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Measuring the chargino parameters

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Abstract. After the supersymmetric particles have been discovered, the priority will be to determine independently the fundamental parameters to reveal the structure of the underlying supersymmetric theory. In my talk I discuss how the chargino sector can be reconstructed completely by measuring the cross-sections with polarized beams at e^+e^- collider experiments: $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ $[i,j=1,2]$. The closure of the two-chargino system can be investigated by analysing sum rules for the production cross-sections.

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1. Introduction

The concept of symmetry between bosons and fermions, supersymmetry (SUSY), has so many attractive features that the supersymmetric extension of the Standard Model is widely considered as a most natural scenario. However, if realized in Nature, supersymmetry must be broken at low energy since no superpartners of ordinary particles have been observed so far. Technically it is achieved by introducing the soft-supersymmetry breaking parameters: gaugino masses M_i, sfermion masses $m_{\tilde{f}}$ and trilinear couplings A^f (gauge group and generation indices are understood).

Enlarging the spectrum of physical states together with the necessity of including the SUSY breaking terms gives rise to a large number of parameters. Even in the minimal supersymmetric model (MSSM) more than 100 new parameters are introduced! This number of parameters, reflecting our ignorance of SUSY breaking mechanism, can be reduced by additional physical assumptions. The most radical reduction is achieved in the so called mSUGRA, by embedding the low-energy supersymmetric theory into a grand unified (SUSY-GUT) framework.

From the fundamental point of view, however, all low-energy parameters should be measured independently of any theoretical assumptions [1]. After discovering supersymmetric particles the priority will be to determine the low-energy Lagrangian parameters. This will allow us to verify the relations among them, if any, in order to distinguish between various SUSY models.

Here I would like to outline how the fundamental SUSY parameters relevant for the chargino sector can be determined from the measurements of chargino pair production cross-sections with polarized beams at future e^+e^- linear colliders [2]. The results summarized here have been worked out in a series of papers [3–6], to which we refer for more detailed discussions and references.

2. Chargino production in e^+e^- collisions

Charginos $\tilde{\chi}^{\pm}_{1,2}$ are mixtures of the spin-1/2 partners of the $W^{\pm}$ gauge bosons, $\tilde{W}^{\pm}$, and of the charged Higgs bosons, $\tilde{H}^{\pm}$. In the $(\tilde{W}^-, \tilde{H}^-)$ basis, the chargino mass matrix

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix} \qquad (1)$$

is given by the fundamental SUSY parameters: the SU(2) gaugino mass M_2, the higgsino mass parameter μ, and the ratio $\tan\beta = v_2/v_1$ of the vev's of the two neutral Higgs fields. In CP-noninvariant SUSY, the reparametrization-invariant CP-violating phase Φ_μ can be attributed to μ, while M_2 is assumed real and positive; μ is real in CP-invariant SUSY sector [7].

Two different unitary matrices acting on the left- and right-chiral $(\tilde{W}, \tilde{H})$ states are needed to diagonalize the chargino mass matrix. They can be parameterized by two rotation angles ϕ_L and ϕ_R, and additional phases $\beta_{L,R}$ and $\beta'_{L,R}$, which are functions of the CP-violating phase Φ_μ. These phases, however, enter the cross-sections only in a combination which can be expressed by $\cos 2\phi_{L,R}$ and chargino masses [4]. As a result, chargino masses and mixing parameters $\cos 2\phi_{L,R}$ can be extracted from physical observables and used to determine the fundamental SUSY parameters $\{M_2, |\mu|, \cos\Phi_\mu, \tan\beta\}$.

The two mixing angles $\phi_{L,R}$ and the phase Φ_μ determine the chargino-chargino-Z and the electron-sneutrino-chargino couplings, while the photon vertices are independent of these angles. The strength of the sneutrino vertex is set by the $e\tilde{\nu}\tilde{W}$ Yukawa coupling $g[e\tilde{\nu}\tilde{W}]$ which is identical to the $e\nu W$ coupling $g[e\nu W]$ in supersymmetric theories: $g[e\tilde{\nu}\tilde{W}] = g[e\nu W] = e/\sin\theta_W$.

In e^+e^- collisions, the charginos are produced in pairs

$$e^+e^- \;\to\; \tilde{\chi}^+_i \, \tilde{\chi}^-_j \quad [i,j=1,2]$$

via three mechanisms: s-channel γ and Z exchanges, and t-channel $\tilde{\nu}_e$ exchange. Since the coupling to the higgsino component, which is proportional to the electron mass, can be neglected in the sneutrino vertex, the sneutrino couples only to left-handed electrons.

From the steep threshold behavior of the cross section, the masses of $\tilde{\chi}^{\pm}_i$ can be determined very accurately [8]. Since the chargino interactions are chiral dependent, the polarization of the electron and positron beams plays a central role in the analysis of the chargino system. If we assume the collider energy to be large enough to produce the entire ensemble of diagonal and mixed chargino pairs $\tilde{\chi}^+_1\tilde{\chi}^-_1$, $\tilde{\chi}^+_1\tilde{\chi}^-_2$ and $\tilde{\chi}^+_2\tilde{\chi}^-_2$, the longitudinal beam polarization is sufficient to verify the completeness of the two-chargino system and to determine the mixing parameters unambiguously.

Let us consider $\sigma_L\{ij\} = \sigma(e^+_R e^-_L \to \tilde{\chi}^+_i\tilde{\chi}^-_j)$, and $\sigma_R\{ij\}$ for opposite longitudinal beam polarization. The fact that all (chiral) production amplitudes are linear in either $c_{2L} \equiv \cos 2\phi_L$, $c_{2R} \equiv \cos 2\phi_R$, $\sin 2\phi_L$ or $\sin 2\phi_R$ [5,6], enables us to write the polarized cross-sections as linear combinations of six formally independent variables

$\vec{z} = [1, c_{2L}, c_{2R}, c_{2L}^2, c_{2R}^2, c_{2L}c_{2R}]$. Restricting ourselves to the left- and right-handed cross-section, and introducing generic notation $\vec{\sigma}$ for the six cross-sections $\sigma_R\{ij\}$ and $\sigma_L\{ij\}$: $\vec{\sigma} = [\,\sigma_R\{11\}, \sigma_R\{12\}, \sigma_R\{22\}, \sigma_L\{11\}, \sigma_L\{12\}, \sigma_L\{22\}\,]$, each cross-section can be decomposed in terms of c_{2L} and c_{2R} as

$$\sigma_i = \sum_{j=1}^{6} f_{ij}[m^2_{\tilde{\chi}^\pm_{1,2}}, m^2_{\tilde{\nu}}]\, z_j. \tag{2}$$

The 6×6 matrix f_{ij} depends on the chargino masses, sneutrino mass (which we assume to be known from e.g. sneutrino pair production) and other known parameters (the scattering energy, SM couplings etc.). It relates the six left/right-handed cross-section and the six variables z_i. Inverting the matrix gives the expressions for the variables z_i, which are not independent, in terms of the observables σ_i. We therefore obtain several non-trivial relations among the observables of the chargino sector:

$$z_1 = 1 \quad : \quad f^{-1}_{1j}\sigma_j = 1, \tag{3}$$

$$z_4 = z_2^2 \quad : \quad f^{-1}_{4j}\sigma_j = \left[f^{-1}_{2j}\sigma_j\right]^2, \tag{4}$$

$$z_5 = z_3^2 \quad : \quad f^{-1}_{5j}\sigma_j = \left[f^{-1}_{3j}\sigma_j\right]^2, \tag{5}$$

$$z_6 = z_2 z_3 : \quad f^{-1}_{6j}\sigma_j = f^{-1}_{2j} f^{-1}_{3k}\sigma_j\sigma_k\,, \tag{6}$$

where summing over repeated indices is understood. The failure of saturating any of these sum rules by the measured cross sections would signal that the chargino two-state $\{\tilde{\chi}^\pm_1, \tilde{\chi}^\pm_2\}$ system is not complete and additional states mix in.

If the sum rules are satisfied, the z_2 and z_3 are the required mixing parameters $\cos 2\phi_L$ and $\cos 2\phi_R$. Alternatively, one can plot $\sigma_L\{ij\}$ and $\sigma_R\{ij\}$ as contours in the c_{2L} and c_{2R} plane. The right-handed cross sections σ_R, due to the absence of the sneutrino exchange diagram, are symmetric in the mixing parameters c_{2L} and c_{2R}. They intersect in exactly two points in the plane which are symmetric under the interchange $c_{2L} \leftrightarrow c_{2R}$.

The cross sections for left-handed electron beams are dominated by the sneutrino contributions. In general, they exhibit quite a different dependence on c_{2L} and c_{2R}. In particular, they are not symmetric with respect to c_{2L} and c_{2R} so that the correct solution for $[c_{2L}, c_{2R}]$ can be singled out of the two solutions obtained from the right-handed cross-sections. At the same time, the identity between the $e\tilde{\nu}\tilde{W}$ Yukawa coupling and the $e\nu W$ gauge coupling can be tested. Varying the Yukawa coupling freely, the contour lines σ_L are shifted through the $[c_{2L}, c_{2R}]$ plane. Only for the supersymmetric solutions the curves σ_L intersect each other and the curves σ_R in exactly one point.

The above program has been worked out [5,6] at a single collider energy $\sqrt{s} = 800$ GeV and an integrated luminosity $\int\mathcal{L} = 1\ \mathrm{ab}^{-1}$ for the two parameter points introduced in ref. [9]. They correspond to a small and a large $\tan\beta$ solution for universal gaugino and scalar masses at the GUT scale (the CP-phase Φ_μ is set to zero):

$$\begin{aligned} &\mathrm{RR1}: (\tan\beta, m_0, M_{\frac{1}{2}}) = (\ 3, 100\,\mathrm{GeV}, 200\,\mathrm{GeV}),\\ &\mathrm{RR2}: (\tan\beta, m_0, M_{\frac{1}{2}}) = (30, 160\,\mathrm{GeV}, 200\,\mathrm{GeV}). \end{aligned} \tag{7}$$

The induced chargino $\tilde{\chi}^\pm_{1,2}$, neutralino $\tilde{\chi}^0_1$ and sneutrino $\tilde{\nu}$ masses are given as follows:

$$m_{\tilde{\chi}_1^\pm} = 128/132\,\mathrm{GeV} \qquad m_{\tilde{\chi}_1^0} = 70/72\,\mathrm{GeV}$$
$$m_{\tilde{\chi}_2^\pm} = 346/295\,\mathrm{GeV} \qquad m_{\tilde{\nu}} = 166/206\,\mathrm{GeV} \tag{8}$$

for the two points RR1/2, respectively. Combining the analyses of σ_R and σ_L, the masses, the mixing parameters and the Yukawa coupling can be determined to quite a high precision. For example, for the scenario RR1 with $\tan\beta = 3$ we find

$$m_{\tilde{\chi}_1^\pm} = 128 \pm 0.04\,\mathrm{GeV} \quad \cos 2\phi_L = 0.645 \pm 0.02 \quad g[e\tilde{\nu}\tilde{W}]/g[e\nu W] = 1 \pm 0.001$$
$$m_{\tilde{\chi}_2^\pm} = 346 \pm 0.25\,\mathrm{GeV} \quad \cos 2\phi_R = 0.844 \pm 0.005, \tag{9}$$

where 1σ statistical errors are given.

If the collider energy, in its first phase, is sufficient to generate only the light $\tilde{\chi}_1^+\tilde{\chi}_1^-$ pair, the transverse polarization of both beams is required [5], or measurement of chargino polarization [3], in order to obtain a unique solution for $\cos 2\phi_{L,R}$ in general. However, additional advantage of having the transverse polarization is that not only the left-handed σ_L but also the transverse σ_T cross section depends on the sneutrino mass and the $e\tilde{\nu}\tilde{W}$ Yukawa coupling. Thus the sneutrino mass and the Yukawa coupling can be determined indirectly from the left-handed and transverse cross sections [5,6].

3. Determining the fundamental chargino parameters

From the measured chargino masses and mixing angles the fundamental supersymmetric parameters $\{M_2, |\mu|, \cos\Phi_\mu, \tan\beta\}$ can be derived in the following way.

Based on the definition $M_2 > 0$, the gaugino mass parameter M_2, the modulus and the phase of the higgsino mass parameter and $\tan\beta$ read as follows:

$$\begin{aligned}
M_2 &= m_W[\Sigma - \Delta(c_{2L} + c_{2R})]^{1/2}, \\
|\mu| &= m_W[\Sigma + \Delta(c_{2L} + c_{2R})]^{1/2}, \\
\cos\Phi_\mu &= [\Delta^2(2 - c_{2L}^2 - c_{2R}^2) - \Sigma] \\
&\quad \{[1 - \Delta^2(c_{2L} - c_{2R})^2][\Sigma^2 - \Delta^2(c_{2L} + c_{2R})^2]\}^{-1/2}, \\
\tan\beta &= \{[1 - \Delta(c_{2L} - c_{2R})]/[1 + \Delta(c_{2L} - c_{2R})]\}^{1/2},
\end{aligned} \tag{10}$$

where we introduced the abbreviations

$$\Sigma = (m_{\tilde{\chi}_2^\pm}^2 + m_{\tilde{\chi}_1^\pm}^2 - 2m_W^2)/2m_W^2 \qquad \Delta = (m_{\tilde{\chi}_2^\pm}^2 - m_{\tilde{\chi}_1^\pm}^2)/4m_W^2. \tag{11}$$

The remaining ambiguity in $\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu$ in CP-noninvariant theories can only be resolved by measuring CP-violating observables related to the normal $\tilde{\chi}_1^-$ or/and $\tilde{\chi}_2^+$ polarization in non-diagonal $\tilde{\chi}_1^-\tilde{\chi}_2^+$ chargino-pair production.

The accuracy which can be expected in such an analysis, for both CP invariant reference points RR1 and RR2 is as follows (errors are statistical only at the 1σ level)

$$\begin{aligned}
M_2 &: \quad 152 \pm 1.75\ \mathrm{GeV} \qquad 150 \pm 1.2\ \mathrm{GeV} \\
\mu &: \quad 316 \pm 0.87\ \mathrm{GeV} \qquad 263 \pm 0.7\ \mathrm{GeV} \\
\tan\beta &: \quad 3 \pm 0.69 \qquad\qquad > 20.2,
\end{aligned} \tag{12}$$

where the first (second) column is for RR1 (RR2). Since the chargino observables depend only on $\cos 2\beta$, the dependence on β is flat for $2\beta \to \pi$ so that it is difficult to derive the value of $\tan\beta$ in the case of large $\tan\beta$ due to error propagation.

An important input in the above discussion is the heavy chargino mass. In the initial stage of the linear collider below the heavy chargino threshold, the parameters M_2, μ and $\tan\beta$ of CP-invariant theory can nevertheless be reconstructed up to a twofold ambiguity from the measured $\chi_1^\pm$ mass and mixing parameters $\cos 2\phi_R$ and $\cos 2\phi_L$ [3].

4. Conclusions

The chargino sector can be analysed independently of the structure of the neutralino sector [10] which is potentially very complex in theories beyond the minimal supersymmetric standard model (MSSM). The analysis presented here is based strictly on low-energy SUSY. Once these basic parameters are determined experimentally, they provide essential components in the reconstruction of the fundamental supersymmetric theory at the grand unification scale. From the chargino masses and mixing angles the underlying fundamental SUSY parameters, $M_2, |\mu|$ and $\tan\beta$, can be extracted *unambiguously*; the phase Φ_μ can be determined up to a twofold ambiguity $\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu$. This ambiguity can only be resolved by measuring manifestly CP-noninvariant observables related to the normal polarization of the charginos. In the first phase of e^+e^- linear collider, i.e. with the heavy chargino mass still unknown, the parameters M_2, μ and $\tan\beta$ can be determined up to at most twofold ambiguity [3].

The reconstruction of the basic SUSY parameters presented here has been carried out at the tree level; the small higher-order corrections [11] include parameters from other sectors of the MSSM demanding iterative higher-order expansions in global analyses at the very end.

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