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Neutrino masses from SUSY: Different contributions and their implications

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Abstract. We discuss the various sources of neutrino masses in supersymmetric standard models with explicit lepton number violation. We show that the bilinear lepton number violating soft terms in models with either bilinear or trilinear lepton number violating couplings in the superpotential, play an important role in determining the neutrino mass spectrum. A comparative study of the neutrino mass spectrum and its implications for the present neutrino anomalies in these models is presented.

Keywords. R-parity violation; neutrino masses.

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1. Introduction

The results from the solar and the atmospheric neutrino experiments indicate the existence of small masses for the neutrinos. For the atmospheric neutrino anomaly, the recent results from the super-Kamiokande [1] favour a neutrino mass squared difference of $\Delta m^2 \sim 10^{-3}\,\text{eV}^2$ with large mixing, whereas the solar neutrino [2] experiments indicate a mass squared difference of $\Delta m^2 \sim 10^{-6}\,\text{eV}^2$ (MSW [3] conversion) with small or large mixing or $\Delta m^2 \sim 10^{-10}\,\text{eV}^2$ (vacuum oscillations) with large mixing. Thus, the neutrino mass spectrum seems to be characterized by hierarchical masses with one or two large mixings. Various theoretical models have been constructed to realize such a mass spectrum for the neutrinos [4].

The minimal supersymmetric standard model (MSSM) [5] motivated for completely different reasons, provides a natural framework where neutrino masses with hierarchical structure and large mixing can be accommodated if lepton number violation is assumed. The minimal supersymmetric extension of the standard model naturally allows for lepton and baryon number violating couplings into the superpotential. These are normally suppressed by imposing a discrete symmetry called R-parity (R_p) so as to save the proton from decaying rapidly. But, the imposition of R-parity removes both the baryon and lepton number violating couplings which is not necessary for a long proton life time. One can safely assume symmetries other than R_p which allow for either lepton number violation or baryon number violation into the superpotential. The lepton number violating couplings are given as

$$W_{\not R_p} = \epsilon_i L_i H_2 + \lambda'_{ijk} L_i Q_j d^c_k + \lambda_{ijk} L_i L_j e^c_k, \quad (1)$$

where standard notation [5] has been used with i, j, k as generation indices and SU(2), colour indices are suppressed. From $W_{\not R_p}$ we see that, these couplings can be either of bilinear (ϵ_i) or trilinear type (λ',λ). The presence of either of these couplings naturally leads to majorana masses for the neutrinos.

One of the most important sources of neutrino masses in the presence of the above couplings arises from the bilinear lepton number violating soft terms. Such terms are generally present in the soft potential when one considers bilinear lepton number violating terms in the superpotential [6–11]. If one considers only trilinear lepton number violating couplings in the superpotential, such bilinear soft terms are generated in the soft potential at the weak scale [12] due to renormalization group (RG) scaling. Thus, in theories where supersymmetry is broken at a high scale, such terms are always present in the soft potential at the weak scale, if one considers either trilinear or bilinear lepton number violating couplings in the superpotential.

The presence of such bilinear soft terms would lead to generation of vacuum expectation values (νevs) for the sneutrinos, which in turn leads to a mixing between the neutrinos and the neutralinos, thus giving rise to a tree level mass to the neutrino. Only one of the neutrinos attains mass in this manner. The other two masses arise from loop diagrams involving the lepton number violating Yukawa couplings and diagrams involving gauge couplings. The neutrino mass spectrum in these models is characterized by the following properties:

- neutrino masses are $\propto R_p$ violating couplings,
- hierarchical masses for the neutrinos and,
- natural large mixing.

In the present paper, we try to understand the above characteristics of the neutrino mass spectrum in these models and study their implications for the neutrino anomalies. The paper is organized as follows. In §2, we review the salient features of the neutrino mass spectrum in the presence of bilinear lepton number violation in the superpotential. We discuss results for two popular models namely, constrained MSSM and the minimal messenger model of the gauge mediated supersymmetry breaking (MMM). In §3, we present the neutrino mass structure in the presence of trilinear couplings and discuss the implications for the neutrino anomalies due to the presence of soft bilinear lepton number violating terms. We end with a discussion in §4.

2. Bilinear R_p violation

Neutrino masses in bilinear R-parity violating models and the associated phenomenology has been reviewed in Mukhopadhyaya's [13] article in this issue. Here, we basically repeat the salient features of these models and in doing so, we follow a slightly different approach which is more analytical.

In these class of models, we assume only bilinear R-parity violating terms to be present in the superpotential. This class of models has been studied well in [6–10] as they are theoretically well motivated. These models are closely related to models where R-parity is broken spontaneously [14] if one assumes $\epsilon \equiv \nu_R h_\nu$, where ν_R are the singlet fields and

h_ν are their Yukawa couplings. Moreover, such terms can be generated by the spontaneous breaking of a Peccei–Quinn symmetry which leads to the generation of μ and ϵ_i at the weak scale through dim 5 operators [9,15].

The most comprehensive work in these models has been done by Hempfling [8] who studies the neutrino mass spectrum and its implications for the neutrino anomalies numerically. We follow here the approach of [9] which is more analytical.

2.1 *The framework*

The superpotential in this model is given by

$$W = h^u_{ij} Q_i u^c_j H_2 + h^d_i Q_i d^c_i H'_1 + h^e_i L'_i e^c_i H'_1 + \mu H'_1 H_2 + \epsilon_i L'_i H_2, \tag{2}$$

where we have used the standard notation [5] and the superfields are in the mass basis of the down quarks and charged leptons in the absence of the bilinear lepton number violating terms. In this basis, the tree level neutrino mass generated comes out to be proportional to the misalignment induced due to RG scaling between the bilinear soft lepton number violating terms and the R-parity violating parameters ϵ_i [8,10]. Alternatively, one can work in the basis where the bilinear R-parity violating terms are absent in the superpotential. This is possible because, the superfields L'_i and H'_1 carry the same quantum numbers under $SU(2)_L \times U(1)_Y$ and a simple redefinition of the fields would remove the bilinear $\not{L}$ terms from the superpotential.

We define

$$\begin{aligned}
H_1 &= c_3 H'_1 + s_3(s_2(c_1 L'_2 + s_1 L'_1) + c_2 L'_3) \\
L_1 &= -s_1 L'_2 + c_1 L'_1 \\
L_2 &= c_2(c_1 L'_2 + s_1 L'_1) - s_2 L'_3 \\
L_3 &= -s_3 H'_1 + c_3(s_2(c_1 L'_2 - s_1 L'_1) + c_2 L'_3)
\end{aligned} \tag{3}$$

$$\begin{aligned}
\epsilon_1 &= \mu s_1 s_2 s_3, \qquad \epsilon_2 = \mu c_1 s_2 s_3, \\
\epsilon_3 &= \mu s_3 c_2, \qquad \mu' = \mu c_3,
\end{aligned} \tag{4}$$

and $\mu \equiv (\mu'^2 + \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)^{1/2}$.

Such a redefinition has the following consequences. (1) The lepton number violation now reappears as trilinear terms in the superpotential. (2) The charged lepton mass matrix is no longer diagonal. In the rotated basis, the superpotential now looks as

$$\begin{aligned}
W = {} & L_i \frac{(M_l)_{ij}}{\langle H_1 \rangle} e^c_j H_1 + \mu H_1 H_2 + \frac{m_i}{\langle H_1 \rangle} Q_i d^c_i (H_1 - \tan\theta_3 L_3) \\
& + L_1 L_3(-h^e_1 c_1 s_3 e^c_1 + h^e_2 s_1 s_3 e^c_2) \\
& + L_2 L_3(-h^e_1 s_1 s_3 c_2 e^c_1 - h^e_2 c_1 c_2 s_3 e^c_2 + h^e_3 s_2 s_3 e^c_3),
\end{aligned} \tag{5}$$

where M_l now represents the non-diagonal mass matrix of the charged leptons. The mass basis for the charged leptons in the presence of non-zero ϵ_i are denoted by $\alpha = e, \mu, \tau$ and are defined as:

$$L_i = (O_L^T)_{i\alpha} L_\alpha, \quad e_i^c = (O_R^T)_{i\alpha} e_\alpha^c, \tag{6}$$

where O_L is determined by requiring

$$O_L M_l O_R^T = \text{diagonal}. \tag{7}$$

The trilinear lepton number violating couplings get the following form in the new basis:

$$W = -\frac{\tan\theta_3}{\langle H_1 \rangle} [(O_L^T)_{3\alpha} L_\alpha] \left(m_\beta^l L_\beta e_\beta^c + m_i^D Q_i d_i^c \right). \tag{8}$$

The soft potential of these models also contains bilinear lepton number violating terms (B_i). But, these terms cannot be removed by the redefinition of the fields due to the inequality of the leptonic and the Higgs mass terms as well as the corresponding B parameters at the weak scale. In general, the B parameters can be typically of $O(M_{\rm SUSY})$ leading to unacceptably large neutrino mass. A natural way of suppressing this large mass is to consider universal soft masses and B terms at the high scale. Exploiting the universality, the scalar potential in the basis where the bilinear $\not{L}$ terms are rotated away from the superpotential can be written as

$$\begin{aligned} V = &(m_{H_1}^2 + \mu^2)|H_1^0|^2 + (m_{H_2}^2 + \mu^2)|H_2^0|^2 + m_{\tilde{\nu}_3}^2 |\tilde{\nu_3}|^2 + m_{\tilde{\nu}_2'}^2 |\tilde{\nu_2}|^2 + m_{\tilde{\nu}_1{}'}^2 |\tilde{\nu_1}|^2 \\ &- \left[\mu H_2^0 \left(B H_1^0 + c_3 s_3 \tilde{\nu}_3 (\Delta B_H - s_2^2 \Delta B_L) - c_2 s_2 s_3 \tilde{\nu}_2 \Delta B_L \right) \ + \text{H.c.} \right] \\ &+ \left[-c_2 s_2 \Delta m_L (s_3 H_1^0 + c_3 \tilde{\nu}_3) \tilde{\nu}_2^* + s_3 c_3 \tilde{\nu}_3 H_1^{0*} (\Delta m_H - s_2^2 \Delta m_L) \ + \ \text{H.c.} \right] \\ &+ \frac{1}{8} (g^2 + g'^2) (|H_1^0|^2 + |\tilde{\nu}_1|^2 + |\tilde{\nu}_2|^2 + |\tilde{\nu}_3|^2 - |H_2^0|^2)^2, \end{aligned} \tag{9}$$

where

$$\begin{aligned} m_{H_1}^2 &= m_{H_1'}^2 + s_3^2 \Delta m_H - s_2^2 s_3^2 \Delta m_L, \\ m_{\tilde{\nu}_3}^2 &= m_{\tilde{\nu}_3'}^2 - s_3^2 \Delta m_H - s_2^2 c_3^2 \Delta m_L, \\ m_{\tilde{\nu}_2}^2 &= m_{\tilde{\nu}_2'}^2 + s_2^2 \Delta m_L, \\ B &= B_\mu + s_3^2 \Delta B_H - s_2^2 s_3^2 \Delta B_L, \end{aligned} \tag{10}$$

and

$$\Delta m_{L,H} \equiv \left(m_{\tilde{\nu}_3'{}^2} - m_{\tilde{\nu}_2', H_1'}^2 \right), \quad \Delta B_{L,H} \equiv (B_3 - B_{2,\mu}). \tag{11}$$

In writing the above, we have neglected the contributions coming from the first two generation Yukawa couplings. The parameters $\Delta m_{L,H}$ and $\Delta B_{L,H}$ are equal to zero at the high scale, due to the universality condition, but are nonzero at the weak scale due to the differences in RG scaling of the soft terms of the Higgs and the leptons and within the leptonic generations. The relevant RG equations for these parameters given in [8,9,11] show that these parameters are proportional to the down quark and the tau lepton Yukawa couplings.

The terms linear in the sneutrino fields in the soft potential would now generate vacuum expectation values for the sneutrinos. Since, these VEVs are found to be much smaller than the Higgs vacuum expectation values, we can determine them analytically as

$$\langle\tilde{\nu_2}\rangle \approx \frac{c_2 s_2 s_3}{(m^2_{\tilde{\nu}_2}+D)}(v_1\Delta m_L - \mu v_2 \Delta B_L)\ ,$$
$$\langle\tilde{\nu_3}\rangle \approx \frac{c_3 s_3}{(m^2_{\tilde{\nu}_3}+D)}(v_1(-\Delta m_H + s_2^2\Delta m_L) - \mu v_2(-\Delta B_H + s_2^2 \Delta B_L))\ . \tag{12}$$

As we have neglected the first two generation Yukawa couplings, only two of the sneutrinos attain *vev*s. Due to these *vev*s, neutrinos now mix with the neutralinos giving rise to a tree level mass to one of the neutrinos. In the limit where the sneutrino *vev*s are much smaller than the Higgs *vev*s, one can use the seesaw approximation [7] to determine the tree level neutrino mass matrix. In the charged lepton mass basis this matrix is given as

$$M_0 = m_0 O_L \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_\phi^2 & s_\phi c_\phi \\ 0 & s_\phi c_\phi & c_\phi^2 \end{pmatrix} O_L^T, \tag{13}$$

where O_L has been defined above and

$$\tan\phi = \frac{\langle\tilde{\nu_2}\rangle}{\langle\tilde{\nu_3}\rangle} \tag{14}$$

and

$$m_0 = \frac{\mu(cg^2+g'^2)(\langle\tilde{\nu}_2\rangle^2 + \langle\tilde{\nu}_3\rangle^2)}{2(-c\mu M + 2M_W^2 c_\beta s_\beta(c+\tan^2\theta_W))}. \tag{15}$$

As noted earlier, the tree level mass matrix eq. (13) gives mass to only one of the neutrinos. The other two masses are generated at the one loop level through the loops involving lepton number violating Yukawa couplings of eq. (8). These masses take the following form in the charged lepton mass basis where we have retained only the dominant contribution [9,16]:

$$(M_{1b})_{\alpha\beta} = m_{1b}(O_L)_{\alpha 3}(O_L)_{\beta 3}, \tag{16}$$

$$M_{1\tau} = m_{1\tau}\begin{pmatrix} O^2_{L_{13}} & O_{L_{13}}O_{L_{23}} & 0 \\ O_{L_{13}}O_{L_{23}} & O^2_{L_{23}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{17}$$

where

$$m_{1b,1\tau} = N_c \frac{m^3_{b,\tau}}{16\pi^2 v_1^2}\tan\theta_3^2 \sin\phi_{b,\tau}\cos\phi_{b,\tau} \quad \ln\left(\frac{M^2_{b_2,\tau_2}}{M^2_{b_1,\tau_1}}\right), \tag{18}$$

and $N_c = 3, 1$ for the $\tilde{b}$ and $\tilde{\tau}$. We see that the 1-loop contribution also depends on the down quark and the tau lepton Yukawa couplings. Thus, in the limit where the down quark and the tau lepton Yukawa couplings tend to zero, there will be no neutrino masses in the model. The total mass matrix including the 1-loop corrections is given by

$$\mathcal{M}_\nu = M_0 + M_{1b} + M_{1\tau}. \tag{19}$$

The above mass matrix is written in the mass basis of the charged leptons. It has the following simple form in the basis:

$$O_L^T \mathcal{M}_\nu O_L \;\approx\; \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_0 s_\phi^2 + m_{1\tau} N_2^{-4} c_2^2 s_2^2 c_3^2 & m_0 s_\phi c_\phi + m_{1\tau} N_2^{-4} c_2 s_2^3 c_3^3 \\ 0 & m_0 s_\phi c_\phi + m_{1\tau} N_2^{-4} c_2 s_2^3 c_3^3 & m_0 c_\phi^2 + m_{1b} + m_{1\tau} N_2^{-4} s_2^4 c_3^4 \end{pmatrix}. \tag{20}$$

From the above, we see that one of the neutrinos remains massless in the limit of vanishing first two generation Yukawa couplings. The eigenvalues of the lower 2×2 block of eq. (20) give the two neutrino masses in this approximation. One of the eigenvalues is essentially the tree level contribution whereas the other is essentially the loop induced contribution. In general, the tree level contribution dominates over the loop contribution giving rise to the hierarchical pattern in the neutrino masses. Addition of the first two generation Yukawa couplings would give rise to a much smaller mass compared to these two masses [8,9,11].

The total mixing is determined by the product of the matrices,

$$U = O_L O_\nu^T, \tag{21}$$

where O_ν is the matrix diagonalizing the RHS of eq. (20). O_L is basically dependent on the ratios of the R-parity violating parameters whereas, O_ν depends both on R-parity violating parameters and the soft supersymmetry breaking parameters. The contribution to the mixing from O_ν is small unless the loop level mass is comparable to the tree level mass. Thus, if all the R-parity violating parameters are of similar magnitude, large mixing is natural in these models. From the above we see that, the neutrino mass spectrum depends crucially on the soft sector of the theory and hence on the way supersymmetry is broken. We consider here two popular models of supersymmetry breaking namely, minimal supergravity and minimal messenger model (MMM) of gauge mediated supersymmetry breaking and study the neutrino mass spectrum in these scenarios.

2.2 *Constrained MSSM*

As mentioned earlier, universality at the high scale for the soft parameters leads naturally to small neutrino masses at the weak scale. Constrained MSSM i.e, minimal supergravity with universal boundary conditions at $M_{\rm GUT}$ provides an appropriate framework from this point of view. The total parameters of the above model in the CMSSM framework are the standard CMSSM parameters m, M_2, $\tan\beta$, A and the sign(μ) along with the R-parity violating parameters ϵ_i or the three angles s_1, s_2, s_3.

It is found that the loop mass $m_{1\tau}$ contributes negligibly to the neutrino mass spectrum, throughout the CMSSM parameter space [8,9]. In the limit $m_{1\tau} \ll m_{1b}, m_0$ approximate expressions for the neutrino masses can be given as [9]

$$\begin{aligned} m_{\nu_3} &\sim m_0 + m_{1b}, \\ m_{\nu_2} &\sim \frac{m_0 m_{1b} s_\phi^2}{m_0 + m_{1b}}, \end{aligned} \tag{22}$$

where the approximation $s_\phi^2 \ll 1$ is used in obtaining the second line of the above. One of the neutrinos remains massless in this approximation. The main features of the neutrino mass spectrum in this framework can be summarized as follows :

- For most of the CMSSM parameter space, the tree level contribution dominates over the loop contribution giving rise to the hierarchical pattern in the mass spectrum [8,9].
- But, there also exist regions in the CMSSM parameter space, where the two contributions to the sneutrino vevs, (eq. (12)) cancel each other for one particular sign of μ. In these regions, the loop contribution, m_{1b} can become comparable to the tree level mass, m_0 and cancellations among m_0 and m_{1b} can take place. Here, the two neutrinos form a pseudo-Dirac pair with a common mass $m_0 s_\phi$ relevant for solutions of the neutrino anomalies [9].
- The mixing matrix is given by the product $O_L O_\nu^T$. Even in the case where O_ν allows only small mixing, large mixing can be generated from O_L, as it depends only on the ratios of the R-parity violating parameters [9].
- Even though the neutrino masses are suppressed in these scenarios, they are typically of the $O(\text{ MeV})$. Thus, the R-parity violating parameters have to be chosen to be much suppressed compared to the typical order of the supersymmetric breaking scale if neutrino masses are to be of the right order to solve the neutrino anomalies [8,9].
- Numerical results from Hempfling [8] show that solutions for solar (either with MSW conversion or vacuum oscillations) and the atmospheric neutrino anomalies can be accommodated naturally within these models.

2.3 *MMM or minimal messenger model*

The minimal messenger model of gauge mediated supersymmetry breaking [17] is characterized by the vanishing of B and A parameters at the high scale $X \sim$ few hundred TeV. The mass terms for the leptons and one of the Higgs (H_1) are equal at the high scale, whereas the B_i terms are assumed to vanish, thus giving rise to universality in the soft terms relevant for the neutrino mass spectrum. The framework is more constrained than the CMSSM, as the neutrino mass spectrum is now essentially controlled by only four parameters namely Λ and the three R-parity violating couplings, ϵ_i.

The parameter $s_\phi^2 \ll 1$ over most of the parameter space. The approximate eigenvalues for the neutrinos are given as [11]:

$$
\begin{aligned}
& m_{\nu_3} \sim m_0 + m_{1b}, \\
& \frac{m_{\nu_2}}{m_{\nu_3}} \sim c_2^2 s_2^2 \frac{m_{1\tau}}{m_0 + m_{1b}}.
\end{aligned}
\tag{23}
$$

The mixing is essentially dictated by O_L as the mixing from O_ν is very small. Thus, we see that the features of the neutrino mass spectrum like hierarchical structure and large mixing continue to stay in this framework also. But, due to the very constrained nature of this framework, it has the following consequences when one tries to find solutions for the neutrino anomalies [11]:

- It has been found that solutions for the both the solar (either with MSW conversion or vacuum oscillations) and atmospheric neutrino problems can be accommodated in this model separately.
- Unlike in the case of CMSSM, there exists no regions in the MMM parameter space where the loop mass becomes comparable to the tree level mass. This gives rise to a strong hierarchy within the mass eigenvalues. Consequently, there is no parameter space where simultaneous solutions for solar (with MSW conversion) and atmospheric neutrino problems can be realized.
- Though the large hierarchy between the mass eigenvalues suits more for simultaneous solutions of solar (vacuum oscillations) and atmospheric neutrino problems, two large mixings are difficult to realize within the allowed parameter space.

It has been pointed out in [11] that a non-minimal version of this model, which allows for a negative μ parameter can reduce the large hierarchy within the mass eigenvalues and in fact one can have regions in the parameter space where simultaneous solutions for the atmospheric and the solar (with MSW conversion) neutrino problems can be realized. These versions of the models are intricately related to models which try to solve the μ problem in gauge mediated supersymmetry breaking scenarios [11].

3. Trilinear R_p violation

Let us now study the structure of neutrino masses in the presence of trilinear lepton number violating couplings in the superpotential. Here, we consider only λ' couplings to be present in the superpotential. We will comment on the presence of λ couplings later. Thus, the R-parity violating part of the superpotential is given as

$$W_{\not R_p} = \lambda'_{ijk} L_i Q_j d^c_k. \tag{24}$$

The λ' couplings are 27 in number and a priori it appears that there is a lot of arbitrariness in the model. It was pointed out by Drees *et al* [18] that this is not the case and a meaningful study of the neutrino masses in these models can be made if one assumes all the λ' to be of the same order. Though this model has much larger number of parameters, the neutrino mass spectrum has several similar features to the bilinear R-parity violating models.

3.1 *The framework*

The trilinear R-parity violating couplings also give rise to neutrino masses both at the tree level as well as at the loop level similar to that of bilinear R-parity violating models. This is because even in the presence of only trilinear lepton number violating couplings in the superpotential, bilinear soft lepton number violating terms are generated in the soft potential due to RG scaling at the weak scale [12,19]. Thus, at the weak scale, these couplings are present in the scalar potential given as

$$\begin{aligned} V = {} & m^2_{\tilde\nu_i} \mid \tilde\nu_i \mid^2 + m^2_{H_1} \mid H^0_1 \mid^2 + m^2_{H_2} \mid H^0_2 \mid^2 + \left[m^2_{\nu_i H_1} \tilde\nu^\star_i H^0_1 \right. \\ & \left. - \mu\, B_\mu H^0_1 H^0_2 - B_i \tilde\nu_i H^0_2 + \text{ H.c} \right] + \frac{1}{8}(g_1^2 + g_2^2)(\mid H^0_1 \mid^2 - \mid H^0_2 \mid^2)^2 + \cdots . \end{aligned} \tag{25}$$

The relevant RG equations which determine the bilinear soft lepton number violating terms at the weak scale are [12,19,20]:

$$\frac{\mathrm{d}B_i}{\mathrm{d}t} = B_{\epsilon_i}\left(-\frac{1}{2}Y^\tau - \frac{3}{2}Y^t + \frac{3}{2}\tilde{\alpha_2} + \frac{3}{10}\tilde{\alpha_1}\right)$$
$$- \frac{3}{16\pi^2}\mu h_k^D \lambda'_{ikk}\left(\frac{1}{2}B_\mu + A^{\lambda'}_{ikk}\right),$$
$$\frac{dm^2_{\nu_i H_1}}{\mathrm{d}t} = m^2_{\nu_i H_1}\left(-2Y^\tau - \frac{3}{2}Y^b\right) - \frac{3}{32\pi^2}\ h^D$$
$$\lambda'_{ikk}\left(m^2_{H_1} + m^2_{L_i} + 2{m^2}^Q_{kk} + 2A^{\lambda'}_{ikk}A^D_{kk} + 2{m^2}^{D^c}_{kk}\right). \tag{26}$$

Since these terms are absent at the M_{GUT} scale and only generated at the weak scale, the solutions of these equations can be represented as

$$B_i = \lambda'_{ipp}h^d_{ip}\kappa_{ip},$$
$$m^2_{\nu_i H_1} = \lambda'_{ipp}h^d_{ip}\kappa'_{ip}, \tag{27}$$

where p is summed over the generations and the parameters κ, κ' represent the running of the standard parameters [5] present in the RGE's from the high scale to the weak scale. The above soft potential eq. (25) would now give rise to sneutrino vevs given as

$$\langle\tilde{\nu_i}\rangle = \frac{B_i v_2 - m^2_{\nu_i H_1}v_1}{m^2_{L_i} + \frac{1}{2}m^2_Z\cos 2\beta}. \tag{28}$$

As noted earlier, the sneutrino vevs so generated will now mix the neutrinos with the neutralinos thus giving rise to a tree level neutrino mass matrix given as

$$\mathcal{M}^0_{ij} = \frac{\mu(cg^2 + g'^2)\ \langle\tilde{\nu_i}\rangle\ \langle\tilde{\nu_j}\rangle}{2(-c\mu M_2 + 2M^2_w c_\beta s_\beta(c + \tan\theta^2_w))}\ . \tag{29}$$

We can factorize the λ' couplings in the above using eq. (27) and rewrite it as

$$\mathcal{M}^0_{ij} = \lambda'_{ikk}h^d_{kk}\lambda'_{jpp}h^d_{pp}m_0. \tag{30}$$

In the above, m_0 contains all the rest of the parameters which are dependent solely on the soft SUSY breaking parameters [21]. The loop induced mass due to the presence of trilinear couplings in the superpotential [16] is given as

$$\mathcal{M}^{\mathrm{loop}}_{ij} = \frac{3}{16\pi^2}\lambda'_{ilk}\lambda'_{jkl}h^d_l \sin\phi_k\cos\phi_k \ln\frac{M^2_{2k}}{M^2_{1k}},$$
$$= \lambda'_{ilk}\lambda'_{jkl}h^d_{ll}h^d_{kk}m_l, \tag{31}$$

where m_l now parameterizes all the quantities which are dependent on the soft SUSY breaking parameters. We see from the eqs (30), (31) that both the tree level and the loop level neutrino masses are proportional to the down quark Yukawa couplings. Moreover, they almost have a similar structure. The only difference being that the tree level mass matrix depends only on diagonal (λ'_{ikk}) couplings, where as loop mass matrix contains contributions from both diagonal and non-diagonal ($\lambda'_{ijk},\ j \neq k$) couplings. Exploiting this fact, the total mass matrix can be written as

$$\mathcal{M}^\nu = (m_0 + m_l)\lambda'_{ikk} h^d_{kk} \lambda'_{jpp} h^d_{pp} + m_l h^d_{22} h^d_{33} A_{ij}, \tag{32}$$

where

$$A_{ij} = \lambda'_{i23}\lambda'_{j32} + \lambda'_{i32}\lambda'_{j23} - \lambda'_{i22}\lambda'_{j33} - \lambda'_{i33}\lambda'_{j22}. \tag{33}$$

In writing the above, we have neglected contributions coming from $O(h^d_1, h^d_2)$ terms. The first matrix in eq. (32) has only one eigenvalue which is basically equivalent to the tree level contribution. The total mass matrix has the following eigenvalues:

$$\begin{aligned} m_{\nu_1} &\approx m_l h^d_{22} h^d_{33} \delta_1, \\ m_{\nu_2} &\approx m_l h^d_{22} h^d_{33} \delta_2, \\ m_{\nu_3} &\approx (m_0 + m_l)(a_1^2 + a_2^2 + a_3^2), \end{aligned} \tag{34}$$

where

$$\begin{aligned} a_i &= \lambda'_{ikk} h^d_{kk}, \\ \delta_1 &= (c_1^2 A'_{11} - 2c_1 s_1 A'_{12} + s_1^2 A'_{22}), \\ \delta_2 &= (c_1^2 A'_{11} - 2c_1 s_1 A'_{12} + s_1^2 A'_{22}). \end{aligned} \tag{35}$$

The A'_{ij} are functions of λ' given in [19]. From eq. (34) we see that one of the mass eigenvalues is basically the tree level mass and the other two are governed by the loop level contributions. The ratio of the two eigenvalues is given by

$$\begin{aligned} \frac{m_{\nu_2}}{m_{\nu_3}} &\sim \frac{m_l h^d_{22}}{m_0 h^d_{33}} \left(\frac{\delta_2}{\sum_i \lambda'^2_{i33}} \right) \\ &\approx \frac{m_l}{m_0} \frac{m_s}{m_b}. \end{aligned} \tag{36}$$

The second line in the above is under the assumption that all the λ''s are of the same order. In this limit, we see that the ratio of the mass eigenvalues is completely independent of the R-parity violating parameters and only depends on the soft SUSY breaking parameters. The total mixing is governed by

$$K = U\,U' = \begin{pmatrix} c_1c_2 - s_1s_2c_3 & s_1c_2 + c_1s_2c_3 & s_2s_3 \\ -s_2c_1 - s_1c_2c_3 & -s_1s_2 + c_1c_2c_3 & c_2s_3 \\ s_1s_3 & -s_3c_1 & c_3 \end{pmatrix}, \tag{37}$$

where

$$\begin{aligned} s_2 &= \frac{a_1}{\sqrt{a_1^2 + a_2^2}}, \\ s_3 &= \frac{(a_1^2 + a_2^2)^{\frac{1}{2}}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \end{aligned} \tag{38}$$

and $\tan 2\theta_1 = (2A'_{12}/A'_{11} - A'_{22})$.

From the above, we see that the mixing is determined by the ratios of the λ' couplings. In the limit, where all the λ' couplings are of the same order large mixing is natural in these models.

3.2 *Neutrino anomalies and constrained MSSM*

Within the CMSSM framework, the tree level contribution dominates over the loop level contribution for large regions of the parameter space leading to hierarchical pattern for the neutrino mass spectrum. However, similar to the case of bilinear R-parity violating scenarios, for one particular sign of μ, there exists regions in the CMSSM parameter space where the two contributions to the tree level mass cancel each other. In these regions, the loop mass becomes comparable to the tree level mass reducing the hierarchy of the mass spectrum. Thus, large hierarchy and large mixing continue to be the main features of these models just as in the case of models with bilinear lepton number violation. The main implications for solutions of the solar and atmospheric neutrino anomalies are summarized as below:

- The ratio m_l/m_0 which is typically of $O(10^{-3})$ [19] together with m_s/m_b produces the right hierarchy in mass eigenvalues (eq. (36)) for simultaneous solutions for atmospheric and solar neutrino problems (vacuum oscillations). Though, the mixing between all the states is naturally large constraints from the CHOOZ experiment can be accommodated without fine tuning of the parameters [19]. The λ' is typically required to be of the order 10^{-4}.
- Solutions for the solar neutrino problem with MSW conversion (large mixing angle) and atmospheric neutrino problem can be simultaneously accommodated in this model for regions of the CMSSM parameter space where, $m_l \approx m_0$. As noted above, this is possible in the parameter space where the two contributions to the sneutrino vev cancel each other [19].

It should be noted that attempts have been made to understand the neutrino anomalies in these models without including the tree level contribution [18]. In such a case, the hierarchy of the mass eigenvalues (eq. (36)) is dependent only on m_s/m_b, which is in the right range for simultaneous solutions of the solar ν problem (MSW conversion) and the atmospheric neutrino problem. Addition of the tree level contribution increases this hierarchy making it more natural to find regions in parameter space where simultaneous solutions for the solar neutrino (with vacuum oscillations) and atmospheric ν problems can be found. Moreover, since large mixing is natural in these models, additional assumptions on the structure of λ' couplings viz., discrete symmetries [18] or hierarchical [22] patterns are required if one looks for solutions for solar neutrino problem with small angle MSW conversion. An analysis of the appropriate values for the products of the trilinear couplings required to explain the existing neutrino anomalies and a comparison with the existing bounds from other experiments has been reported in [23].

We have so far concentrated on λ' couplings alone. The same discussion will go through if one considers λ couplings in the superpotential. The tree level mass would continue to dominate over the loop mass, but the structure of mixing matrix would be different from here due to the antisymmetry of the λ couplings.

4. Discussion

We have considered here supersymmetric theories with both bilinear and trilinear lepton number violation separately and studied the structure of neutrino masses and mixing in these models. Natural large mixing and hierarchical masses are the main features of the neutrino mass spectrum in these models. The main results are:

- The bilinear soft $\not{L}$ terms play an important role in determining the neutrino mass spectrum in both the bilinear and trilinear lepton number violation scenarios. They give rise to sneutrino vevs which then lead to the generation of tree level mass to the neutrino. Since this mass dominates over the loop mass for most of the parameter space, these terms play a key role in determining the hierarchy of the mass spectrum.
- The mixing is essentially controlled by ratios of the R-parity violating parameters. Large mixing can be typically achieved in both trilinear and the bilinear R-parity violating scenarios, by choosing the R-parity violating parameters to be of the same order in magnitude.
- Assuming universal boundary conditions at the SUSY breaking scale gives rise to small neutrino masses. In these cases, the neutrino masses are proportional to bottom and the tau Yukawa couplings. In the limit where these Yukawa couplings tend to zero, neutrino masses vanish in these models. Additional suppression is required through R-parity violating parameters, if the masses are required to be in the right range to explain the neutrino anomalies.
- In the CMSSM framework, there exists regions in the parameter space where the two contributions to the sneutrino vev cancel each other. In these regions, the loop level contribution becomes comparable to the tree level contribution. In the bilinear R_p violating scenario, this leads to the formation of a pseudo-Dirac pair, whereas in the trilinear R_p violating scenario, this reduces the hierarchy between the masses, making it possible to incorporate simultaneous solutions for MSW and atmospheric ν problems.
- The CMSSM framework with bilinear R-parity violation provides a natural model where solutions for solar and atmospheric neutrino problems can be realized. Simultaneous solutions for solar and atmospheric neutrino problems are difficult to realize in much more constrained framework of MMM of gauge mediated models with bilinear R-parity violation, though one can solve them separately.
- In the CMSSM framework with trilinear lepton number violation, it is more natural to find parameter space where simultaneous solutions for the solar (with vacuum oscillations) and atmospheric neutrino problems can be realized. Simultaneous solutions for solar (MSW, large angle) and atmospheric ν problems can be found in regions where the tree level mass is comparable to the loop level mass. To incorporate MSW conversion (small angle) one has to resort to additional conditions on the λ' couplings such as discrete symmetries or hierarchical patterns.

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