

## $B_c$ meson properties and its leptonic radiative decays

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**Abstract.** The properties of the meson  $B_c$  are outlined. The leptonic radiative decays for  $B_c$  meson are presented. An outlook on the studies of the meson is given.

**Keywords.**  $B_c$ -meson; properties; radiative decays.

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### 1. Introduction

According to the terminology of PDG, the meson  $B_c$  is the groundstate of the binding system ( $c\bar{b}$ ). Its components  $c$ -quark and  $\bar{b}$ -quark both are heavy ( $m_c, m_b \gg \Lambda_{\text{QCD}}$ ), so it and heavy quarkonia ( $c\bar{c}$ ) and ( $b\bar{b}$ ) may be called as double heavy mesons. It, being very different from the heavy quarkonia (flavor-hidden), is explicitly flavored. It is observed very recently [1].

The meson  $B_c$  and the known heavy quarkonia ( $c\bar{c}$ ), ( $b\bar{b}$ ) as well are non-relativistic bound states, but the  $B_c$ -meson production is comparatively more difficult than that of the heavy quarkonia [2–6]. It decays through weak interaction only, contrary to the heavy quarkonia may decay through annihilation into gluons (strong interaction) and/or into photons (electromagnetic interaction) and/or through weak decay as well, so the decay feature of the meson  $B_c$  is very different from that of the heavy quarkonia [7–13]. Therefore  $B_c$  is one of a specially interesting mesons due to the properties. We may have a better understanding not only on the production mechanisms by comparative study of the production for the mesons  $B_c$  and the flavor-hidden heavy quarkonia, but also on the decays of the two heavy flavors  $b$  and  $c$ . For instance, having the good properties, such as a sizable branching ratio of various weak decays due to its nature etc, the meson  $B_c$  not only adds to  $B$  mesons for studying the flavor  $b$  and to  $D$  mesons for studying flavor  $c$  but also offers a unique place, where one can study the two heavy flavors  $b$  and  $c$  in a meson simultaneously. Now there is no one who doubt the color-singlet mechanism for double heavy meson production. Whereas the so-called color-octet mechanism still need to be tested, although it has been proposed for years and obtained a few experimental supports.

Besides the reasons mentioned here precisely, the first observation of the meson  $B_c$  [1] starts a new stage for studying the meson  $B_c$  and stimulates more people to be interested in the studies than before.

## 2. The properties of the meson $B_c$

The meson  $B_c$  is a non-relativistic bound state, so its mass can be computed not only by Monte Carlo simulation on the relevant quantity of lattice QCD but also by potential models.

The situation about the mass now is as follows: (i) The Monte Carlo simulation (with ‘quenched approximation’) [15] reaches a result  $m_{B_c} = 6.389(9)(98)(15)$  GeV. (ii) Potential models predict the mass in a region:  $m_{B_c} = 6.1 \sim 6.3$  GeV [16,12,13]. (iii) The experimental value now is  $m_{B_c} = 6.4 \pm 0.39 \pm 0.13$  GeV (with quite big errors) [1]. Considering the experimental errors and the theoretical uncertainties as indicated by the above values themselves, we may conclude that they are consistent with each other at the present beginning stage.

Potential models and lattice QCD can also calculate out the decay constant  $f_{B_c}$ . It is a very important quantity thus we quote it from potential models [16,12,13]:

$$f_{B_c} = 400 \sim 500 \text{ MeV.} \quad (1)$$

Leading order estimates of the production of  $B_c$  meson are available in refs [2–6], which all are based on perturbative QCD (pQCD). In pQCD framework for the estimates the fragmentation functions, which involve all the non-perturbative effects from a ‘parton’ to the meson as in the general cases for production of any one of the other mesons, play a key role. Whereas it was realized first in 1992 that such fragmentation functions of the double heavy mesons, being different from that of a light meson or a heavy meson, can be further factorized out a perturbative part, which can be reliably calculated by pQCD [2].

As an important consequence of the ‘further’ factorization, the ‘survived’ non-perturbative part from the factorization is related to the wave function at origin (the decay constant or say the relevant color-singlet matrix element in NRQCD language) directly, when a color-singlet fragmentation function is concerned, so that the whole fragmentation function is calculable. For the so-called color-octet fragmentation functions the essential difference from those of the color-singlet ones is just that the ‘survived’ non-perturbative part, being in color-octet, is related to a color-octet matrix element, and is not calculable with present skills but may be determined by fitting experimental measurements so far. Being different from that in the cases of the heavy quarkonia  $\eta_c, \eta_b, J/\psi, \Upsilon$  etc (flavor hidden), the color-singlet production of the meson  $B_c$  is always much greater than the color-octet production due to the fact that the color-octet matrix element is much smaller than a color-singlet one, and  $B_c$ -meson being flavored explicitly cannot gain any enhancement factor from pQCD by one or two orders of QCD as in the case of the heavy quarkonia.

There are different approaches to the estimate of the  $B_c$ -production even with pQCD, e.g., the so-called leading fragmentation computations [5] or the lowest order completed computations of pQCD [4] etc, the results are consistent in the ‘overlap’ region where the approaches work well, and generally speaking, the result is  $\Gamma(Z \rightarrow B_c + X)/\Gamma(Z \rightarrow b\bar{b})$  or  $\sigma(hh \rightarrow B_c + X)/\sigma(hh \rightarrow b\bar{b})$  roughly is  $10^{-4} \sim 10^{-3}$  [2–5]. We cannot show the details of the estimation on  $B_c$ -production here due to the page restriction for the proceedings, but instead, we would like to take an example of a fragmentation function for the color-singlet one of the meson  $B_c$  at the energy scale of the  $(c\bar{c})$ -pair threshold (i.e.  $Q^2 = 4m_c^2$ ) [2], so as to have some idea about the general feature of the fragmentation functions:

$$D_b^{B_c}(z, 4m_c^2) \propto \alpha_s^2(4m_c^2) |\psi_{B_c}(0)|^2 \frac{z(1-z)^2}{(a_2z-1)^6} \cdot \{[2a_1z - 3(a_2 - a_1)(1 - a_2z) \cdot (2-z)](1 - a_2z)z + 6(1 + a_1z)^2(1 - a_2z) - 8a_1a_2z^2(1 - z)\}, \quad (2)$$

where  $|\psi_{B_c}(0)|$  is wave function at original;  $\alpha_s^2(4m_c^2)$  is the QCD coupling constant; and  $a_1 = m_c/m_{B_c}$ ,  $a_2 = m_b/m_{B_c}$ . Here we would like to point out that in general there are two components for  $B_c$ -production: (a)  $c$ -quark fragmentation with  $Q^2 \geq 4m_b^2 \gg \Lambda_{\text{QCD}}^2$ , where  $Q^2$  is the momentum squared of the gluon which produces a pair of heavy quark-antiquark  $b\bar{b}$ , and (b)  $b$ -quark fragmentation with  $Q^2 \geq 4m_c^2 \gg \Lambda_{\text{QCD}}^2$ , where the gluon produces a  $c\bar{c}$  pair instead. Except in photon production, of the two, (b) is much greater than (a), that (a) is negligible comparatively. It is one reason why we only take the example  $D_b^{B_c}(z, Q^2 = 4m_c^2)$  in eq. (2).

According to the estimates [2–6] on the production, we may conclude that only in hadronic colliders such as Tevatron and LHC the events of the meson  $B_c$  may be produced copious enough for experimental investigation. The first of observation of  $B_c$  meson happens in CDF [1] is a good confirmation of the conclusion. As foreseeing experimental progress will be made, more accurate theoretical calculations are requested from now on.

As for the decays of the meson  $B_c$ , all of them may be attributed to the components:  $\bar{b}$ -quark decay with  $c$ -quark as a spectator,  $c$ -quark decay with  $\bar{b}$ -quark as a spectator and  $W$ -boson annihilation. If using  $D$ -mesons' and  $B$ -mesons' lifetime as input to estimate the components of  $\bar{b}$ ,  $c$ -quark decays, and with the decay constant  $f_{B_c}$  of eq. (1) to compute the  $W$ -annihilation component, the lifetime (full width) of the meson  $B_c$  is obtained about 0.4 ps [7,9] that is in the region of the measured value

$$\tau_{B_c} = 0.46_{-0.16}^{+0.18}(\text{stat}) \pm 0.063(\text{sys}) \text{ ps.}$$

As for the decays, there are several approaches to computing them in the literature [7,8,10–14]. Here we will not review all of them but recommend one of them [7,14].

In many cases, in the decays of a  $B_c$ -meson into a charmonium ( $c\bar{c}$ ), the decay products obtain a great momentum recoil due to the great mass difference  $\Delta m = m_{B_c} - m_{(c\bar{c})}$ . Furthermore,  $B_c$  and charmonium are double heavy mesons so their wave functions are calculable by potential model. Therefore, considering such great momentum recoils, it is an interesting problem how to calculate the decays and how to guarantee the accuracy for the calculation with the wave functions, because the wave functions are well tested, so trustable. To solve the problem, it was suggested that first of all to calculate the weak-current matrix element by means of the formalism of Mandelstam [17] which is constructed on the Bathe–Salpeter (BS) for bound states. So the weak current matrix element is Lorentz covariant precisely and the recoil effects are managed properly at the beginning. As known, Schrödinger equation can be set on a more solid foundation of quantum field theories by means of establishing a relation to a corresponding BS equation with an instantaneous approximation. Thus by taking a similar instantaneous approximation (the so-called generalized instantaneous approximation [7,14]) onto the whole weak-current matrix element formulated by the Mandelstam formalism, instead onto the BS equation (as the original one), finally as the result, the weak-current matrix element turns a formulation, where the BS wave functions appearing in Mandelstam formulation at beginning turn to the Schrödinger ones properly but with some 'extra' operators being sandwiched [7,14]. Indeed it has been shown that the recoil effects are treated in the suggested approach quite well by comparing with a typical and wide-adopted one, where the recoil effects are taken by intuition and

by hand [10]. Since there is page restriction for the proceedings too, we cannot write the details here, but simply to quote some results into table 1 so as to see the typical feature of the results. CDF group has observed the semileptonic decays of  $B_c \rightarrow J/\psi + e + \nu_e$  and  $B_c \rightarrow J/\psi + \mu + \nu_\mu$  only, and the combination of the decay branching ratio with the hadronic production cross section is compared with theoretical prediction [1]. It indicates that the theoretical prediction is in agreement with measurement within the experimental errors and theoretical uncertainties.

### 3. The leptonic radiative decays

Since there is potential possibility to measure the decay constant  $f_{B_c}$  via the leptonic radiative decays, we would like to outline the estimate of the decays for a while.

The  $W$ -boson annihilation component in the weak decay of the meson  $B_c$  is not CKM suppressed, but the pure leptonic decays  $B_c \rightarrow e + \nu_e$  and  $B_c \rightarrow \mu + \nu_\mu$  are suppressed by chiral symmetry strongly. Namely they are so small that it is hopeless to be observed experimentally. Since  $\tau$ -lepton is quite heavy that the decay  $B_c \rightarrow \tau + \nu_\tau$  is escaped from the strong suppression. Whereas the  $\tau$  mode is very difficult to be identified, especially in a hadronic environment.

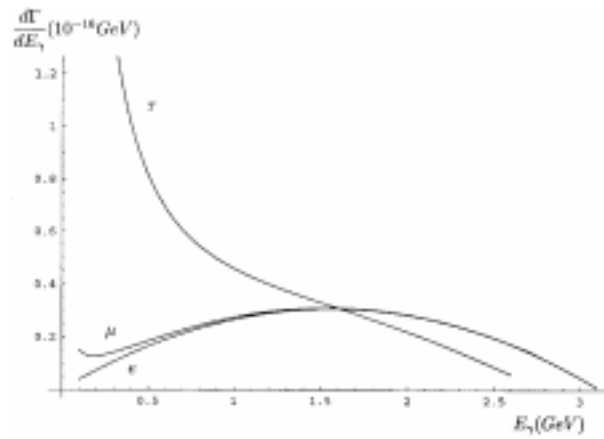
Let us turn to the leptonic radiative decays  $B_c \rightarrow e + \nu_e + \gamma$  and  $B_c \rightarrow \mu + \nu_\mu + \gamma$ . We find that although they are in a higher order of QED, they escape from the chiral suppression that they are much bigger than those of their corresponding pure leptonic ones and are accessible in experiments (especially lepton and photon can be measured directly) [18]. With the ‘extra’ photon, they become comparatively easier to be identified experimentally in the hadronic environments such as Tevatron and LHC. Therefore the study of the radiative decays has been carried out by many authors with various methods [18–20]. In one of them the leptonic radiative decays together with the corresponding pure leptonic decays up-to one-loop order are computed and certain ‘long distance’ effects are estimated

**Table 1.** Exclusive semileptonic decay width (in  $10^{-6}$  eV) for various modes.

Mode	Ref. [7]	Ref. [8]	Ref. [10]	Ref. [13]
$\Gamma(B_c \rightarrow \eta_c + l^+ \nu_l)$	14.2	20.4	10.6	11.1
$\Gamma(B_c \rightarrow J/\psi + l^+ \nu_l)$	34.4	37.3	38.5	30.2
$\Gamma(B_c \rightarrow B_s + l^+ \nu_l)$	26.6	15.2	16.4	14.3
$\Gamma(B_c \rightarrow B_s^* + l^+ \nu_l)$	44.0	45.2	40.9	50.4

**Table 2.** Branching ratios of the ‘whole’ leptonic decays for the different sets of parameters  $(1 - a)$ ,  $(1 - b)$ ,  $(2 - a)$  and  $(2 - b)$ .

	$(1 - a)$	$(2 - a)$	$(1 - b)$	$(2 - b)$
$\text{Br}(B_c \rightarrow e \nu \gamma)$	$5.09 \cdot 10^{-5}$	$5.45 \cdot 10^{-5}$	$4.5 \cdot 10^{-5}$	$4.82 \cdot 10^{-5}$
$\text{Br}(B_c \rightarrow \mu \nu \gamma)$	$10.93 \cdot 10^{-5}$	$10.98 \cdot 10^{-5}$	$9.69 \cdot 10^{-5}$	$9.76 \cdot 10^{-5}$
$\text{Br}(B_c \rightarrow \tau \nu \gamma)$	$1.477 \cdot 10^{-2}$	$1.407 \cdot 10^{-2}$	$1.306 \cdot 10^{-2}$	$1.246 \cdot 10^{-2}$



**Figure 1.** The electron spectroscopy of the leptonic radiative decays of the meson  $B_c$ .

in ref. [20]. It is concluded that the long distance effects is ignorable. Since ref. [20] is a comparative completed calculation on the problem thus here for further experiments as a reference we quote the numerical results of decay branching ratios and the spectroscopy of the photon from it and put them in table 2 and figure 1 respectively.

In the table ‘whole leptonic decays’ means to put pure leptonic decays and its corresponding leptonic radiative decays together. The values of the sets of parameters  $2 - a$ ,  $2 - b$ ,  $2 - a$  and  $2 - b$  in table 2 are different from each other but all reasonable. It is just for seeing the uncertainties and the details can be found in ref. [20], so we will not repeat them here.

#### 4. Outlook

The meson  $B_c$  has been observed just in the semileptonic decay modes  $B_c \rightarrow J/\psi + e + \nu_e$  and  $B_c \rightarrow J/\psi + \mu + \nu_\mu$  only. A new stage for  $B_c$  study is opening. It is expected that more decay modes will be observed and better measurements on the production cross section are available in RUN-II of Tevatron and in LHC soon.

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