PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 55, Nos 1 & 2
July & August 2000
pp. 171–182

Large compact dimensions and high energy experiments

SREERUP RAYCHAUDHURI
Department of Physics, Indian Institute of Technology, Kanpur 208 016, India

Abstract. Models of spacetime with extra compact dimensions and having the Standard Model fields confined to a narrow slice of 4-dimensional spacetime can have strong gravitational effects at the TeV scale as well as electroweak-strength interactions at present-day colliders. Phenomenological consequences of such models are reviewed, with special emphasis on collider signatures.

Keywords. Extra dimensions; Kaluza–Klein; graviton.

PACS Nos 11.10.Kk; 11.25.Mj; 12.90.+b

1. Compact extra dimensions

One of the greatest discoveries of the last century was the fact that relativistic physics can be most elegantly formulated in terms of a four-dimensional spacetime, whose geometrical properties provide a theory of gravity [1]. However, in order to quantize this theory, it has been found necessary to assume that the fundamental objects are not particles but strings [2] living in a spacetime where, in addition to the (1+3) canonical dimensions of Minkowski, there are d additional spatial dimensions which are curled up into circles, i.e. compact. The radius of the circle(s) is R_c and is referred to as the *compactification radius*. The simplest compactification scheme, viz., on a d-torus, will suffice for the present discussion.

Why do we not see any effects of these extra dimensions? Folklore tells us that the typical radii of compactification are as small as the Planck length ($\sim 10^{-33}$ cm), which means that they will not be observable in experiments performed at energies less than the Planck scale ($\sim 10^{19}$ GeV), which is the inverse of the Planck length. We examine the actual evidence for this. If there are d extra (compact) dimensions, then the gravitational interaction will give rise to a two-body potential

$$V(r) = G_{\rm N}\left(\frac{1}{r} + \alpha_d \, \frac{{\rm e}^{-r/R_c}}{r}\right) \quad \text{for} \;\; r \sim R_c, \tag{1}$$

where $G_{\rm N}$ is Newton's constant and α_d measures the strength of deviations from the inverse square law. If the additional Yukawa interactions for $r \sim R_c$ are purely gravitational it can be shown [3,4] that $\alpha_d \sim 1$. Current experiments (essentially of the Eötvös type) measuring the strength of gravitational interactions place bounds [5] on the plane of α_d–R_c, and these basically tell us that any effect with $\alpha_d \sim 1$ and $R_c < 1$ cm would essentially go undetected.

It has been pointed out [6] that a straightforward application of Gauss' law leads to

$$R_c = 2 \times 10^{-17+\frac{31}{d}} \mathrm{cm} \times \left(\frac{1 \text{ TeV}}{M_S} \right)^{1+\frac{2}{d}} , \qquad (2)$$

where M_S is the Planck scale in (4+d) dimensions and represents the scale at which gravity becomes strong. In the framework of string theory, this is also related to the string tension and hence we expect stringy excitations to appear at this scale. Since stringy excitations have not been seen at the electroweak scale — which is the energy attainable with state-of-the-art accelerators — it is clear that M_S must be at least as large as $\mathcal{O}$(1 TeV). Setting M_S to 1 TeV it is trivial to show that $d = 1$ is ruled out by the experimental data mentioned above. However, $M_S \sim 1$ TeV is compatible with experimental evidence for $d > 1$. Models with $d > 6$ are uninteresting from the string-theoretic point of view.

2. Domain walls, D-branes and the ADD model

If indeed spacetime consists of more than 4 dimensions, with large radii of compactification, then one should see startling deviations from the predictions of the Standard Model, which is formulated purely in 4 dimensions. As is well-known, precision tests of the Standard Model have verified the correctness of the Standard Model *in 4 dimensions* with great accuracy. Naively, this seems to tell us that there are no extra dimensions — at least with compactification radii greater than 10^{-16} cm $\sim M_{\rm EW}^{-1}$, where $M_{\rm EW}$ is the electroweak scale. However, all that this tells us is that the Standard Model fields must be constrained to live in a subspace of the full (4+d)-dimensional spacetime (henceforth called the *bulk*). This subspace should extend infinitely in the four canonical directions and have a thickness less than 10^{-16} cm. This is equivalent to saying that the Standard Model is constrained by experiment to live on a 'wall' of three spatial dimensions embedded in the bulk. Thus it is possible to have TeV-scale strings and strong quantum gravity at the TeV scale provided we have a model where the Standard Model lives on such a wall.

The possibility of a higher dimensional bulk with the Standard Model living on a wall has been suggested several times earlier, but the most succint statement has been the recent one by Arkani–Hamed, Dimopoulos and Dvali [6] (ADD). A more natural construction [7], is to identify the wall with a D_3-brane. In fact, it is the discovery that D-branes represent an alternative description of string theories that has rendered ADD-type models so exciting.

D_p-branes, represent solitonic configurations in string theories, in which p of the coordinate fields have fixed values. Open strings are constrained to end on such D-branes, and moreover, carry SU(N) or O(N) gauge theories on the branes. Obviously, this is an excellent framework in which to place the ADD theory. We accordingly assume that the Standard Model fields correspond to the low-lying excitations of open superstrings whose ends are confined to a D_3-brane, identified with the observable Universe. Gravitons correspond to low-lying excitations of *closed* strings, which (having no ends) are free to propagate in the bulk. These closed strings would also have scalar and vector excitations, of which the scalar can be identified with a dilaton field, whose classical value is the radius of compactification R_c [8]. Thus, one can describe the ADD model in terms of the picture drawn in figure 1.

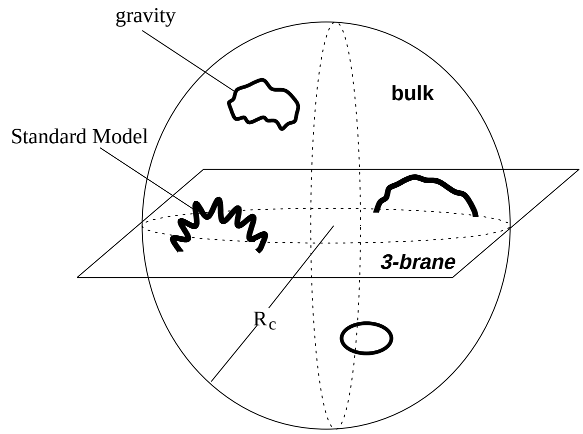

Figure 1. Illustrative sketch showing spacetime as envisaged in the ADD model, with the SM living on a wafer-thin hypersurface in the compactified bulk. Closed strings can propagate freely on the 3-brane as well as in the bulk.

The ADD model has the following interesting features:

- The scale of quantum gravity comes down from the Planck scale to a few TeV; this means that it should be possible [8] to see residual effects of gravitational interactions at the energies currently attainable.
- At TeV machines like the LHC and perhaps high-energy electron or muon colliders, stringy excitations should make their appearance.
- Since new physics comes into play at scales of a few TeV, M_S can be treated as a cutoff for the Standard Model; hence the usual hierarchy problem disappears [6].
- No grand unification is required, since all interactions should unify within a string theory at a few TeV; hence the proton is stable and the negative result of all experiments which seek proton decay is explained.
- Supersymmetry appears naturally on the D_3 brane, since the gauge theory induced by the ends of the open *super*strings is a supersymmetric one.
- If one assumes a hidden sector consisting of fields living on a different brane from the observable one, it is possible to have *anomaly-mediated* supersymmetry-breaking [9]; this provides a natural solution to the supersymmetric flavour problem and predicts an experimentally falsifiable low-energy sparticle spectrum.

However, the ADD model also has some important drawbacks:

- The model introduces two scales, namely the compactification scale $R_c \sim 1$ mm $\sim 10^{-13}$ GeV and the string scale $M_S \sim 1$ TeV $\sim 10^{-17}$ cm, whose separation is comparable to that between the electroweak scale and the Planck scale.

- If $d < 6$, the rest of the 10 extra dimensions must be compactified to the Planck scale; this means the introduction of another scale, with as wide a separation as the other two.

- Radiative corrections tend to draw the compactification scale to the string scale; this is referred to as the *radius stabilization* problem, and is completely analogous to the usual hierarchy problem in the Standard Model.
- Even the presence of supersymmetry does not help stabilize the radius, since the supersymmetry-breaking scale, which is expected to be of the order of a few TeV, will serve to shrink the compactification radii to $\sim 10^{-17}$ cm.

Thus, it is premature to describe the ADD model as a solution to the hierarchy problem, as it was described in the initial stages [6,7]. One may, therefore, regard the ADD model in the following light. If the fundamental theory underlying all physics is indeed a string theory, it is a valid question to try to find the energy scale at which these stringy effects become apparent. As the above arguments show, if the Standard Model fields are confined to a wall or a D-brane, it is possible for this string scale to be as low as a few TeV, in which case gravitational effects, including stringy effects, may become observable in high-energy experiments. These are interesting to look for and hence form the substance of the remaining discussion.

3. Gravitational interactions of (almost) electroweak strength

For energies much below the string scale M_S, the effective theory of gravitation on the D_3-brane is just a Kaluza–Klein theory with the important difference that matter fields are confined to the brane and cannot propagate freely in the bulk, as gravitational fields can. However, because of the fact that matter is confined to a hypersurface, one can see that the spin-1 'graviton' sector decouples from matter and is hence unobservable. This marks an important departure from the traditional Kaluza–Klein framework, where the spin-1 fields play a crucial role.

As usual in Kaluza–Klein compactification, for each massless field in the bulk, there is a set of massive fields in 4 dimensions and this is called a Kaluza–Klein tower. The massive Kaluza–Klein graviton states are responsible for most of the new interactions expected between matter fields on the brane. Since R_c is large, the masses of the particles in the tower are small, in fact, as small as 10^{-13} GeV. The existence of multitudes of superlight graviton (and dilaton) states, with mass spacing $\sim 10^{-13}$ GeV, turns out to be crucial in the observation of gravitational effects in high energy processes.

Feynman rules for the interactions of the graviton and dilaton states with matter fields on the brane have been obtained to the lowest order in $\kappa = \sqrt{16\pi G_N}$ by Giudice, Rattazzi and Wells [10] and, independently, by Han, Lykken and Zhang [11]. These tell us that the gravitons couple to any pair of (like) particles with finite energy-momentum, as indeed, we should expect in a theory of gravity. In fact, the strength of the coupling is proportional to the energy-momentum, which is also a standard feature of gravitation. The dilaton, however, does not couple to a pair of on-shell particles if they are massless. Both graviton and dilaton interactions are blind to all Standard Model quantum numbers such as flavour, colour and charge. A full list of the Feynman rules to $\mathcal{O}(\kappa)$ may be found in Appendix A of ref. [11].

The Feynman rules derived for the ADD theory in refs [10] and [11] show that each graviton or dilaton in the Kaluza–Klein tower couples to matter fields with the usual gravitational coupling $\kappa\sqrt{s} \sim \frac{\sqrt{s}}{M_{\rm Pl}}$ — which is minuscule for energies achievable in the laboratory. This is a usual feature of gravitational interactions and immediately raises the question as to why, then, the ADD model is phenomenologically interesting. The answer is that though *each* individual graviton couples very weakly, there are, as explained above, huge numbers of accessible graviton states in the Kaluza–Klein tower, whose *collective* interactions build up to observable strength. Let us see how this works.

Exchange of virtual gravitons: Since the (massive) gravitons in the Kaluza–Kein tower couple to any pair of particles with finite energy-momentum, one can have, for example, graviton-mediated contributions to well-known processes such as, for example, Bhabha scattering $e^+e^- \to G^*_{\vec{n}} \to e^+e^-$. Dilaton contributions may be neglected since the initial and final state fermions are treated as massless. The crucial point is that *all* the gravitons in the Kaluza–Klein tower are equally capable of mediating interactions which contribute to the above process. Hence, to obtain the observable cross-section, it is necessary to sum over all the graviton states in the propagator. This sum must be a *coherent* one, i.e. the amplitudes must be summed over all graviton propagators before adding to the Standard Model contribution and squaring the resultant amplitude. The amplitude for an individual graviton state is proportional to $\kappa^2 = 16\pi G_N$, since there are two graviton vertices, each coupling with strength κ. The sum over all graviton propagators should be cut off at $M_{\vec{n}} \simeq M_S$, since the effective theory itself breaks down beyond this scale. Since the mass spacing between neighbouring graviton modes is vanishingly small $\sim 10^{-13}$ GeV, one can now treat the mass spectrum as forming a quasicontinuum. It follows that the Bhabha amplitude is proportional to the product $\kappa^2 R^d \sim M_S^{-4}$, the last relation following from Gauss' law. This means that the density of states is so large that the sum over states cancels out the $s/M_{\rm Pl}^2$ suppression of the individual states and replaces it by a suppression by s^2/M_S^4, which is much softer since $M_S \sim 1$ TeV. In fact, the strength of this interaction is just below the electroweak strength.

Production of real gravitons: Once produced, a real (massive) graviton must escape detection, since it interacts extremely weakly with matter. Thus, if produced, real gravitons will lead to missing energy and momentum signals. As before, to be specific, let us consider the 'gravistrahlung' process [12] $e^+e^- \to \gamma^* \to \gamma G_{\vec{n}}$. The final states contain different graviton modes and hence, are technically distinct. However, all gravitons $G_{\vec{n}}$ will be *observationally identical*, since they will all lead to missing energy and momentum. In order to calculate the observable cross-section for gravistrahlung, then, we must perform an *incoherent* sum over all the graviton states i.e. we must calculate the individual cross-sections — each of which is proportional to $\kappa^2 s$ — and then sum over all the states which are kinematically accessible. This sum is similar to the one performed for virtual gravitons in that the factor of $\kappa\sqrt{s}$ cancels out when multiplied by the phase space. It is interesting to note that this happens because the sum is an incoherent one rather than a coherent one.

Low-energy approximation for virtual gravitons: For processes involving exchange of virtual gravitons, the sum over propagators, converted into an integral, has been worked out in detail in refs [10,11]. Following the analysis of Han *et al* we can write this sum as

$$D(s) \;=\; \kappa^2 \sum_{\vec{n}} \frac{1}{s - M_{\vec{n}}^2 + i\epsilon} \;\simeq\; \frac{1}{M_S^4}\lambda_s\left(d, \sqrt{s}/M_S\right) \qquad (3)$$

for s-channel processes, since the width of the (ultralight) gravitons is negligible. The $\lambda_s(d,x)$ function appears on using the quasicontinuum approximation and performing appropriate replacements as explained above. The explicit form is given in Appendix B of ref. [11]. In the limit $x \to 0$, i.e. $\sqrt{s} \ll M_S$ it turns out that $\lambda_s(d, x \to 0)$ is either constant or has a very slow (logarithmic) variation with x. One is, therefore, justified at low energies, in taking λ_s to be almost constant. One can go a step further [13] and absorb its magnitude into the value of M_S, writing $D(s) \simeq \mathrm{sgn}(\lambda_s)\widetilde{M}_S^{-4}$. This parametrization is very useful in that it reduces the number of parameters in the theory to a single one $\widetilde{M}_S$, apart from a sign. A single phenomenological analysis (for each sign) then suffices to derive bounds on the effective M_S for any process, irrespective of the number of extra dimensions.

As may be expected, the constant λ_s approximation is good only for small x and may be expected to break down as $x = \to 1$. Numerical evaluation suggests that the low-energy approximation is more or less valid, up to $x \simeq 0.5$; for higher values of x, it is better to use the full expression for λ_s. For space-like propagators, such as are found in t-channel processes, a similar sum can be performed. Interestingly, in low-energy limit, $\sqrt{|t|} \ll M_S$ we get the same approximate form (up to a sign), which means that we can again make a constant–λ_t approximation and even absorb the constant into $\widetilde{M}_S^4$. However, the t-channel $\widetilde{M}_S^4$ is somewhat different from the s-channel $\widetilde{M}_S^4$.

4. Astrophysical constraints on the ADD model

We are now in a position to discuss phenomenological studies. Since gravitons couple more strongly to matter as the energy-momentum increases we should look to really high-energy systems to see significant deviations from the Standard Model. Not surprisingly, the most dramatic constraints come from astrophysics. The basic effect one should observe in stars is simply the fact that gravitons, once produced, will escape from the interior of the star, carrying away energy, and tending, therefore, to cool the star. The most important processes which have been studied are: (1) graviton Compton scattering: $e\gamma \to eG_{\vec{n}}$; (2) graviton bremsstrahlung: $e(N_Z^A) \to eG_{\vec{n}}(N_Z^A)$, where (N_Z^A) denotes a nucleus of atomic number Z and atomic weight A; (3) photon fusion: $\gamma\gamma(N_Z^A) \to G_{\vec{n}}(N_Z^A)$; (4) graviton Primakoff process: $\gamma(N_Z^A) \to G_{\vec{n}}(N_Z^A)$ and (5) graviton radiation: $NN \to NNG_{\vec{n}}$, which comprises both initial and final-state radiation of gravitons in a two-nucleon scattering process.

For the first four processes, the cross-section varies as T^d, where T is the temperature of the stellar core, for the last it varies as T^{d+2}. All these cross-sections are suppressed by M_S^{-4} and are somewhat smaller than corresponding electroweak cross-sections. A simple calculation shows that the energy losses through gravitons are very small when one considers ordinary stars, like the Sun or even superhot stars like red giants. For interesting cross-sections, it is necessary to consider a supernova explosion, such as that of SN1987A, which is believed to have had a core temperature around 30 MeV during the explosion. At this temperature, graviton radiation dominates the energy loss; this turns out to be so severe as to cool the supernova core to an unacceptable level. To keep the energy loss acceptable, therefore, the cross-section must be small enough: this means that there is a lower bound on M_S. An order-of-magnitude estimate of the constraint made by ADD [8] comes out to be $M_S > 10^{\frac{30-9d}{4+2d}}$ TeV. These results have been refined by Cullen and Perelstein [14] and by Barger *et al* [15]. These bounds restrict $M_S >$ 30–130 TeV for $d = 2$, $M_S > 4$ TeV for $d = 3$ and $M_S > 1$ TeV for $d = 4$. These are stronger than collider bounds – in fact it is difficult to imagine collider data providing a better constraint for $d = 2$.

A different point of view has been studied [16] by Jain, McKay, Panda and Ralston, who use graviton exchange contributions to explain the existence of the ultra-high energy (UHE) component in cosmic ray neutrinos. They start off from the observation that at energies of the order of 10^{11} GeV, UHE neutrinos are the only component of cosmic rays which can really reach the earth from distant cosmic sources without attenuation by the microwave background. This attenuation leads, in fact, to the well-known Greisen–Kuzmin–Zatsepin (GKZ) bound of about 10^{10} GeV on cosmic ray primaries. However, neutrino cross-sections in the SM are too small to account for the observed GKZ-violating air-showers, and the identification of neutrinos with UHE primaries requires some new interaction(s). Jain *et al* pointed out that in order for the neutrino–nucleon scattering cross-section to grow with energy at the proper rate it is necessary to have exchange of spin-2 particles. A tower of KK graviton states is, therefore, just what is needed to obtain cross-sections of the desired order. The requirement that the cross-section be sufficiently large leads then to an *upper bound* of 1–2 TeV on the string scale M_S. In fact, this study is the only *positive* hint for the existence of graviton interactions.

5. Low and intermediate energy effects

Another possibility at low and intermediate energies is to study one-loop effects with gravitons in the loop. Since these virtual gravitons can have all possible energies (up to the cutoff scale M_S), one may expect fairly large effects when all the gravitons in the Kaluza–Klein tower are summed over. In these calculations, dilaton propagators contribute as well as the graviton ones, since the dilaton usually couples to at least one off-shell particle. Loop diagrams with graviton propagators have both ultraviolet (UV) and infrared (IR) divergences. The UV divergences are well-known; it is further known that they render the linearized theory of gravity non-renormalizable. This does not bother us, however, since in the ADD model, the linearized theory is just an effective one, valid up to the string scale $M_S \sim 1$ TeV. On the other hand, gravity is well-known to be an infrared-safe theory, in fact, more so in higher dimensions. Hence the presence of IR divergences is an artefact of treating the masses as a quasicontinuum and we should expect them to cancel out when the full set of Feynman diagrams for a particular process is computed.

One of the earliest studies of one-loop effects involving gravitons was performed by Graesser [17], who studied gravitational contributions to the anomalous magnetic moment of the muon. and was able to show successfully that IR divergences do cancel out when the full set of diagrams is evaluated. He further found that the final result is not UV divergent either, the gravity-induced magnetic moment operator being proportional to m_μ^2 rather than M_S^2. While the cancellation of IR divergences is expected and natural, the cancellation of UV divergences comes as something of a surprise. The extra contribution to the muon anomalous magnetic moment leads to a rather small value, which is not very useful in deriving bounds on M_S.

A more ambitious calculation, that of the ρ-parameter, has been attempted by Das and Raychaudhuri [18]. Here there arises the serious problem that, in the unrenormalized theory, there are tadpole diagrams involving self-couplings in the gravity sector as well as vertices to order κ^2, which contribute at the lowest level to the self-energies of the W

and Z bosons. (These tadpoles also give rise to a finite cosmological constant.) Most of these couplings are neither available in the literature, nor trivial to derive. The authors of ref. [18] therefore adopt a halfway measure by adding on the tadpole contribution, with an unknown multiplicative form factor ξ. The mass R_c^{-1} of the first massive mode acts as an IR regulator. Das and Raychaudhuri then tune ξ numerically to achieve cancellation of the dependence on this regulator R_c^{-1}. Using this prescription they are then able to show that for $d = 5$, the gravity-induced contribution to the ρ-parameter is too large to be compatible with experiment; $d = 6$ is marginally allowed. While this result is interesting, it requires to be substantiated by exact analytical calculations, for which the first step is to derive the Feynman rules to $\mathcal{O}(\kappa^2)$ in the ADD model.

Another point of view has been taken by Han, Marfatia and Zhang [19]. They have evaluated the exact Feynman rules to order κ^2 relevant for the W and Z self-energies and used this to calculate the oblique parameters analytically. They are able to show that, as in the case of the muon anomalous magnetic moment, the final result is free of both IR and UV divergences and, in fact, shows a decoupling behaviour with large M_S. However, their calculation, though elegant, does not take any tadpole graphs into consideration. The inclusion of these, which is unavoidable in a non-renormalizable effective theory like linearized gravity, would probably bring back both the IR and UV divergences which appear in ref. [18].

In general, one-loop calculations involving gravitons are messy and replete with conceptual and technical pitfalls. Very little work has been done till date in this area and it would be interesting to see more studies in the literature.

6. Collider studies

The greatest efforts in determining phenomenological consequences of the ADD model have been made in the area of collider searches. Some of the important effects are described below, grouped by the collider, rather than the nature of the process.

Graviton effects at LEP-1: At LEP-1 energies, graviton effects may be expected to be rather small, since they are suppressed by a factor $(M_Z/M_S)^4 \sim 10^{-4}$. One of the processes of interest is the process $Z \to f\bar{f}G_n(\Phi_n)$, which has been evaluated by Balazs, He, Repko, Yuan and Dicus [20] who find that the branching ratios for such processes are typically of order 10^{-7}, which means very few events, if at all, can be expected in the data collected by LEP-1. In fact, LEP-1 is not the best machine to search for gravity effects, since the energy is rather small.

Graviton effects at LEP-2: One of the earliest processes with real graviton production to be studied [12,10] was the ‘gravistrahlung’ process $e^+e^- \to \gamma^* \to \gamma G_{\vec{n}}$ discussed above. Since the gravitons escape the detector, the signal for this process would be single-photon events, which are well-documented. Using the data already published by the LEP Collaborations, one can obtain bounds on M_S ranging from 1.2 TeV to 520 GeV as d increases from 2 to 6. Virtual graviton exchanges can also lead to a number of interesting processes at LEP-2. Hewett [13] perfomed an early study of Bhabha scattering and analogous processes, including dijet production, and claims a lower bound $\widetilde{M}_S = 985$ GeV. Agashe and Deshpande [21] have studied processes of the form $e^+e^- \to G_{\vec{n}} \to VV$ where

$V = \gamma, Z, W$. Of these, the clearest signals come from the $ZZ \to 4\ell$ 'gold-plated' signal. These authors obtain lower bounds on $\widetilde{M}_S$ of 720 GeV, 665 GeV and 980 GeV from studies of final states with $V = \gamma, Z, W$ respectively. Subsequent to these early studies, more detailed studies at LEP-2 have been performed [22–24], mainly by experimental physicists with direct access to LEP data. These are naturally more refined; however, their results are in general agreement with the earlier studies.

Graviton effects at HERA: At HERA, running in the $e^{\pm}p$ mode, one can expect graviton-induced contributions to the deep inelastic process $e^{\pm}p \to e^{\pm}X$, described as the neutral current (NC) process. At the parton level, apart from a t-channel graviton exchange in $e^{\pm}q \to e^{\pm}q$ which adds coherently to the Standard Model contributions, there is a process with $e^{\pm}g \to e^{\pm}g$ which also arises from t-channel graviton exchange. These processes were first described by Mathews, Raychaudhuri and Sridhar [26] and subsequently, in somewhat greater detail, by Rizzo [27]. Interestingly, the graviton-induced contributions lead to an excess of events predicted in the high-Q^2 region. Though the early excitement about this has died down, the *integrated* data on NC events from both the H1 and ZEUS collaborations still show a moderate excess in high-Q^2 events. Since the graviton contributions seem to match the trend of the data, we get rather weak bounds (~ 450 GeV) on $\widetilde{M}_S$ from HERA. Two alternative approaches exist. One is to take the view that the high-Q^2 data are unreliable and restrict our analysis to the region $Q^2 < 10^4$ GeV^2; this provides a bound of about 900 GeV on $\widetilde{M}_S$ and is in agreement with bounds from other processes. The other [27] is to consider polarized beams and study the decay distributions; this may improve the bound by another 500 GeV or so. The H1 collaboration has recently presented a detailed study of the ADD model [28] which essentially substantiates these conclusions; however, they get a somewhat better bound ~ 700 GeV for $\lambda = -1$.

Graviton effects at the Tevatron: When we come to $p\bar{p}$ collisions at the Tevatron it is easy to see that a large number of processes can occur. Among processes with real graviton production the simplest [10,12] is $q\bar{q} \to g^* \to gG_{\vec{n}}$, which is analogous to the gravisstrahlung process. The signal will be a mono-jet and missing energy. However, when all contributions are fully computed, one still predicts a very small signal at Run-1, which would hardly be observable. Real graviton processes with associated W and Z production have been studied by Balazs *et al* [20] who find rather weak bounds $M_S > 500(300)$ GeV for final states with an associated $W(Z)$ boson (for $\lambda = -1$). This is for the best case $d = 2$; for larger values of d, the bounds are even weaker. Another possible signal can come from Drell–Yan production of leptons with a graviton radiated from any one of the legs of the Feynman diagram(s). This has been studied by Han, Rainwater and Zeppenfeld [29]. The final result is somewhat disappointing, since it leads to another weak bound of about 500 GeV on M_S (in the best case). It is thus apparent that real graviton processes are not the most profitable way to look for Kaluza–Klein gravitons of the ADD model, at least in Run-1 of the Tevatron.

The earliest study of virtual graviton processes at the Tevatron was the study of Drell–Yan dileptons by Hewett [13]. From her studies, Hewett concluded that one can get a lower bound on $\widetilde{M}_S$ of 920 (980) GeV for $\lambda = +1(-1)$. However, the details of this study were not made explicit. A more detailed study of dileptons was made by Gupta, Mondal and Raychaudhuri [30]. They considered the published results on dielectron data from the D0 collaboration and dimuon data from the CDF collaboration and compared it with the predictions of the ADD model using Bayesian statistics. The final result, is a bound of about 1 TeV, with an uncertainty of about 50 GeV either way — which improves only marginally over that of Hewett, though the study is more thorough. A similar study of the $t\bar{t}$ production cross-section was carried out by Mathews, Raychaudhuri and Sridhar [31] and independently by Rizzo [32] yielding a rather weak bound of about $\widetilde{M}_S > 660$ GeV. Pair-production of the electroweak vector bosons W and Z has been studied by Balasz *et al* [20]: the best bounds are $\widetilde{M}_S > 570$ GeV from W^+W^- production, for $\lambda = +1$ and $\widetilde{M}_S > 390$ GeV from ZZ production, for $\lambda = -1$.

The most interesting processes at the Tevatron in the context of the ADD model are those which give rise to dijet final states. These were first computed by Mathews, Raychaudhuri and Sridhar [33] and subsequently by Atwood, Bar-Shalom and Soni [34]. There are several sub-processes at the lowest order in QCD which lead to the production of dijet final states in $p\bar{p}$ collisions. To each QCD diagram with a gluon exchange, there corresponds a similar diagram with a spin-2 Kaluza–Klein graviton exchange. The results of this long, but straightforward, calculation have been used in ref. [33] to find the extra contributions to dijet production at the Tevatron and obtain bounds on them. Two dynamical variables are of interest for the comparison, namely the angular distribution and the dijet invariant mass. The angular distribution of the final-state partons is expected to be sensitive to the spin of the exchanged particle and hence to show considerable deviation from the lowest-order QCD prediction when spin-2 gravitons are exchanged. In fact, by fitting the data given by the CDF and D0 collaborations, a bound of $\widetilde{M}_S > 1.1$ TeV is obtained. However, when the invariant mass distribution is concerned, the maximum deviations arise in the higher bins, where the exact forms of λ_s and λ_t need to be used. Despite the larger errors in the higher invariant mass bins, Mathews *et al* have been able to derive lower bounds on M_S of about 1.7, 1.2, 1.4, 1.45 and 1.5 TeV for $d =$ 2, 3, 4, 5 and 6 respectively. These are better than the bounds obtained by looking at the angular distributions and, in fact, turn out to be higher than any other bounds obtained using current data from colliders.

Most of the studies discussed above have focussed on data available from Run 1 of the Tevatron. Since Run 2 is scheduled to begin soon, it is imperative that a *detailed* study be made of the results expected in this context.

Graviton effects at the LHC: We now turn from the analyses done at presently running machines to expectations for the future. The LHC is obviously the machine which is expected to yield the most promising results because of its large energy. Matters are somewhat complicated here because the typical parton-level centre-of-mass energy is likely to be 2–3 TeV, which is comparable to the values attachable to the string scale. One may, therefore, actually expect stringy excitations to be seen! No concrete predications can be made for these, since (a) the effective theory of graviton-matter interactions described above breaks down, and (b) the actual string theory is still not known. Three possibilities suggest themselves: (i) The analysis can be restricted to lower invariant mass bins alone, where the low-energy effective theory is valid, (ii) the analysis can be restricted to the case of M_S in the range of 5 TeV or more, so that again the low-energy effective theory is valid; or (iii) an extrapolation, using unitarity, can be made to the case when $\sqrt{s} > M_S$, as has been done for UHE cosmic rays [16], and the LHC analysis done using these formulae. The first of these options is a sound one, but it seems a pity to waste data which will be freely available. The second option is also theoretically sound, but it leaves a large range in M_S uncovered. The third option is perhaps the most interesting one, but has not been attempted in any studies so far. Till date, most of the analyses assume the second of the three options, which allows extrapolation of the Tevatron studies discussed above. Bounds on M_S range from 3 to 6 TeV.

In the context of the LHC, it is fair to say that the study of graviton-induced processes is still in its infancy. In view of the fact that this machine will run at energies comparable to the string scale and may well produce stringy excitations, there may be great surprises in store for us when the LHC data start coming in.

Graviton effects at lepton colliders of the future: Several studies have been performed of graviton-induced processes at a 500 GeV e^+e^- (or $\mu^+\mu^-$) collider. These essentially extrapolate the processes at LEP to higher energies and luminosities. This is also true, in a sense, of the work done involving $\gamma\gamma$ colliders, assuming a laser back-scattering mechanism. Of more interest are $e\gamma$ collisions, since they can lead to Compton-type effects, disussed in the context of astrophysics. There have been several studies, but these will become really interesting only in the future. It may be noted, however, that if one wishes to make studies at TeV-range lepton colliders, it might be necessary to use some of the extrapolation techniques necessary at the LHC. In all these studies, polarization of the incident beam can be used with profit to enhance the graviton signal relative to Standard Model backgrounds.

7. Summary

We have seen some of the phenomenological consequences of the ADD model, which envisages multiple graviton exchanges at low energies building up to electroweak-strength interactions in current collider experiments. The ADD model is motivated by a desire to explore the consequences of having TeV-scale strings, which are allowed by present experimental data, provided the Standard Model particles are confined to a 3-dimensional 'wall'. A number of interesting lower bounds — and on upper bound — on the string scale have already been obtained and it is of great interest to see if future collider experiments can find a signal to show that our world consists of more dimensions than four.

Acknowledgement

The author wishes to express his gratitude to K Sridhar and P Mathews, who introduced him to extra dimensions, and to his other collaborators, P Das, A Gupta and N K Mondal.

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