PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 55, Nos 1 & 2
July & August 2000
pp. 241–245

Light front quantum chromodynamics: Towards phenomenology

A HARINDRANATH
Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Calcutta 700 064, India

Abstract. We briefly review the application of light front QCD to inclusive deep inelastic scattering.

Keywords. Light front dynamics; quantum chromodynamics; deep inelastic scattering.

PACS Nos 11.10.Ef; 11.30.Cp; 12.38.Aw; 13.88.+e

1. Introduction

In this talk we briefly review the salient features of light front dynamics [1] and its application to inclusive deep inelastic scattering. For applications of light front techniques to exclusive processes, see ref. [2].

2. Special features of light front dynamics relevant for phenomenology

2.1 *Kinematics*

The idea of light front quantization was introduced by Dirac in 1949. Introduce the variables $x^{\pm} = x^0 \pm x^3$, $x^{\perp} = (x^1, x^2)$. We have, $x^2 = x^+x^- - (x^{\perp})^2$. Further $k.x = 1/2k^+x^- + 1/2k^-x^+ - k^{\perp} \cdot x^{\perp}$. An example of a light front is the surface $x^+ = 0$. Time is chosen to be x^+. Then x^- is a longitudinal coordinate, k^- is the energy and k^+ is the longitudinal momentum. For an on mass-shell particle, longitudinal momentum $k^+ \geq 0$ and energy $k^- = (k^{\perp})^2 + m^2/k^+$. Note the nonrelativistic structure of the dispersion relation in the transverse plane.

What makes light front dynamics appealing from high energy phenomenology point of view?

- Boost are kinematical: Longitudinal boost becomes a scale transformation and transverse boosts are Galilean boosts.
- With a cutoff $k_i^+ > 0$ on constituent momentum, vacuum is trivial and hence parton picture makes sense.

- Light front power counting [3] forces one to treat x^- and $x^\perp$ differently, only $x^\perp$ carry inverse mass dimension. This is convenient for the description and the understanding of DIS phenomena.
- Relativistic fermion and gauge boson ($A^+ = 0$ gauge) have only two dynamical degrees of freedom. This makes it possible to completely unravel the dynamical dependence of various operators.

2.2 *Multi-parton wavefunctions*

The bound state of a hadron on light-front can be simply expanded in terms of the Fock states

$$|PS\rangle = \sum_{n,\lambda_i} \int' \mathrm{d}x_i \mathrm{d}^2\kappa_{\perp i} |n, x_i P^+, x_i P_\perp + \kappa_{\perp i}, \lambda_i\rangle \Phi_n^S(x_i, \kappa_{\perp i}, \lambda_i)\,, \tag{1}$$

where n represents n constituents contained in the Fock state $|n, x_i P^+, x_i P_\perp + \kappa_{\perp i}, \lambda_i\rangle$, λ_i is the helicity of the i-th constituent, $\int'$ denotes the integral over the space $\sum_i x_i = 1$, and $\sum_i \kappa_{\perp i} = 0$ while x_i is the fraction of the total longitudinal momentum carried by the i-th constituent, and $\kappa_{\perp i}$ is its relative transverse momentum with respect to the center mass frame, $x_i = p_i^+/P^+$, $\kappa_{i\perp} = p_{i\perp} - x_i P_\perp$ with $p_i^+, p_{i\perp}$ the longitudinal and transverse momenta of the i-th constituent. $\Phi_n^S(x_i, \kappa_{\perp i}, \lambda_i)$ is the amplitude of the Fock state $|n, x_i P^+, x_i P_\perp + \kappa_{\perp i}, \lambda_i\rangle$, i.e., the *multi-parton wave function*, which is boost invariant and satisfy the normalization condition $\sum_{n,\lambda_i} \int' \mathrm{d}x_i\ \mathrm{d}^2\kappa_{\perp i} |\Phi_n^S(x_i, \kappa_{\perp i}, \lambda_i)|^2 = 1$, and is, in principle, determined from the light-front bound state equation. For a meson, for example,

$$\Big(M^2 - \sum_{i=1}^{n} \frac{\kappa_{i\perp}^2 + m_i^2}{x_i}\Big) \begin{bmatrix} \Phi^S_{q\bar q} \\ \Phi^S_{q\bar q g} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar q|H_{\rm int}|q\bar q\rangle & \langle q\bar q|H_{\rm int}|q\bar q g\rangle & \cdots \\ \langle q\bar q g|H_{\rm int}|q\bar q\rangle & \cdots & \\ \vdots & \ddots & \end{bmatrix} \begin{bmatrix} \Phi^S_{q\bar q} \\ \Phi^S_{q\bar q g} \\ \vdots \end{bmatrix}. \tag{2}$$

Here $H_{\rm int}$ is the interaction part of the light-front QCD Hamiltonian. $\Phi_n^S(x_i, \kappa_{\perp i}, \lambda_i)$ play a crucial role in high energy inclusive and exclusive reactions.

3. Light front QCD Hamiltonian approach to deep inelastic structure functions

Taking clue from Bjorken's original derivation of scaling, we make use of the Bjorken–Johnson–Low (BJL) expansion of scattering amplitudes specialized to light front kinematics and light-front current commutators. Since the current commutators are evaluated in the interacting theory of QCD, we overcome some of the original shortcomings. The analysis directly leads to expressions for various quark distribution functions (which are related to structure functions in the leading logarithmic approximation) as Fourier transforms of *equal light front time* correlation functions which involve bilocal vector and axial vector currents. Since the bilocality is only in the longitudinal direction we can immediately exploit the whole machinery of Fock space expansion techniques and bring in multi-parton wave functions [4].

Since matrix elements relevant for deep inelastic scattering are measured at the scale Q, a major question is whether we can consistently carry out the renormalization program. We are able to achieve this [5] using the tools of old fashioned perturbation theory, the elements of which are transition matrix elements and energy denominators. The use of light-front gauge $A^+ = 0$ greatly simplifies matters. For example, the path ordered exponential involving the gauge field between the fermion field operators in the bilocal currents reduces to unity in this gauge. In our method, the physical picture is transparent at each stage of the calculation since one is using techniques that closely resembles those in nonrelativistic many body theory.

Since multi-parton wave functions carry both perturbative and non-perturbative information on the structure of hadrons, both aspects are treated in the same framework in our formalism and we are thus able to provide a unified picture. This is a unique feature of our program which is lacking in alternative methods currently in practice namely operator product expansion method and QCD factorization method.

To resolve various outstanding issues associated with the twist four longitudinal structure function $F_L^{\tau=4}(x)$, an analysis [6] was performed based on the BJL expansion for the forward virtual photon-hadron Compton scattering amplitude and equal (light-front) time current commutators. We showed that the integral of $F_L^{\tau=4}/x$ is related to the expectation value of the fermionic part of the *light-front Hamiltonian density* at fixed momentum transfer. Using the Fock space expansion for states and operators, we have evaluated the twist four longitudinal structure function for dressed quark and gluon targets in perturbation theory. The new relation, in addition to providing physical intuition on $F_L^{\tau=4}$, relates the quadratic and logarithmic divergences of $F_L^{\tau=4}$ to mass corrections in light-front QCD and hence provides a new pathway for the renormalization of the corresponding twist four operator. The mixing of quark and gluon operators in QCD naturally leads to a twist four longitudinal gluon structure function and to a new sum rule $\int \mathrm{d}x F_L/x = 4M^2/Q^2$, which is the first sum rule obtained for a twist four observable. The validity of the sum rule in a non-perturbative context is explicitly verified in two-dimensional QCD. We have presented numerical results for the F_2 and F_L structure functions for the meson in two-dimensional QCD in the one pair approximation. We have pointed out the relevance of our results for the problem of the partitioning of hadron mass in QCD.

4. Spin operators in light front QCD and polarized DIS

We have investigated important issues in both longitudinally and transversely polarized structure functions. For the transversely polarized structure function g_2, if the twist three contributions are ignored, one gets an expression purely in terms of the longitudinally polarized structure function g_1. In order to examine the validity of this Wandzura–Wilczek relation for the polarized DIS structure function $g_2(x, Q^2)$, we have used [7] the light-front time-ordering perturbative (p)QCD to calculate $g_2(x, Q^2)$ at order α_s on a quark target. In contrast to the folklore in pQCD, we found that the study of the transversely polarized structure function in pQCD is meaningful only if we begin with massive quarks. The result showed that the Wandzura–Wilczek relation for $g_2(x, Q^2)$ is strongly violated in pQCD.

In the case of longitudinally polarized scattering, the nucleon spin crisis has attracted lot of attention. To address various issues that arise in polarized scattering, a proper understanding of spin operators in QCD is mandatory. In ref. [8] we have addressed the long standing problem of the construction of relativistic spin operators for a composite system in QCD. Consider the Pauli–Lubanski spin operators

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} M_{\nu\rho} P_\sigma$$

with $\epsilon^{+-12} = -2$. For a massive particle, the spin operators $\mathcal{J}^i$ in light front theory are given in terms of Poincaré generators by

$$M\mathcal{J}^i = W^i - P^i \mathcal{J}^3 \quad (i=1,2) = \epsilon^{ij}\left(K^3 P^j + \frac{1}{2}(F^j P^+ - E^j P^-)\right) - P^i \mathcal{J}^3.$$

$$\mathcal{J}^3 = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+}(E^1 P^2 - E^2 P^1). \tag{3}$$

Exploiting the kinematical boost symmetry in light front theory, we showed that transverse spin operators for massless particles can be introduced in an arbitrary reference frame, in analogy with those for massive particles.

We were able to show that [9], in light-front quantization, with $A^+ = 0$ gauge, J^3 is equal to the naive canonical form independent of interactions at the operator level, provided the fields vanish at the boundary. Explicitly,

$$J^3 = J^3_{f(o)} + J^3_{f(i)} + J^3_{g(o)} + J^3_{g(i)}. \tag{4}$$

Having constructed the gauge fixed light-front helicity operator, quark and gluon orbital helicity distribution functions relevant for polarized deep inelastic scattering were introduced as Fourier transform of the forward hadron matrix elements of appropriate bilocal operators. The utility of these definitions was illustrated with the calculation of anomalous dimensions in perturbation theory. The helicity sum rule for dressed quark and gluon targets in light-front perturbation theory was explicitly verified.

The proton helicity sum rule is given by

$$\langle PS^{\parallel} \mid \mathcal{J}^3 \mid PS^{\parallel}\rangle = \langle PS^{\parallel} \mid \mathcal{J}^3_{q(i)} \mid PS^{\parallel}\rangle + \langle PS^{\parallel} \mid \mathcal{J}^3_{q(o)} \mid PS^{\parallel}\rangle + \langle PS^{\parallel} \mid \mathcal{J}^3_{g(i)} \mid PS^{\parallel}\rangle + \langle PS^{\parallel} \mid \mathcal{J}^3_{g(o)} \mid PS^{\parallel}\rangle. \tag{5}$$

The flavour singlet part of the helicity structure function

$$\int_0^1 \mathrm{d}x g_1(x, Q^2) \propto \langle PS^{\parallel} \mid \mathcal{J}^3_{q(i)} \mid PS^{\parallel}\rangle. \tag{6}$$

The transverse rotation operator

$$F^2 = F^2_{\mathrm{I}} + F^2_{\mathrm{II}} + F^2_{\mathrm{III}}.$$

The operators F^2_{II} and F^2_{III} which do not explicitly depend upon coordinates arise from the fermionic and bosonic parts respectively of the gauge invariant, symmetric, energy momentum tensor in QCD. It follows that the transverse spin operators $\mathcal{J}^i$, $(i = 1, 2)$ can also be written as the sum of three parts, $\mathcal{J}^i_{\mathrm{I}}$ which has explicit coordinate dependence, $\mathcal{J}^i_{\mathrm{II}}$

which arises from the fermionic part, and $\mathcal{J}^i_{\rm III}$ which arises from the bosonic part of the energy momentum tensor. In light front QCD, the complete set of transverse spin operators are identified for the first time, which are responsible for the helicity flip of the nucleon. We can write down a proton transverse spin sum rule

$$\langle PS^\perp \mid \mathcal{J}^i \mid PS^\perp \rangle = \langle PS^\perp \mid \mathcal{J}^i_{\rm I} \mid PS^\perp \rangle + \langle PS^\perp \mid \mathcal{J}^i_{\rm II} \mid PS^\perp \rangle + \langle PS^\perp \mid \mathcal{J}^i_{\rm III} \mid PS^\perp \rangle. \tag{7}$$

The flavour singlet part of the transverse polarized structure function

$$\int_0^1 {\rm d}x g_T(x, Q^2) \propto \langle PS^\perp \mid \mathcal{J}^i_{\rm II} \mid PS^\perp \rangle. \tag{8}$$

Thus we establish the direct connection between transverse spin in light front QCD and transverse polarized deep inelastic scattering.

In summary, we have shown that an approach to deep inelastic structure functions based on light front QCD Hamiltonian can provide a very clear physical picture as well as a well-defined calculational tool to investigate various perturbative and non-perturbative issues.

Acknowledgement

This talk is mainly based on work done in collaboration with Rajen Kundu, Asmita Mukherjee, Raghunath Ratabole, James P Vary, and Wei-Min Zhang. We also acknowledge helpful discussions with Stan Brodsky and Dipankar Chakrabarti.

References

[1] P A M Dirac, *Rev. Mod. Phys.* **21**, 321 (1949); For resources on light front dynamics, visit the URL: http://tnp.saha.ernet.in/ hari/light/light.html

[2] S J Brodsky and G P Lepage, in Perturbative Quantum Chromodynamics edited by A Mueller (World Scientific, Singapore, 1989) and references therein
A recent overview is given in S J Brodsky, hep-ph/9911368

[3] K G Wilson, T S Walhout, A Harindranath, Wei-Min Zhang, R J Perry and St D Głazek, *Phys. Rev.* **D49**, 6720 (1994)

[4] A Harindranath, Rajen Kundu and Wei-Min Zhang, *Phys. Rev.* **D59**, 094012 (1999)

[5] A Harindranath, Rajen Kundu, and Wei-Min Zhang, *Phys. Rev.* **D59**, 094013 (1999)

[6] A Harindranath, Rajen Kundu, Asmita Mukherjee and James P Vary, *Phys. Lett.* **B417**, 361 (1998)
A Harindranath, Rajen Kundu, Asmita Mukherjee and James P Vary, *Phys. Rev.* **D58**, 114022 (1998)

[7] A Harindranath and Wei-Min Zhang, *Phys. Lett.* **B408**, 347 (1997)

[8] A Harindranath, Asmita Mukherjee and Raghunath Ratabole, *Transverse Spin in QCD and Transverse Polarized Deep Inelastic Scattering*, to appear in *Phys. Lett.*

[9] A Harindranath and Rajen Kundu, *Phys. Rev.* **D59**, 116013 (1999)