

## Non-minimal and non-universal supersymmetry

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**Abstract.** I motivate and discuss non-minimal and non-universal models of supersymmetry and supergravity consistent with string unification at  $10^{16}$  GeV.

**Keywords.** Supersymmetry; SUSY models; SUGRA models.

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### 1. Introduction

The motivations for TeV scale supersymmetry (SUSY) [1] remain as good as ever:

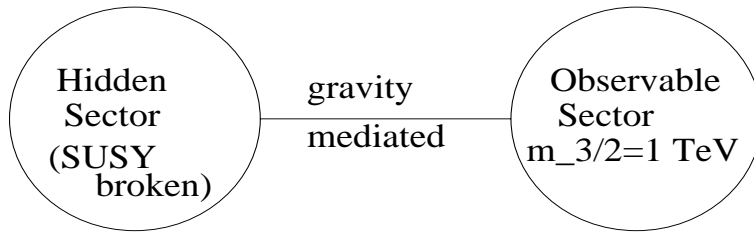
1. TeV scale SUSY cancels the quadratic divergences in the Higgs mass (hierarchy problem),
2. TeV scale SUSY ensures that the gauge couplings meet at  $M_{\text{GUT}} \sim 10^{16}$  GeV.

The discovery of  $D$ -branes allows gravity live in extra dimensions not felt by the gauge forces, and permits the effective gravitational coupling  $G_N E^2$  to be unified with the other gauge couplings at  $M_{\text{GUT}} \sim 10^{16}$  GeV [2]. It also opens a Pandora's box of many other higher-dimensional scenarios for the unification of gravity and gauge interactions, possibly at scales as low as 1 TeV [3]. However in many such scenarios there is no convincing explanation for the apparent merging of the gauge couplings at  $10^{16}$  GeV. In this talk we shall therefore discuss scenarios consistent with conventional high energy gauge unification, and ignore all large compact dimension scenarios.

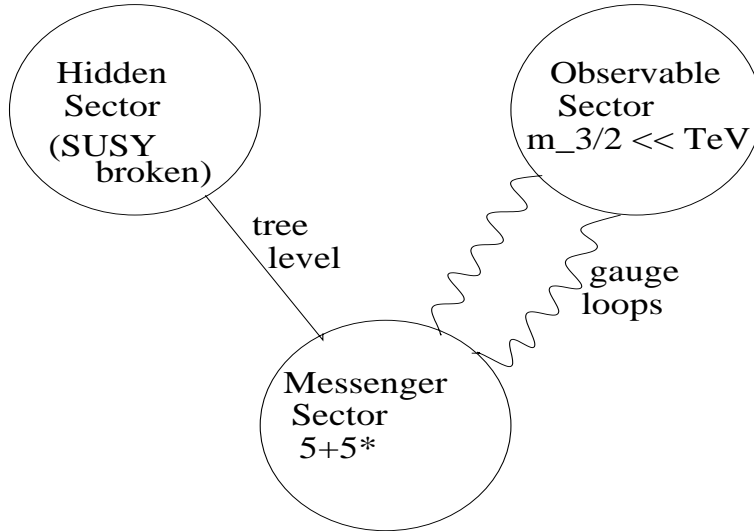
Since the vacuum is not manifestly supersymmetric, it must be broken. Mechanisms for SUSY breaking may be classified according to the magnitude of the gravitino mass  $m_{3/2}$ :

- $m_{3/2} \sim 1$  TeV (gravity mediated [4])
- $m_{3/2} \ll 1$  TeV (gauge mediated [5])
- $m_{3/2} \gg 1$  TeV (anomaly mediated [6])

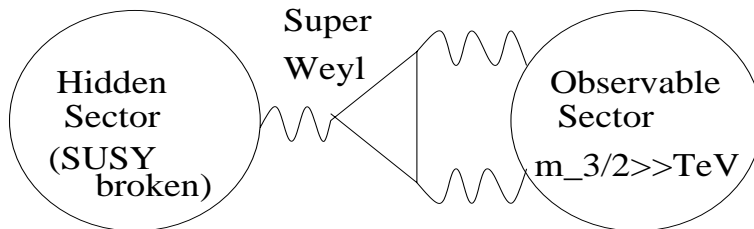
The three mechanism are illustrated schematically in figures 1–3. In this talk we shall concentrate mainly on the gravity mediated scenario, with the soft masses generated from an effective supergravity (SUGRA) theory and, according to our previous discussion, we assume these soft masses to be generated at a scale of about  $10^{16}$  GeV.



**Figure 1.** Gravity mediated SUSY breaking. There are operators  $\sim 1/M_P$  connecting the hidden sector to the observable sector which communicate the SUSY breaking.



**Figure 2.** Gauge mediated SUSY breaking. There may be operators  $\sim 1/M_P$  connecting the hidden sector to the observable sector but they play no important role since  $m_{3/2} \ll 1 \text{ TeV}$ .



**Figure 3.** Anomaly mediated SUSY breaking. There are *no* operators  $\sim 1/M_P$  connecting the hidden sector to the observable sector in this scenario, which allows the super Weyl anomaly contributions to dominate.

## 2. Elements of a SUGRA theory

As indicated in figures 1–3 the superpotential and Kahler potential [4] have the form

$$\begin{aligned} W &= W_{\text{hid}} + W_{\text{obs}} \\ K &= K_{\text{hid}} + K_{\text{obs}} \end{aligned} \quad (1)$$

and the Kahler function

$$G = K/M_P^2 + \ln |W/M_P^3|^2 \quad (2)$$

when inserted into the  $F$ -terms of the SUGRA potential

$$V_F = M_P^4 e^G [G^i (G^{-1})^j_i G_j - 3], \quad (3)$$

where  $G^i = \frac{\delta G}{\delta \phi_i}$ ,  $G_i = \frac{\delta G}{\delta \phi^{*i}}$ ,  $(G^{-1})^k_i G_k^j = \delta_i^j$ , leads to a hidden sector SUSY breaking order parameter  $F_i = -M_P^2 e^{G/2} (G^{-1})^j_i G_j$  and a gravitino mass

$$m_{3/2}^2 = \frac{1}{3M_P^2} \langle K^j_i F_i F^{*j} \rangle = M_P^2 e^{\langle G \rangle}, \quad (4)$$

where the last equality assumes  $\langle V_F \rangle = 0$ . Terms in the expansion of  $V_F$  then lead to soft SUSY breaking masses in the observable sector depending on the details of the Kahler potential. The common feature of the models we consider is that the gravitino mass is  $m_{3/2} \sim \text{TeV}$  which corresponds to a SUSY breaking scale in the hidden sector of  $\langle F_i \rangle \sim (3 \times 10^{10} \text{ GeV})^2$ .

In minimal SUGRA the Kahler potential is postulated to have the form

$$K = K_{\text{hid}} + \tilde{Q}^{*i} \tilde{Q}^i + \dots \quad (5)$$

where  $\tilde{Q}^i$  represents one of the squarks or sleptons, which are thereby assumed to have diagonal metric and minimal kinetic terms. This results in universal soft scalar masses

$$V_{\text{soft}} = m_0^2 (\tilde{Q}^{*i} \tilde{Q}^i + \dots) \quad (6)$$

where  $m_0^2 = m_{3/2}^2$ , and universal soft trilinear parameters of order  $m_{3/2}$  proportional to the Yukawa couplings in the superpotential. If the gauge kinetic functions  $f_a$  are independent of  $a$  then this results in universal gaugino masses  $M_{1/2}$  of order  $m_{3/2}$  at high energies.

In string theory the hidden sector consists of the dilaton  $S$  and moduli  $T_i$ , which get vacuum expectation values (VEVs) of order  $M_P$ . Although the string mechanism of SUSY breaking is unclear it may be parametrized in terms of the  $F$ -terms of  $S, T_i$  [7],

$$\begin{aligned} F^S &= \sqrt{3} m_{3/2} (S + S^*) \sin \theta e^{-i\gamma_S} \\ F^i &= \sqrt{3} m_{3/2} (T_i + T_i^*) \cos \theta \Theta_i e^{-i\gamma_i}, \end{aligned} \quad (7)$$

where  $\sum \Theta_i^2 = 1$ . The dilaton and moduli couple directly to the MSSM multiplets in a very complicated non-universal way in the Kahler potential

$$K = K_{\text{hid}} + \tilde{Q}^{*i} \tilde{Q}^i (T_1 + T_1^*)^{n_{Q_i}^{T_1}} (T_2 + T_2^*)^{n_{Q_i}^{T_2}} (T_3 + T_3^*)^{n_{Q_i}^{T_3}} (S + S^*)^{n_{Q_i}^S} + \dots, \quad (8)$$

where  $n_{Q_i}^{T_1}$ , etc. are the half-integer valued modular weights, whose  $i$  dependence leads to non-universality in the soft scalar masses

$$m_{Q_i}^2 = m_{3/2}^2 (1 + 3 \sin^2 \theta n_{Q_i}^S + 3 \cos^2 \theta n_{Q_i}^{T_a} \Theta_a^2). \quad (9)$$

The trilinears are similarly non-universal. The gauge kinetic functions in a  $D$ -brane theory [7] involving gauge groups on intersecting 9-branes and 5-branes with  $f_9 = S$ ,  $f_{5_a} = T_a$  leads to gaugino masses

$$M_9 = \sqrt{3} m_{3/2} \sin \theta e^{-i\gamma_S} \\ M_{5_i} = \sqrt{3} m_{3/2} \cos \theta \Theta_i e^{-i\gamma_i}. \quad (10)$$

Thus if different standard model gauge factors are on different branes the generic expectation is non-universal gaugino masses. In general the message is clear: from a modern string perspective non-universality is the natural expectation.

### 3. Fine-tuning in SUGRA models

An issue amongst SUSY phenomenologists at the moment is the absence of Higgs and superpartners at currently accessible experimental energies. Is this really a concern? The question revolves around the issue of how much fine-tuning one is prepared to tolerate. Although fine-tuning is not a well defined concept, the general notion of fine-tuning is unavoidable since it is the existence of fine-tuning in the standard model which provides the strongest motivation for low energy supersymmetry, and the widespread belief that superpartners should be found before or at the LHC. Although a precise measure of *absolute* fine-tuning is impossible, the idea of *relative fine-tuning* can be helpful in selecting certain models and regions of parameter space over others. It is useful to compare different models using a common definition of fine-tuning [8]

$$\Delta_a = \text{abs} \left( \frac{a}{M_Z^2} \frac{\partial M_Z^2}{\partial a} \right) \quad (11)$$

where  $a$  is an input parameter, and fine-tuning  $\Delta^{\text{max}}$  is defined to be the maximum of all the  $\Delta_a$ .

In a recent paper [9] we compared the fine-tuning of several different SUGRA models as listed below:

#### 1. Minimal supergravity

$$a_{\text{mSUGRA}} \in \{m_0^2, M_{1/2}, A(0), B(0), \mu(0)\}, \quad (12)$$

where as usual  $m_0$ ,  $M_{1/2}$  and  $A(0)$  are the universal scalar mass, gaugino mass and trilinear coupling respectively,  $B(0)$  is the soft breaking bilinear coupling in the Higgs potential and  $\mu(0)$  is the Higgsino mass parameter.

2. No-scale supergravity with non-universal gaugino masses

$$a_{\text{no-scale}} \in \{M_1(0), M_2(0), M_3(0), B(0), \mu(0)\}. \quad (13)$$

3.  $D$ -brane model

$$a_{D\text{-brane}} \in \{m_{3/2}, \theta, \Theta_1, \Theta_2, \Theta_3, B(0), \mu(0)\}, \quad (14)$$

where  $\theta$  and  $\Theta_i$  are the goldstino angles, with  $\Theta_1^2 + \Theta_2^2 + \Theta_3^2 = 1$ , and  $m_{3/2}$  is the gravitino mass. The gaugino masses are given by

$$\begin{aligned} M_1(0) &= M_3(0) = \sqrt{3}m_{3/2} \cos \theta \Theta_1 e^{-i\alpha_1}, \\ M_2(0) &= \sqrt{3}m_{3/2} \cos \theta \Theta_2 e^{-i\alpha_2}, \end{aligned} \quad (15)$$

and there are two types of soft scalar masses

$$\begin{aligned} m_{5152}^2 &= m_{3/2}^2 \left[ 1 - \frac{3}{2}(\sin^2 \theta + \cos^2 \theta \Theta_3^2) \right], \\ m_{51}^2 &= m_{3/2}^2 [1 - 3 \sin^2 \theta]. \end{aligned} \quad (16)$$

4. Anomaly mediated supersymmetry breaking

$$a_{\text{AMSB}} \in \{m_{3/2}, m_0^2, B(0), \mu(0)\}. \quad (17)$$

Our main results are shown in figures 4–7, corresponding to SUGRA models 1–4 above. In all models, fine-tuning is reduced as  $\tan \beta$  is increased, with  $\tan \beta = 10$  preferred over  $\tan \beta = 2, 3$ . Nevertheless, the present LEP2 limit on the Higgs and chargino mass of about 100 GeV and the gluino mass limit of about 250 GeV implies that  $\Delta^{\text{max}}$  is of order 10 or higher.

The fine-tuning increases most sharply with the Higgs mass. The reasons for this are spelled out in the analytic treatment in ref. [10]. The starting point for understanding fine-tuning are the minimal supersymmetric standard model (MSSM) minimization conditions

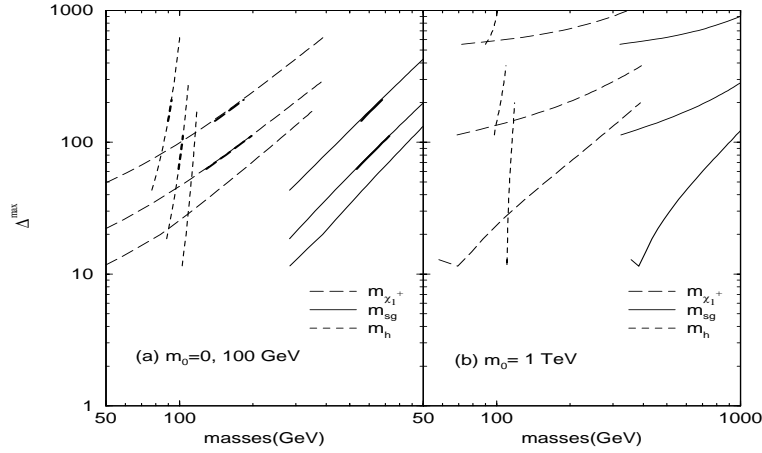
$$\frac{M_Z^2}{2} = -\mu^2(t) + \left( \frac{m_{H_D}^2(t) - m_{H_U}^2(t) \tan^2 \beta}{\tan^2 \beta - 1} \right), \quad (18)$$

$$\sin 2\beta = \frac{2m_3^2(t)}{m_1^2(t) + m_2^2(t)}, \quad (19)$$

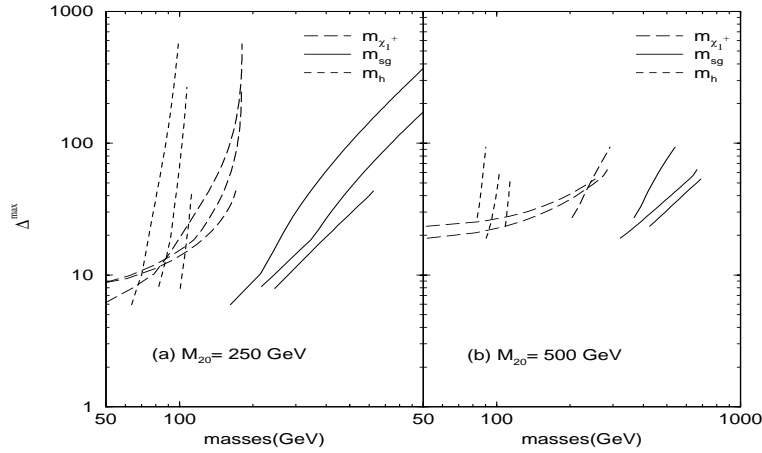
where

$$\tan \beta = v_2/v_1, \quad M_Z^2 = \frac{1}{2}(g'^2 + g_2^2)(v_1^2 + v_2^2). \quad (20)$$

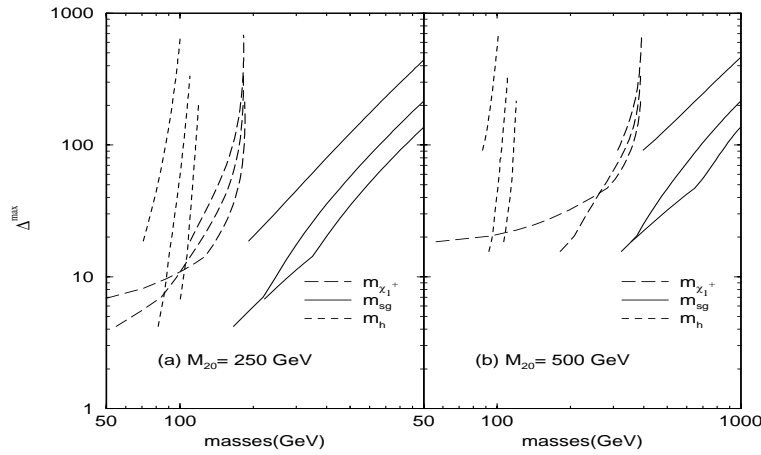
From these conditions the  $Z$  mass may be expanded in terms of high energy input parameters. For example for  $\tan \beta = 2.5$  we find [10]



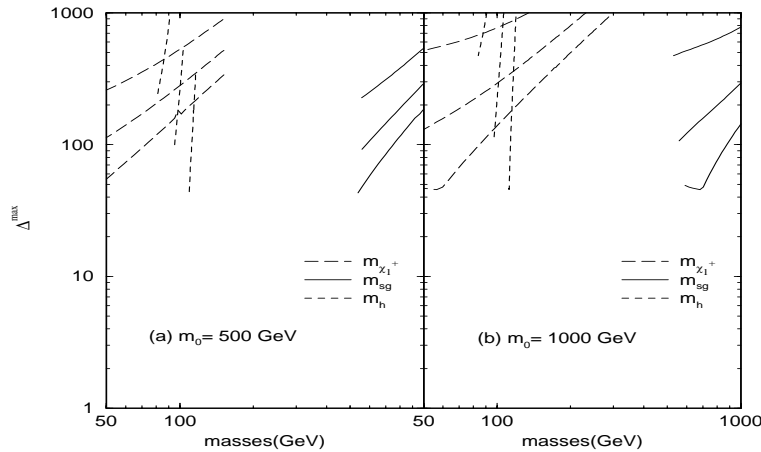
**Figure 4.** Results for the minimal SUGRA model. The maximum sensitivity parameter  $\Delta^{\max}$  is plotted as a function of the lightest CP even Higgs mass (short dashes), gluino mass (solid line) and lightest chargino (long dashes). For each particle type, the three sets of curves correspond to  $\tan\beta = 2, 3, 10$ , from top left to bottom right, respectively. In panel (a) the shorter, thicker lines correspond to  $m_0 = 0$ , while the longer lines are those for  $m_0 = 100$  GeV. In panel (b) the results correspond to  $m_0 = 1000$  GeV.



**Figure 5.** Results for the no-scale with non-universal gaugino masses. The maximum sensitivity parameter  $\Delta^{\max}$  is plotted as a function of the lightest CP even Higgs mass (short dashes), gluino mass (solid line) and lightest chargino (long dashes). For each particle type, the three sets of curves correspond to  $\tan\beta = 2, 3, 10$ , from top left to bottom right, respectively. In panel (a) we fix  $M_2(0) = 250$  GeV, while in panel (b)  $M_2(0) = 500$  GeV.



**Figure 6.** Results for the  $D$ -brane model. The maximum sensitivity parameter  $\Delta^{\max}$  is plotted as a function of the lightest CP even Higgs mass (short dashes), gluino mass (solid line) and lightest chargino (long dashes). For each particle type, the three sets of curves correspond to  $\tan \beta = 2, 3, 10$ , from top left to bottom right, respectively. In panel (a) we fix  $M_2(0) = 250$  GeV, while in panel (b)  $M_2(0) = 500$  GeV.



**Figure 7.** Results for the anomaly mediated supersymmetry breaking model. The maximum sensitivity parameter  $\Delta^{\max}$  is plotted as a function of the lightest CP even Higgs mass (short dashes), gluino mass (solid line) and lightest chargino (long dashes). For each particle type, the three sets of curves correspond to  $\tan \beta = 2, 3, 10$ , from top left to bottom right, respectively. In panel (a) we fix  $m_0 = 500$  GeV, while in panel (b)  $m_0 = 1000$  GeV.

$$\begin{aligned} \frac{M_Z^2}{2} = & -0.87 \mu^2(0) + 3.6 M_3^2(0) - 0.12 M_2^2(0) + 0.007 M_1^2(0) \\ & - 0.71 m_{H_U}^2(0) + 0.19 m_{H_D}^2(0) + 0.48 (m_Q^2(0) + m_U^2(0)) \\ & - 0.34 A_t(0) M_3(0) - 0.07 A_t(0) M_2(0) - 0.01 A_t(0) M_1(0) + 0.09 A_t^2(0) \\ & + 0.25 M_2(0) M_3(0) + 0.03 M_1(0) M_3(0) + 0.007 M_1(0) M_2(0). \end{aligned} \quad (21)$$

It is clear that  $M_3(0)$  dominates the right-hand side, so to reduce fine-tuning we should make  $M_3(0)$  as small as possible. However  $M_3(0)$  cannot be made too small otherwise the gluino mass becomes too light. More importantly  $M_3(0)$  contributes to the radiative corrections to the Higgs mass. For a given tree-level Higgs mass the experimental bound implies a lower bound on  $M_3(0)$  which may be stronger than that coming from the gluino mass bound. Moreover this bound grows exponentially with the Higgs mass since the radiatively corrected Higgs mass includes the terms

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + (34 \text{ GeV})^2 \ln \left( \frac{m_{t_1}^2 m_{t_2}^2}{m_t^4} \right) + \dots \quad (22)$$

and the product of stop masses may be expanded as

$$\begin{aligned} m_{t_1}^2 m_{t_2}^2 \approx & (170 \text{ GeV})^4 + 26 M_3^4(0) + 1.5 A_t(0) M_3^3(0) \\ & + (2.5 m_Q^2(0) + 3.5 m_U^2(0) - 2.1 m_{H_U}^2(0)) M_3^2(0) \\ & + (134 \text{ GeV})^2 m_Q^2(0) + (130 \text{ GeV})^2 m_U^2(0) - (108 \text{ GeV})^2 m_{H_U}^2(0) \\ & + (430 \text{ GeV})^2 M_3^2(0) - (180 \text{ GeV})^2 M_2(0) M_3(0) \\ & + (180 \text{ GeV})^2 A_t(0) M_3(0) - (59 \text{ GeV})^2 A_t^2(0) - (65 \text{ GeV})^2 \mu^2(0) \\ & + (67 \text{ GeV})^2 A_t(0) \mu(0) - (210 \text{ GeV})^2 M_3(0) \mu(0). \end{aligned} \quad (23)$$

Taken together eqs (22) and (23) show that any shortfall in the tree-level contribution to the Higgs mass must be compensated by exponential increases in stop masses, which in turn involves exponential increases in  $M_3(0)$ , and hence exponential increases in fine-tuning.

The Higgs fine-tuning curves are fairly model independent, and as the Higgs mass limit rises above 100 GeV come to quickly dominate the fine-tuning. We conclude that the prospects for the discovery of the Higgs boson at LEP2 are good.

For each model there is a correlation between the Higgs, chargino and gluino mass, for a given value of fine-tuning. For example if the Higgs is discovered at a particular mass value, then the corresponding chargino and gluino mass for each  $\tan \beta$  can be read off from figures 4–7.

The new general features of the results may then be summarized as follows:

- The gluino mass curves are less model dependent than the chargino curves, and this implies that in all models if the fine-tuning is not too large then the prospects for the discovery of the gluino at the Tevatron are good.
- The fine-tuning due to the chargino mass is model dependent. For example in the no-scale model with non-universal gaugino masses and the  $D$ -brane scenario the charginos may be relatively heavy compared to mSUGRA.
- Some models have less fine-tuning than others. We may order the models on the basis of fine-tuning from the lowest fine-tuning to the highest fine-tuning:  $D$ -brane scenario < generalized no-scale SUGRA < mSUGRA < AMSB.

- The  $D$ -brane model is less fine-tuned partly because the gaugino masses are non-universal, and partly because there are large regions where  $\Delta_{m_{3/2}}$ ,  $\Delta_{\mu(0)}$ , and  $\Delta_\theta$  are all close to zero. However in these regions the fine tuning is dominated by  $\Delta_\Theta$ , and this leads to an inescapable fine-tuning constraint on the Higgs and gluino mass.

#### 4. Can supersymmetric soft phases be the source of all CP violation?

A further motivation for non-universal SUSY comes from the possibility that all of CP violation arises from the phases present on soft SUSY masses. With universal soft masses this is impossible, but with non-universal soft masses it has recently been shown that even if the CKM phase is zero, it is possible to account for  $\epsilon$  and  $\epsilon'$  via gluino exchange diagrams involving left-handed down squarks mixing with right-handed strange squarks via a complex soft mass term [11]. We have shown [12] that CP violation in the  $B$  sector may also arise from chargino-stop box diagrams with a complex stop mixing mass. A particular structure is required to avoid excessive contributions from similar diagrams to  $\epsilon$ . An important feature is that only *flavour independent* phases (for example phases on the non-universal gaugino masses) are required. The model predicts  $\sin 2\beta = -\sin 2\alpha$  which implies that  $B \rightarrow \psi K_S$  and  $B \rightarrow \pi^+ \pi^-$  are related. Further details are summarized in table 1.

**Table 1.**

Observable	Dominant contribution	Flavor content
nEDM	$\tilde{g}, \tilde{\chi}^+, \tilde{\chi}^0$	$(\delta_{dd})_{LR}, \sim \tilde{K}_{ud} \tilde{K}_{ud}^*$
$\epsilon$	$\tilde{g}$	$(\delta_{ds})_{LR}$
$\epsilon'$	$\tilde{g}$	$(\delta_{ds})_{LR}$
$\Delta m_K$	SM	SM
$K_L \rightarrow \pi \nu \bar{\nu}$	SM, $\tilde{g}$	$(\delta_{ds})_{LR}$
$\Delta m_{B_d}$	$\tilde{\chi}^+$	$ \tilde{K}_{tb} \tilde{K}_{td}^* $
$\Delta m_{B_s}$	SM, $\tilde{\chi}^+$	$ \tilde{K}_{tb} \tilde{K}_{ts}^* $
$\sin 2\beta$	$\tilde{\chi}^+$	$\tilde{K}_{tb} \tilde{K}_{td}^*$
$\sin 2\alpha$	$\tilde{\chi}^+$	$\tilde{K}_{tb} \tilde{K}_{td}^*$
$\sin 2\gamma$	$\tilde{\chi}^+$	$\tilde{K}_{tb} \tilde{K}_{ts}^*$
$\mathcal{A}_{CP}(b \rightarrow s\gamma)$	$\tilde{\chi}^+$	$\sim \tilde{K}_{tb} \tilde{K}_{ts}^*$
$\Delta m_D$	$\tilde{g}$	$\sim  \tilde{K}_{tc} \tilde{K}_{tu}^* $
$n_B/n_\gamma$	$\tilde{\chi}^+, \tilde{\chi}^0, \tilde{t}_R$	–

## 5. Conclusion

We have concentrated on conventional theories in which gauge unification occurs at a high energy scale  $10^{16}$  GeV, and the possibility that full string unification, including gravity, occurs at this scale. We have further concentrated on the conventional scenario for SUSY breaking, namely gravity mediated SUSY breaking. In this framework, non-universal soft SUSY masses are the natural expectation, and the absence of FCNC's remains an important challenge for SUSY. However there are phenomenological advantages to having non-universal soft masses which we have highlighted, namely the reductions in fine-tuning which arise from having  $M_3(0) < M_2(0)$ , and also the resultant lowering of the prediction for  $\alpha_s(M_Z)$  in this case. Also the possibility that all of CP-violation could arise from flavour-independent phases of soft SUSY masses is interesting.

The particle mass most sensitive to fine-tuning is the lightest Higgs boson mass, with fine-tuning growing exponentially with the Higgs mass. In this respect it is worth remembering that we have been assuming the MSSM based on two Higgs doublets with a mass term  $\mu H_1 H_2$ . In the next-to-minimal supersymmetric standard model (NMSSM) [13] the mass term  $\mu$  is replaced by the vacuum expectation value (VEV) of a Higgs singlet  $N$ ,  $\mu H_1 H_2 \rightarrow \lambda N H_1 H_2$ . In the NMSSM the lightest physical CP even Higgs boson may be heavier than in the MSSM. The NMSSM also solves the  $\mu$ -problem and opens the electroweak baryogenesis window which is tightly constrained in the MSSM by LEP bounds.

## Acknowledgements

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