having the canonical Newtonian value inside. Such a spherical domain, which may be said to be a gravity ball characterized by an effective index of refraction which causes bending of light incident on it. It so acts as a spherical lense. The gravity ball is assumed to be of a size of a typical cluster of galaxies and it is shown that even empty gravity ball can produce arc like images of the background source galaxy. There may be distortions of the field, however, if the backgound has random galaxy field. Constraints on the size of the large gravity ball are also obtained from the existing observations of clusters with arcs.

The paper 'Classical and quantum cosmology with R^2 gravity' written by A K Sanyal and B Modak was presented by Sanyal. The authors refer to the work of Horowitz [1] to show that the field equations for R^2 gravity can be arrived at from the action, which is written in the canonical form by introducing a new variable such as the negative of the first derivative of the action with respect to the highest existing derivative of the field variable. For FRW metric it is $Q = -\mathrm{d}S/\mathrm{d}p''$, where p stands for log a, a being the scale factor. When the canonical quantization is performed with the basic variables p and p' instead of Q, one finds that the Wheeler–deWitt equation looks very similar to the Schrödinger equation. Pollock [2] extended this technique to ($R + \beta R^2$) gravity and induced gravity theory. Wheeler–deWitt equation in these cases turns out to be the Schrödinger like equation. In the present paper the authors introduce slightly different choices of the canonical variables leading, of course, to the semiclassical field equations. The quantum mechanical counterpart that is the Wheeler–deWitt equation is, however, quite different from that found originally by Horowitz although the equation appears to be similar to Schrödinger equation. The Wheeler–deWitt equation in these choices yields an effective potential and at the weak energy limit the Hamiltonian is mostly dominated by the effective potential. The extremum of this potential yields classical Einstein's equations, which should be a desirable feature of higher derivative gravity theory.

References

[1] G T Horowitz, *Phys. Rev.* **D31**, 1169 (1985)
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