PRAMANA
— journal of
physics

Vol. 55, No. 4
October 2000
pp. 591–598

Workshop II – Black holes and compact objects: Quantum aspects

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Abstract. This is a summary of the papers presented in session W2 on a fairly wide-ranging variety of topics in the area of black hole physics and quantum aspects of gravity, including quantum field and string theory in curved spacetimes. In addition, experts in a couple of topical subjects were invited to present short surveys on the subjects of their specialization. The invited speakers were: Mitra, who surveys recent research on the very topical area of AdS black holes, and Date, who presents a comparative perspective on trapping and isolated horizons. Among the contributed papers, the first, by Jassal, is an attempt to understand the dynamics of strings near a black hole horizon. This is followed by a paper by Barve *et al* on a calculation of the quantum stress tensor for a background that includes a naked singularity. Following this we have Singh on radial oscillations of quark stars in strong magnetic fields. The next paper by Goyal and Dahiya, discusses chiral symmetry restoration in a linear sigma model in the presence of a magnetic field. The following paper, by Horwitz, offers new perspectives on the intriguing question of primordial black holes. Finally, Madhavan discusses issues pertaining to the classical limit of kinematical quantum gravity.

Asymptotic anti-de Sitter black holes

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The most familiar black hole spacetimes are asymptotically flat and satisfy Einstein's equations without a cosmological constant. However, asymptotically anti-de Sitter versions of these black holes also exist, corresponding to a negative cosmological constant. The possibility of transitions between asymptotically anti-de Sitter Schwarzschild black holes and pure anti-de Sitter spacetime has been known for some time. Recently there has been a lot of interest in charged black holes with similar asymptotic behaviour. In the following we briefly discuss their

For the asymptotically anti-de Sitter Schwarzschild black hole, given by the metric

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r} + \frac{r^2}{l^2}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\left(1 - \frac{2M}{r} + \frac{r^2}{l^2}\right)} + r^2\mathrm{d}\Omega^2, \tag{1}$$

the on-shell Euclidean action diverges in infinite space because the curvature is constant. One may subtract out the contribution due to pure anti-de Sitter space and obtain a finite result [1] in the infinite volume limit. The euclidean anti-de Sitter spacetime has to be defined with the periodicity of time (i.e., the temperature) determined by the black hole. The temperatures are set to be equal at the boundary of space (which is taken to infinity).

Hawking and Page [1] considered the competition in free energy or action between the black hole and anti-de Sitter space. At $T < T_{\rm min} = \frac{\sqrt{3}}{2\pi l}$, a black hole of this kind cannot be defined, so that only the anti-de Sitter space is allowed. At $T > T_{\rm min}$, there can be two black holes of different sizes. The smaller black hole always has positive action compared to anti-de Sitter space. For this branch or phase the specific heat is negative. Thus such black holes are thermodynamically unstable. The bigger black hole has positive specific heat and lower action and it is this which should be compared with the anti-de Sitter space. This bigger black hole is stable if $T > T_c = \frac{1}{l\pi} > T_{\rm min}$. Thus, between $T_{\rm min}$ and T_c the anti-de Sitter space is still the thermodynamically stable phase, but a change takes place at T_c.

The Reissner–Nordström black hole solution of the Einstein–Maxwell equations in free space with a negative cosmological constant $\Lambda = -\frac{3}{l^2}$ is given by

$$\mathrm{d}s^2 = -h\mathrm{d}t^2 + h^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2,\ A = \frac{Q}{r}\mathrm{d}t, \tag{2}$$

with

$$h = 1 - \frac{r_+}{r} - \frac{r_+^3}{l^2 r} - \frac{Q^2}{r_+ r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}. \tag{3}$$

In general r_+, Q are independent, but in the extremal case they get related:

$$1 - \frac{Q^2}{r_+^2} + \frac{3r_+^2}{l^2} = 0. \tag{4}$$

Unlike the Schwarzschild case, where there are two black hole phases, three phases occur for Reissner–Nordström black holes. Transitions between these phases have been discussed in the literature [2]. The analogue of the main Hawking–Page transition however is a transition between a charged black hole and the charged analogue of anti-de Sitter space, which is a charged extremal black hole quantized after extremalization [3], and thus with arbitrary temperature [4].

A study of the action for off-shell configurations near the black hole solution – for simplicity only in a class of spherically symmetric metrics [5] – was made for comparison of the actions of non-extremal and extremal black holes:

$$I_{\text{non-ex}} - I_{\rm ex} = \frac{\pi(r_+ - r_0)}{1 - \frac{q^2}{r_+^2} + \frac{3r_+^2}{l^2}}\left(r_+ - 3r_0 - \frac{r_+^3 + r_+^2 r_0 + r_+ r_0^2 + 9r_0^3}{l^2}\right). \tag{5}$$

For a black hole with positive temperature, $r_+ > r_0$, the extremal value, so that the sign of this difference depends on the last factor involving cubics in r_+ and r_0. For large enough $r_0 (> \frac{l}{3})$, non-extremal black holes are stable against decay into the extremal black hole. However, for small r_0, i.e., for small charge, there exists a range of values of r_+ for which the factor is positive, corresponding to the occurrence of non-extremal black holes capable of decaying to extremal black holes [6]. The transition to this small charge behaviour from the large charge behaviour occurs when $r_0 \approx 0.1105l$, corresponding to a charge of $|q_0| \approx 0.1125l < \frac{l}{6}$.

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Trapping horizons and isolated horizons: A comparison

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The interpretation of black holes as thermodynamic systems is based on the following main ingredients:

(i) Classical processes involving stationary black holes in asymptotically flat space times obey the ‘laws of black hole mechanics’ which happen to be formally identifiable with the laws of thermodynamics. (ii) With *quantum mechanical* input (Hawking effect), the formal similarity is strengthened to a literal interpretation. (iii) The suggested relation between the area of a black hole and its entropy, is strengthened by *non-perturbative* quantum gravity effects.

This mechanics-thermodynamics correspondence is really established for event horizons of stationary black holes which happen to be Killing horizons. A natural question is whether a similar correspondence can be established for more general gravitational systems. The first step in such an exploration is to seek generalization of ‘laws of black hole mechanics’ to cover more general systems. Two such generalizations have been proposed, the ‘trapping horizons’ introduced by Hayward [1] a while ago and the recently introduced ‘isolated horizons’ of Ashtekar *et al* [2]. A brief summary of a comparison is attempted.

The ‘trapping horizon’ proposal seeks to exercise all reference to asymptotics and still have a framework obeying a *generalized* version of the usual laws of black hole mechanics. For this purpose a key property that is abstracted from apparent horizons is that trapping horizons are characterized in terms of double null foliations of marginally trapped surfaces. The definitions naturally lead to four different types outer/inner and future/past, black holes being of the future-outer type. For these types, Hayward establishes (a) topology law restricting the topology of the leaves e.g. spherical or planar for outer-future; (b) signature law stipulating the geometry of horizon hypersurface e.g., null/space-like/ for outer-future; (c) for the spherical topology, the area 2-form must be non-decreasing subject to energy conditions (point-wise statement); (d) ‘surface gravity’ defined in a certain way obeys an integral inequality which becomes an equality only for null horizons and finally (e) a generalized first law (also point-wise). While the ideas look very natural, lack of action principle formulation appears to have hampered further progress regarding quantum inputs.

The ‘isolated horizon’ proposal primararily seeks to relax stationarity feature of the usual black holes. A realistic collapse may be expected to pass through phases wherein nothing crosses the inner boundary. This is the regime for which the isolated horizon ideas are applicable. Evidently, in general, one is not referring to an event horizon though these are not excluded. The earlier discussions of ‘non-rotating’ isolated horizons are in a sense special case of the trapping horizon (null hypersurface). These are foliated by marginally trapped surfaces of spherical topology. The definitions of surface gravity, mass, charges etc are different though. The surface gravity is identified as the acceleration of the (non-affinely parametrized) null generators of the horizon with an ambiguity of constant scaling. This is fixed by appealing to known solutions within each class e.g. Schwarzschild for no matter, Reissner–Nordstrom for Maxwell matter etc. The mass is identified, by appealing to an action principle, as the value of the Hamiltonian on the isolated horizon boundary. Dilatonic matter case is also analysed in this framework. For these the zeroth and the first laws are esablished. The idea of isolation excludes the possibility of increasing area. Being explicitly based on an action principle in terms of self dual connection, computation of entropy [3] is also done.

Recently however, the definitions have been further refined [4] and a sub-case of non-rotating, distorted horizons is also analysed. Full details of rotating case are not yet available.

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W2.1 Null strings near a higher dimensional black hole

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The investigation of string propagation in curved spacetime is important with a view to eventually understand and interpret string quantization in curved spacetimes. To obtain four-dimensional spacetime from string theory, the extra dimensions have to be compactified. The mechanism of this compactification is an important issue in string theory, and needs to be studied using a variety of approaches. One way could be to study the dynamics of a string probe near a black hole, i.e., in the strong gravity regime, where these extra dimensions are expected to contribute nontrivially. The problem is complicated as it involves solving equations of motion in D-dimensional spacetime, which includes the compact manifold. As a simplifying approach, string propagation is studied in the background of a five-dimensional Kaluza-Klein black hole, a minimal extension to four-dimensional spacetime. The Kaluza-Klein backgrounds are solutions to the five-dimensional Einstein equations, and include regular four-dimensional black holes which carry, in general, electric and magnetic charge as well as a scalar charge. This study of string propagation in Kaluza-Klein backgrounds receives an additional motivation because it has recently been shown that such 5-dimensional metrics are in fact solutions of the higher-dimensional string theory.

Several approximation schemes exist to solve the highly nonlinear string equations. This paper uses the null string expansion, in which string coordinates are expanded perturbatively with the world-sheet velocity of light c as an expansion parameter. If $c << 1$ (null string limit), the coordinate expansion is suitable to describe strings in a strong gravitational background. This is the case considered here, since the interest is in probing the dynamical behaviour of the extra dimensions.

In the present study, the motion of a null string near a 5-dimensional black hole is investigated. For simplicity, the electrically and magnetically charged cases are considered separately. The zeroth order equations of motion are solved in the region just outside the horizon and in the equatorial plane. Although complicated, the equations for a magnetic black hole can be reduced to quadratures and solved analytically. The solutions can be expressed in terms of elliptical integrals, depending on the relative values of the parameters. In the limit of small scalar charge, the elliptical integrals solve to elementary functions. This is also true for the special case of the Pollard–Gross–Perry–Sorkin magnetic black hole even though the scalar charge is not small. In this case, the string decelerates as the string approaches the horizon. This ‘anti-gravity’ effect of extremal black holes has already been commented on in the literature.

The numerical solutions of the equations of motion show that the extra fifth coordinate increases monotonically in the magnetic case, while in the electric case it first increases and then starts decreasing. The picture is easier to interpret in terms of the Kaluza-Klein radius. The latter decreases as the string probe approaches a magnetic black hole, while it increases in the electrically charged case. However, the true string dynamics are expected to manifest themselves in higher orders: the effect of the background on the string itself is not explicit in the zeroth order. The first-order string equations of motion are set up and analysed with a view to study the effect of the extra dimension on the string as it approaches the horizon. In this case the equations do not reduce to quadratures and have to be solved numerically. Preliminary results indicate changes in the shape of the string which are different in the two cases.

W2.2 Quantum stress tensor in 2d dust collapse

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We have calculated the two dimensional quantum stress tensor for a minimally coupled massless scalar field in self-similar spherical dust collapse [1], in a manner analogous to the calculation of Hiscock *et al* [2], by suppressing the angular co-ordinates. In two dimensions, the trace anomaly permits an exact calculation of the expectation value without having to resort to geometric optics. Once again, the assumption that the spacetime is self-similar is made. Our results are identical to those of Hiscock *et al* – we also find a divergence of the outgoing flux on the Cauchy horizon, and the form of the divergence is same as that for the null dust model.

We have shown further that the quantum stress tensor diverges on the Cauchy horizon, if it exists, in non-self-similar Tolman Bondi dust and null dust collapse in two dimensions [3]. There is no direct way of deducing this by analytical calculations of the expressions in the general case. However, we employ a limiting process of approaching the required spacetime metric via patching up tractable solutions (one of them being the self-similar one). Using this technique, it appears that the divergence results at the outset from the metric rendered non-invertible at the Cauchy horizon in the self-similar portion. This does not seem to be the ultimate reason for the divergence though, for it is seen that the divergence persists even after the limit to the actual metric is taken. Needless to say, the divergence does not seem to be ultimately a result of the self-similar nature although that does help in making the divergence evident in the calculations.

The divergence on the Cauchy horizon suggests that when the back-reaction of the flux on the metric is taken into account, the formation of the naked singularity will be avoided. This is an example of the instability of the Cauchy horizon and a possible way of preserving cosmic censorship.

On the other hand, if naked singularities do persist and the Cauchy horizon does come into existence in spite of a back-reaction calculation, then the divergence suggests that a burst of radiation is likely to occur on the Cauchy horizon. A remarkable feature of the divergence is that it occurs all over the horizon even in the low curvature regions. This may very well serve to identify the Cauchy horizon which is a null ray without any special local geometrical features to an asymptotic observer from the point of view of classical relativity.

It is possible to calculate the quantum stress tensor in case of the covered singularities in the self-similar dust and null dust models as well, and it turns out to be finite. One may argue that the non-self-similar cases also yield finite quantum stress tensors using the technique presented in [3], contrasting with the divergence in the naked cases.

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W2.3 Primordial black holes, new perspectives

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Our subject is a re-examination of the formation of primordial black holes by means of density fluctuations. A common statement has been small black holes formed in whatever are the appropriate numbers by density fluctuations are very hot and thus have very intense radiation and that this can be a problem in terms of observation. We will argue that there are two considerations about this process which have not been seriously considered in previous discussion of the matter. The first is that any process which is not a virtual process must conserve that total energy of the universe gravitational plus matter energies. Since the spontaneous creation of a black hole, via a density fluctuation, occurs in a small finite region and there arises a particle horizon, the rest of the universe outside of which is unaffected by our fluctuation, the energy of our test region must have been zero and remain zero – the energy involved since we are dealing with a thermal state, must be the mean thermal energy. This imposes constraints on the creation of the black holes; we must explore the consequences of remaining on the energy shell. Any other state is a finite lifetime fluctuation and not a real physical state which can decay with Hawking radiation. This has not been considered in the standard treatments. Furthermore, the fluctuations of gravity are slower than the matter; a matter fluctuation which disappears before the gravity responds to create a horizon will not be a black hole at all.

The second consideration is that fluctuations leading to concentrating some appropriate mass within its Schwarzschild radius, even if they satisfy the on the energy shell conditions, to be the physical thermal black hole that we know, it must have undergone conditions of thermalization. Since the entropy of a black hole is much greater than that of the radiation in the volume emptied out to make the black hole, there must have occurred some process whereby the entropy is produced. In our perspective this is effected in a decoherence process which produces the Schwarzschild classical metric coupled to appropriately thermalized radiation. In general, it is clear that this kind of process is needed to explain the appearance of the black hole thermal state. The consequence of the above is that we require either a thermodynamic transition leading to the new thermal metastable state or a dynamic analysis, which leads to the thermalization via a decoherence process. We discuss a number of examples of the above, the formation of a black hole on the energy shell and the transition to a metastable thermal equilibrium black hole in cosmology.

W2.4 Radial oscillations of quark stars in strong magnetic fields

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The eigenfunctions of radial pulsations of quark stars are calculated in a general relativistic formulation given by Chandrasekhar in the density-dependent quark mass model in a strong magnetic field. It is found that the square of the frequencies are always decreasing functions of the central density of the strange star. The maximum mass, the radius and the gravitational redshift of the star increase with the magnetic field.

W2.5 Chiral symmetry restoration in a linear sigma model

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We study the chiral symmetry structure in a linear sigma model with fermions in the presence of an external, uniform magnetic field in the ‘effective potential’ approach at the one loop level. We know that at high temperatures and/or at high densities, the quark condensates melts at some critical point and chiral symmetry is restored.

It has also been suggested that systems with spontaneously broken symmetries may make a transition from broken symmetric to restored symmetric phase in the presence of external fields. Large magnetic fields with a strength up to 10^{18} Gauss have been conceived to exist at the time of supernova collapse inside neutron stars and in other astrophysical compact objects and in the early Universe. Effect of such a strong magnetic field on chiral phase transition is thus of great interest for baryon free quark matter in the early universe and for high density baryon matter in the core of neutron stars. A particularly attractive frame work to study such systems is the linear sigma model. We have considered this as an effective model for low energy phase of QCD and have examined the chiral symmetry properties at finite density and in the presence of external magnetic field.

An elegant and efficient way to study symmetry properties of the vacuum at finite temperature, density and in the presence of external fields is through the 'effective potential' approach. We have calculated the effective potential in the presence of magnetic field and at finite densities for two flavor SU(2) $\times$ SU(2) chiral quark model. We obtain the well known result that the magnetic field enhances chiral symmetry breaking in free baryon density case. However in the presence of high density we find that the magnetic field continues to enhance chiral symmetry breaking at low densities as expected but as the magnetic field is raised the chiral symmetry is restored at a much lower density compared to the free field finite density case.

This would have implications on the dynamics of neutron star evolution where such high densities and magnetic fields may be present. It is likely that in the core of neutron stars the nuclear matter may undergo a transition to deconfined quark matter. Existence of chirally broken quark phase would imply the presence of massive quark matter in the core of such stars. If the core is magnetized, the chiral symmetry will get restored at densities lower than what they would be if no magnetic field were present. This would affect the equation of state and will have astrophysical implications.

W2.6 On the classical limit of kinematical loop gravity

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We analyse the classical limit of kinematic loop quantum gravity in which the diffeomorphism and hamiltonian constraints are ignored. We show that there are no quantum states in which the primary variables of the loop approach, namely the SU(2) holonomies along all possible loops, approximate to their classical counterparts. Instead of the set of all holonomies, we construct a new set of 'macroscopic' operators based on physical lattices specified by the quasi-classical states themselves. We propose that these operators be used to analyse the classical limit. Thus, our aim is to approximate classical data using states in which appropriate macroscopic operators have low quantum fluctuations.

We explicitly construct candidate quasi-classical states in 2 spatial dimensions and indicate how these constructions may generalize to 3d. We discuss the less robust aspects of our proposal with a view towards possible modifications. Finally, we show that our proposal also applies to the diffeomorphism invariant Rovelli model which couples a matter reference system to the Hussain Kuchař model.