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Constraints on m_s and ϵ'/ϵ from lattice quantum chromodynamics

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Abstract. Results for light quark masses obtained from lattice QCD simulations are compared and contrasted with other determinations. Relevance of these results to estimates of ϵ'/ϵ is discussed.

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1. Determination of light quark masses

Quark masses are not physical observables in QCD, rather they enter the theory as parameters in the Lagrangian. Their values depend on the QCD renormalization scale, and three quantitative approaches have been used to determine them.

1.1 *Chiral perturbation theory*

This is a low energy ($E \leq 1\ \mathrm{GeV}$) effective field theory of QCD in presence of spontaneous chiral symmetry breaking. With $m_q \ll \Lambda_{\mathrm{QCD}}$, the pseudo-Nambu–Goldstone bosons are the dominant fields at low energy. $O(p^2, m_q)$ terms in the effective Lagrangian are fixed by the spectrum. $O(p^4, p^2 m_q, m_q^2)$ terms are estimated using resonance saturation and large N_c power counting, as well as from phenomenological fits to various form factors. Small electromagnetic and isospin breaking effects are systematically included, and bounds are obtained on quark mass ratios.

The range of validity of this effective field theory expansion cannot be convincingly established. Also the absolute mass scale has to be fixed from experimental data. Yet renormalization group invariant dimensionless mass ratios can be tightly constrained [1].

$$\frac{m_u}{m_d} = 0.553 \pm 0.043\,, \quad \frac{m_s}{m_d} = 18.9 \pm 0.8\,, \quad \frac{m_s}{m_u} = 34.4 \pm 3.7\,, \tag{1}$$

$$\frac{2m_s}{m_u+m_d} \equiv \frac{m_s}{m_{ud}} = 24.4 \pm 1.5 \, , \; \frac{m_s - m_{ud}}{m_d - m_u} = 40.8 \pm 3.2 \, . \tag{2}$$

These mass ratios are typically combined with $m_s(1 \text{ GeV})$ extracted from QCD sum rules to give individual quark masses.

1.2 *QCD sum rules*

In this approach, two-point hadronic current correlators are evaluated using the operator product expansion (OPE) and perturbative QCD in the Euclidean region $q^2 < 0$. The expansion in inverse powers of $-q^2$ is then analytically continued to the physical region $q^2 > 0$, and matched to the experimental data using dispersion relations. The experimental data is organized in terms of contributions from well-known leading poles and subleading branch-cuts from the QCD continuum. Subtractions are used in the dispersion relations to suppress large continuum contributions, while threshold factors and moments help in suppressing contribution of regions near the poles where perturbative QCD is inapplicable. The results critically depend on proper choice of the boundary conditions.

Pseudoscalar, scalar, e^+e^--annihilation and τ-decay sum rules have been used by various groups to extract m_s. In the $\overline{MS}$ scheme, the required β and γ functions are known to 4-loop precision, giving the most accurate results [2]:

$$\overline{m}_s(1 \text{ GeV}) = 162.5 \pm 15.5 \text{ MeV} \, , \; \overline{m}_s(2 \text{ GeV}) = 117.8 \pm 12.3 \text{ MeV} \, . \tag{3}$$

Analysis of Cabbibo-suppressed τ-decay gives $\overline{m}_s(M_\tau) = 119 \pm 24$ MeV [3].

The major uncertainty in the sum rule approach is the asymptotic nature of the perturbative QCD expansion—in the longitudinal channel it cannot be pushed beyond $O(\alpha_s^3)$.

1.3 *Lattice QCD simulations*

These provide a first principle non-perturbative determination of quark masses without additional assumptions. It is convenient to convert the running quark masses to renormalization group invariant quark masses, completely within the lattice regularization framework.

$$m_q^{\rm RGI} = \lim_{\mu \to \infty} \{ (2\beta_0 g^2(\mu))^{-\gamma_0/2\beta_0} m_q(\mu) \} \, . \tag{4}$$

With improvements in simulation algorithms and computers, the systematic errors from finite lattice spacing, finite lattice size and the quenched approximation are gradually coming under control. Most simulations are in the range $m_s/3 \le m_q \le 3m_s$, and $m_u = m_d$ is assumed. Results from several simulations are combined to extrapolate to $\mu \to \infty$.

Perturbative relations connecting lattice and $\overline{MS}$ values show that including higher order corrections decreases $\overline{m}_q$. Moreover, at $\mu = 2$ GeV, the N^3LO correction is comparable to the N^2LO one, and decreases $\overline{m}_q$ by about 10% [4]. A rigorous comparison of lattice and $\overline{MS}$ values should therefore be made at $\mu > 2$ GeV, contrary to common practice.

Here is a summary of results presented by various groups at LAT99.

- *CP-PACS*: RG-improved gauge field action and tadpole-improved clover fermion action are used. Ward identities are used to extract m_q. $N_f = 0$ and $N_f = 2$ results are compared, demonstrating that going to $N_f = 2$ reduces the systematic error in the lattice scale. The results converted to the $\overline{MS}$ scheme are [5] (see figure 1):

$$\overline{m}_{ud}(2\ \mathrm{GeV}) = 3.3(4)\ \mathrm{MeV}\ ,\ \overline{m}_s(2\ \mathrm{GeV}) = 84(7)\ \mathrm{MeV}\ . \tag{5}$$

- *ALPHA/UKQCD*: Schrödinger functional method is used with $O(a)$ improved action and $N_f = 0$. PCAC relation is used to extract m_q. With $(M_K r_0)^2 = 1.5736$ fixing the reference scale, the results are [6] (see figure 2):

$$2M_{\mathrm{ref}} = m_s^{\mathrm{RGI}} + m_{ud}^{\mathrm{RGI}} = 143(5)\ \mathrm{MeV}, \overline{m}_s(2\ \mathrm{GeV}) = 94(4)\ \mathrm{MeV}. \tag{6}$$

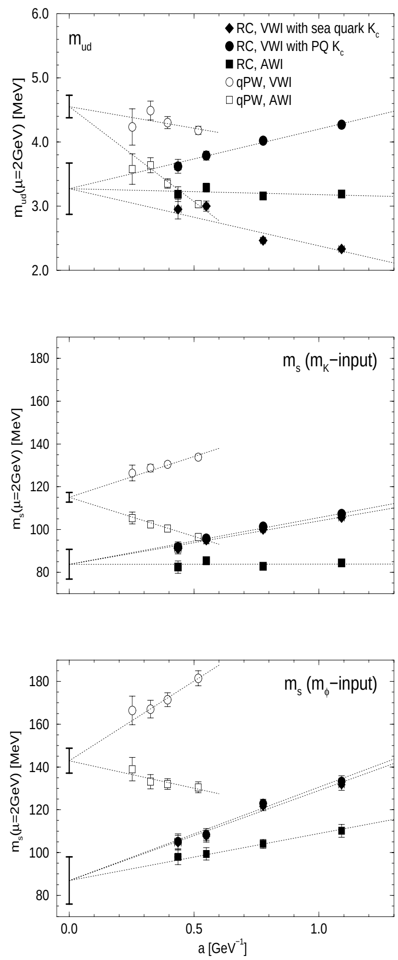

Figure 1. Continuum extrapolations of m_{ud} (top), and m_s with m_K-input (middle) and m_ϕ-input (bottom) [5]. Open and filled symbols are results for $N_f = 0$ and 2.

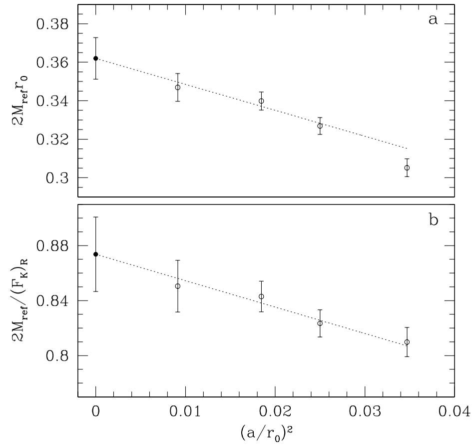

Figure 2. Continuum extrapolations of M_{ref} in units of (a) r_0, and (b) $(F_K)_{\rm R}$ [6]. The largest value of a is not used in the (dashed) extrapolation to $a = 0$.

- *QCDSF*: Schrödinger functional method is used with $O(a)$ improved action and $N_f = 0$. PCAC relation is used to extract m_q. Using $r_0 = 0.5$ fm as the reference scale yields [7]:

$$\overline{m}_{ud}(2\ \mathrm{GeV}) = 4.4(2)\ \mathrm{MeV}\ ,\ \overline{m}_s(2\ \mathrm{GeV}) = 105(4)\ \mathrm{MeV}\ . \tag{7}$$

- *Rome group*: $O(a)$ improved action is used with $N_f = 0$. Ward identity for the renormalized quark propagator is used to extract m_q. With pseudoscalar and vector meson masses fixing the lattice scale, the results are [8]:

$$\overline{m}_{ud}(2\ \mathrm{GeV}) = 4.8(5)\ \mathrm{MeV}\ ,\ \overline{m}_s(2\ \mathrm{GeV}) = 111(9)\ \mathrm{MeV}\ . \tag{8}$$

Numerical results show that, for a fixed value of the lattice scale, increasing N_f increases the coupling g^2 through vacuum polarization. This translates into $m_q(\mu)$ decreasing with increasing N_f. My educated guess, including the effect of a relatively light s-quark, is

$$\overline{m}_s(2\ \mathrm{GeV}, N_f = 2.5)\ =\ 90 \pm 10\ \mathrm{MeV}\ . \tag{9}$$

2. Determination of ϵ'/ϵ

ϵ and ϵ' parametrise indirect and direct CP-violation effects in neutral kaon decays into two pions. ϵ arises from the mixing between CP-eigenstates K_L and K_S, and is experimentally measured to be

$$\epsilon = 2.280(13) \cdot 10^{-3}\ \mathrm{e}^{i\phi_\epsilon}\ ,\ \phi_\epsilon \approx \pi/4\ . \tag{10}$$

ϵ' parametrizes CP-violation in the decay amplitudes without $K_L - K_S$ mixing, and in the Standard Model it arises from the complex phase in the CKM quark mixing matrix. $\pi\pi$-

scattering data establish that the phases of ϵ and ϵ' are almost identical. Recent results from NA31, E731, KTeV and NA48 experiments have determined (see ref. [9] for details)

$$\mathrm{Re}(\epsilon'/\epsilon) \ = \ (21.2 \pm 4.6) \cdot 10^{-4} \ . \tag{11}$$

2.1 *Parametrization using OPE*

All phenomenological explanations of ϵ'/ϵ take the CP-conserving data from experiments and estimate the CP-violating part. In the Standard Model, ϵ is found by calculating the box diagram, while ϵ' is found by calculating the g, γ, Z^0 penguin diagrams. Operator mixing and RG-evolution are crucial in the analysis, and electroweak contributions have become important due to large m_t. The dominant components are the $K \to \pi\pi$ matrix elements of the 4-fermion operators Q_6 and Q_8, in the $\Delta I = 1/2$ and $\Delta I = 3/2$ channels. Let $B_{i,\Delta I}$ denote the ratios of the actual matrix elements to their values in the vacuum saturation approximation (VSA). Using commonly accepted scale parameters, and including isospin breaking effects with $\Omega_{\eta+\eta'} = 0.25(8)$, an approximate formula is [9]

$$\frac{\epsilon'}{\epsilon} \ \approx \ 13 \cdot \mathrm{Im}(V_{td}V_{ts}^*) \cdot \left(\frac{110 \ \mathrm{MeV}}{\overline{m}_s(2 \ \mathrm{GeV})}\right)^2 \cdot \left(\frac{\Lambda^{(4)}_{\overline{MS}}}{340 \ \mathrm{MeV}}\right) \cdot \cdot \left[B_{6,(1/2)}(1-\Omega_{\eta+\eta'}) - 0.4 B_{8,(3/2)}\right] \ . \tag{12}$$

I want to emphasize that the appearance of m_s in the above formula is spurious; it arises because the VSA matrix element values are accompanied by quark mass factors in the chiral limit. It is silly to express one unknown matrix element in terms of two other unknowns, m_s and $B_{i,\Delta I}$, but that has become commonplace. To avoid unwanted systematic errors, one should therefore either (a) calculate the matrix elements directly, or (b) use the same calculational framework to evaluate both m_s and the $B_{i,\Delta I}$. Much of the confusion in the literature has arisen from not following this simple guideline consistently.

ϵ'/ϵ has been evaluated in three different frameworks: lattice QCD, large-N_c approximation, and chiral quark model. Lattice QCD simulations show that $B_{8,(3/2)}$ is suppressed below 1, but provide no clean result for $B_{6,(1/2)}$. With $B_{6,(1/2)} = 1.0(3)$ and $B_{8,(3/2)} = 0.8(2)$, the estimates for ϵ'/ϵ are about a factor 2 below its experimental value.

2.2 *Final state interactions*

Final state interactions (FSI) strongly influence $K \to \pi\pi$ decays, as shown by the experimental phase shifts. They are absent in VSA, lattice QCD, large-N_c approximation, and appear at subleading order in chiral perturbation theory. Their contribution to the decay amplitudes can be estimated using dispersion relation analysis of the experimentally measured phase shifts in the elastic $\pi\pi$ channel. There is no doubt that incorporating the FSI boosts the $\Delta I = 1/2$ amplitude and suppresses the $\Delta I = 3/2$ one, and hence increases ϵ'/ϵ. The exact value of the enhancement is debatable, since it depends on the boundary conditions used to analyse the experimental data. In a specific analysis [10], the enhancement is the required factor of 2. Higher order chiral quark model calculations, which automatically include the FSI, also enhance ϵ'/ϵ close to its experimental value [11].

In conclusion, the Standard Model, with a proper framework to handle non-perturbative QCD effects, is fully capable of explaining the observed value of ϵ'/ϵ; contributions to ϵ'/ϵ from new effects beyond the Standard Model must be kept $\leq 5 \cdot 10^{-4}$.

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