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Polarization of the cosmic microwave background radiation

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Abstract. In re-ionized models, the measurement of polarization of CMBR can be a good criterion to narrow down the parameter space for cosmological models. A Vishniac-type effect in second order polarization over arc minute scales has been calculated. It has been shown that while the effect is very small ($\sim 10^{-2}\mu$K) for CDM models, it can be significant ($\sim 0.3\mu$K) for some isocurvature models.

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1. Introduction

Reionization of matter in the Universe in the post-decoupling era can affect the cosmic microwave background photons through scattering processes. While Compton scattering can lead to distortion in spectrum of the CMB photons, Thomson scattering can distort the anisotropy as well as polarization of the CMB. Such distortions play a diagnostic role in the study of the state of matter after the standard decoupling epoch. This in turn can shed light on the physical processes taking place in the Universe in the post recombination era.

There are a number of plausible physical processes that can re-ionize the matter content in the Universe in the post-decoupling era. Although at present one is not sure as to precisely which are the physical processes that actually caused re-ionization, one knows from observations that the present Universe around us is in an ionized state. The absence of 'Gunn-Peterson' dips in the spectra of distant quasars indicates that the Universe was probably reionised at some redshift $z = z_* > 5$ ([6]). The value of z_* is not known observationally, while different theoretical models have different predictions for this redshift.

Here, we will discuss the nature of polarization signals generated by Thomson scattering from this re-ionized matter in the Universe. In our model we will assume that the Universe underwent standard recombination at an epoch τ_D corresponding to a red-shift of about 1100. After this epoch, the Universe was in an unionized state till an epoch τ_{RI} when the matter content in the Universe got re-ionized. Let us denote the red-shift of this epoch by z_{RI}. We assume that the Universe remained ionized after this till the present epoch. The CMB photons coming from the surface of last scatter (SLS) get Thomson scattered by the ionized matter and hence get a non-zero degree of polarization, much like the way the light rays from the morning sun Thomson scatter from the electrons in the ionosphere producing a non-zero degree of polarization except in the direction around the sun. Hence, in the re-ionized Universe, we can effectively view ourselves to be located at the center of a cosmic ionosphere. The source of incident radiation is the CMBR.

The plan of presentation is as follows: We introduce the concept of optical depth and discuss the related parameter called the visibility function in §2. In §3 we discuss the first and second order polarization. Lastly, in §4 we present the results of numerical estimates, and discuss the expected polarization from specific models.

2. Optical depth and the visibility function

Consider a distribution of charged particles, say electrons, in space. Consider a photon traveling through it along a trajectory $\mathbf{l}(t)$. The optical depth along a path is a measure of the number of scattering of the photon as it traverses the path. It is given by

$$\kappa = \int_{\mathcal{C}} n_e(\mathbf{l})\sigma_T \, \mathrm{d}l \tag{1}$$

where, $\mathcal{C}$ is the path of the photon. A related quantity, the visibility function, g is given by,

$$g = \dot{\kappa} \exp^{-\kappa}, \tag{2}$$

where the dot represents derivative with respect to time. In the cosmological context, the visibility function is given by,

$$g(\tau, \tau') = n_e(\tau')a(\tau')\sigma_T \exp^{-\int_{\tau'}^{\tau} n_e(\tau'')a(\tau'')\sigma_T \, \mathrm{d}\tau''}, \tag{3}$$

where τ' is the conformal time and τ is the conformal time corresponding to the present epoch. Physically $g(\tau, \tau')\,\mathrm{d}\tau'$ measures the probability that the photon received by the observer at the epoch τ got scattered for the last time between τ' and $\tau' + \mathrm{d}\tau'$.

In the case of models in which the Universe did not undergo a phase of re-ionization in the post recombination era, $g(\tau, \tau')$ has a single sharp peak at $\tau' = \tau_D$. In the case of reionization, the visibility function gets an additional peak at the epoch of reionization. This peak is smaller than the first and its width is larger. We model the second peak as a Lorentzian given by

$$g(\tau, \tau') = N \frac{1}{\sigma} \mathrm{e}^{-\frac{(\tau' - \tau_{RI})}{\sigma}} \theta(\tau' - \tau_{RI}). \tag{4}$$

3. Polarization from re-ionization

The equations governing the evolution of polarization perturbation $\Delta_P(\mathbf{x}, \gamma, \tau)$ for scalar modes can be derived from the moments of the Boltzmann equation for photons. They are given by [3]

$$\dot{\Delta}_P + \gamma_i \partial_i \Delta_P = n_e \sigma_T a(\tau) \left(-\Delta_P + \frac{1}{2}[1 - P_2(\mu)]\Pi \right). \tag{5}$$

Here $\mathbf{x}$ is the comoving co-ordinate, τ is conformal time, n_e the electron density, **v** the velocity field of the fluid of charged particles, γ is the direction of photon propagation, and the gradient in the polarization perturbation. A dot represents derivative with respect to conformal time, $\mu = \cos(\theta)$, where θ is the angle between the direction of the photon propagation vector and the gradient. We have also defined

$$\Pi(\mathbf{x},\tau) = -\Delta_{T2}(\mathbf{x},\tau) - \Delta_{P2}(\mathbf{x},\tau) + \Delta_{P0}(\mathbf{x},\tau), \tag{6}$$

with Δ_{T0}, Δ_{P0} the monopole, and Δ_{T2}, Δ_{P2} the quadrupole temperature and polarization perturbations, respectively. We define these angular moments by

$$\Delta_{P,T}(\mathbf{x},\gamma,\tau) = \sum_l (2l+1) P_l(\mu)\Delta_{Pl,Tl}(\mathbf{x},\tau);$$

$$\Delta_{Pl,Tl} = \int \frac{\mathrm{d}\mu}{2} P_l(\mu)\Delta_{P,T}(\mu). \tag{7}$$

The polarization terms on the right hand side of eq. (5) are negligible. Thus the dominant source term for the polarization equation arises from the quadrupole anisotropy of temperature. The required equation in real space finally turns out to be

$$\dot{\Delta}_P + \gamma_i \partial_i \Delta_P = -\overline{n}_e(1+\delta_e)\sigma_T a(\tau)\frac{1}{2}[1-P_2(\mu)]\Delta_{T_2}. \tag{8}$$

We had mentioned earlier that anisotropic unpolarized radiation can through Thomson scattering produce linearly polarized light. The above equation shows the nature of anisotropy required to produce this effect. From symmetry one can notice that isotropic radiation cannot produce polarization. It also turns out that radiation with dipole anisotropy also cannot produce polarization. As we see from equation (8), polarization can be caused by scattering of radiation which has a quadrupole anisotropy.

In terms of Fourier modes, the equation governing the evolution of polarization perturbation can be expressed as

$$\dot{\Delta}_P + ik\mu\Delta_P = -\overline{n_e}\sigma_T a \frac{1}{2}(1-P_2(\mu))\left[\Delta_{T_2} + S(\mathbf{k},\tau)\right] \tag{9}$$

where $S(\mathbf{k},\tau)$ is given by,

$$S(\mathbf{k},\tau) = \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\delta_e(\mathbf{k}-\mathbf{p},\tau)\left[\Delta_{T_2}(\mathbf{p},\tau)\right]. \tag{10}$$

The term in the square brackets, in eq. (9) depends on $\vec{k}$ as well as τ.

The formal solution of the polarization perturbation equation is given by

$$\Delta_P(\mathbf{k},\tau) = -\frac{1}{2}(1-P_2(\mu))\int_0^\tau \left[\Delta_{T_2}(\mathbf{k},\tau') + S(\mathbf{k},\tau')\right] \times g(\tau,\tau')\mathrm{e}^{ik\mu(\tau'-\tau)}\mathrm{d}\tau'. \tag{11}$$

We now express Δ_{T_2} as a sum of first and second order terms. As has been shown in [2], the dominant second order temperature term (the Vishniac term) arises from the coupling of the first-order temperature anisotropy and the electron density perturbation and hence this is the only second-order temperature term we consider. So we have,

$$\Delta_T = \Delta_T^{(1)} + \Delta_T^{(V)}, \tag{12}$$

where $\Delta_T^{(V)}$ is the Vishniac term. Similarly the polarization term can also be expressed as a sum of first and second order terms

$$\Delta_P = \Delta_P^{(1)} + \Delta_P^{(2)}, \tag{13}$$

where

$$\Delta_P^{(1)}(\mathbf{k},\tau) = -\frac{1}{2}(1-P_2(\mu))\int_0^\tau \left[\Delta_{T_2}^{(1)}(\mathbf{k},\tau')\right] g(\tau,\tau')\mathrm{e}^{ik\mu(\tau'-\tau)}\mathrm{d}\tau' \tag{14}$$

$$\begin{aligned}\Delta_P^{(2)}(\mathbf{k},\tau) = &-\frac{1}{2}(1-P_2(\mu))\int_0^\tau \left[\Delta_{T_2}^{(V)}(\mathbf{k},\tau')\right] g(\tau,\tau')\mathrm{e}^{ik\mu(\tau'-\tau)}\mathrm{d}\tau' \\ &-\frac{1}{2}(1-P_2(\mu))\int_0^\tau \left[S(\mathbf{k},\tau')\right] g(\tau,\tau')\mathrm{e}^{ik\mu(\tau'-\tau)}\mathrm{d}\tau'.\end{aligned} \tag{15}$$

In (15) the right hand side comprises of two terms. The first one arises due to the product of the Vishniac temperature term and the uniform component of the electron density. The second term is also a second order term but arises because of the product of the first order temparature anisotropy term and the perturbations in the electron density. Thus the origin of the second term closely resembles the origin of the Vishniac temperature term. Hence we will call this the Vishniac type polarization term and denote it by $\Delta_P^{(V)}$. It has been shown in [4] that the first term on the right hand side of eq. (15) is negligible especially for small angular scales. Thus the only second order polarization term which need to be considered is $\Delta_P^{(V)}$

$$\Delta_P^{(V)} = -\frac{1}{2}(1-P_2(\mu))\int_0^\tau \left[S(\mathbf{k},\tau')\right] g(\tau,\tau')\mathrm{e}^{ik\mu(\tau'-\tau)}\mathrm{d}\tau' \tag{16}$$

and $\Delta_P = \Delta_P^{(1)} + \Delta_P^{(V)}$.

We note that only the first order temperature anisotropy is of relevance to the calculation of the first and the dominant second order polarization term. The quadrupole component of the first order temperature perturbation at an epoch $\tau > \tau_{DC}$ can be expressed in terms of Bessel function in the following way [5]:

$$\Delta_{T_2}(k,\tau) = \Delta_{T_0}(k,\tau_0) j_2[k(\tau-\tau_D)]. \tag{17}$$

We now discuss the contribution of the first order and the Vishniac type polarization terms.

3.1 *First order polarization term*

As pointed out, this term arises because of the coupling of the first order temperature perturbations with the uniform component of the electron energy density. A semi-analytic calculation of this term has been done in ref. [1]. When there is no re-ionization, the visibility function has a single peak around the decoupling epoch. This ensures that the Bessel function in the integrand (obtained by substituting (17) in (15)) has a dominant contribution for large values of k. As pointed out earlier the visibility function has two peaks in the models with re-ionization. One of the peaks is at the same value of τ as in the case of the models with no re-ionization. The extra peak that appears in the case with reionization appears around a value of $\tau' = \tau_{RI}$. The second peak is generally of lesser height and is also broader than the first one. The net effect is that, the Bessel function in the integrand is non-zero at a smaller value of k also. This is over-and-above contribution at large k as in the case with no re-ionization. As a result of all this, in reionized models, the first order polarization has an extra peak at large angular scales.

3.2 *Vishniac-type polarization term*

This term involves the product of the first order temperature anisotropy and the electron density perturbation in real space [7]. Hence, in Fourier space there is a mode coupling between δ_e and $\Delta_{T_2}^{(1)}$ as is clear from the expression for S in (10). Because of this mode-coupling, $\Delta_{T_2}^{(1)}$ over one scale can lead to polarization perturbation in an entirely different scale depending on the form of the electron density perturbation. This is in contrast to the case of $\Delta_P^{(1)}$ where its contribution at a particular scale is directly related to the k-dependence of $\Delta_{T_2}^{(1)}$. Thus in the case of $\Delta_P^{(V)}$, the evolution of δ_e plays a crucial role.

We are interested in calculating second order polarization over arc-minute scales, These correspond to k-values of about $10\,\mathrm{Mpc}^{-1}$ to $1\,\mathrm{Mpc}^{-1}$. For $\tau_{RI} - \tau_D \sim 100\,\mathrm{Mpc}$, $j_2[p(\tau' - \tau_D)]$ peaks at $p = p_0 \sim (300\,\mathrm{Mpc})^{-1}$ to $(1000\,\mathrm{Mpc})^{-1}$ for $\tau > \tau_D$. For $k \gg p_0$, $\delta_e(\mathbf{k} - \mathbf{p}) \sim \delta_e(\mathbf{k})$ in Δ_P^V. Using this form for δ_e the mode coupling term in S in eq. (10), we get,

$$S(\mathbf{k}, \tau') = \delta_e(\mathbf{k}, \tau') Q_2(\tau'), \tag{18}$$

where

$$Q_2(\tau') = \int (\mathrm{d}^3 p/(2\pi)^3) \Delta_{T_2}^{(1)}(\mathbf{p}, \tau').$$

(For more details the reader is refered to [7] and references therein.)

4. Numerical estimate of Vishniac type polarization

In the previous section we have derived the expressions for the computation of Vishniac type polarization perturbations. As we saw, the degree of second order polarization over different angular scales depends on the electron density perturbation spectrum. After the epoch of decoupling, at a red-shift of about 1100, matter and radiation decouple from each other and perturbations in matter grow freely via gravitational collapse. This is dictated by the scenarios of structure formation. When the matter in the Universe gets ionized, the spectrum of matter density perturbations translates to the spectrum of electron density perturbations. It is this spectrum of electron density perturbation that governs the Vishniac type polarization discussed in the last section. Thus, the polarization term, Δ_P^V, depends on the scenarios of structure formation. Faster the growth of density perturbations, the larger the value of Δ_P^V over small angular scales. The electron density perturbation at any epoch in the post decoupling era is given by,

$$\delta_e(\mathbf{k},\tau) = \left(\frac{\tau}{\tau_0}\right)^2 \delta_e(\mathbf{k},\tau_0)\frac{f(\Omega(z(\tau)))}{f(\Omega_0)}, \tag{19}$$

and the power spectrum of electron density perturbation by

$$\Delta_e^2 = \frac{k^3\langle|\ \delta_e\ |^2\rangle}{2\pi^2}. \tag{20}$$

Here, $(\tau^2)f(\Omega(z(\tau)))$ is the growth factor of density perturbation with time. Defining the Vishniac-type contribution to the polarization power spectrum by

$$Q_P^V(k) = \frac{k^3}{2\pi^2}\frac{1}{2}\int_{+1}^{-1}\langle|\Delta_P^V|^2\rangle\mathrm{d}\mu \tag{21}$$

and substituting from the above, we get

$$Q_P^V(k) = \frac{\Delta_k^2}{k\sigma}\frac{1}{5}\left(\frac{\Delta T}{T}\right)^2. \tag{22}$$

Here,

$$\left(\frac{\Delta T}{T}\right)^2_Q(\tau_{RI}) \equiv \frac{5C_{T2}(\tau_{RI})}{4\pi} = 5\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\langle|\Delta_{T2}(\mathbf{p},\tau_{RI})|^2\rangle. \tag{23}$$

As pointed out earlier, different scenarios of structure formation lead to different types of polarization signals. We will broadly consider cold dark matter models and isocurvature models.

4.1 *CDM models*

In the standard cold dark matter models ($\Omega_0 = 1$, $\Omega_{\rm bar} = 0.05$ and $h = 0.5$), $\sqrt{(Q_P^V)}\sim 2.2\times\ 10^{-2}\mu$K. In Λ + CDM models ($\Omega_0 = 0.35$, $\Omega_{\rm bar} = 0.04$, $\Omega_\Lambda = 0.65$, $h = 0.7$ and the spectral index, $n = 1$), $\sqrt{(Q_P^V)}\sim 1.9\times 10^{-2}\mu$K. The peak of the polarization signal in these cases occurs typically at a multi-pole moment $l\sim 5000$ corresponding to an angular scale of about 0.66 arc-minutes.

4.2 *Isocurvature models*

These models involve a relatively early collapse of density perturbation leading to a relatively larger value of δ_e and hence a larger value of Vishniac type polarization. For

$\Omega_0 = 0.2$, $\Omega_{bar} = 0.05$ and $h = 0.8$ we get $\sqrt{(Q_P^V)} \sim 0.3\mu$K. The peak occurs at a l value of about 10000 corresponding to an angular scale of about 0.33 arc-minutes.

We see that the isocurvature models are the best cases for possible detection of Vishniac type polarization of the CMBR. Future experiments can use this measurement to narrow down the parameter space for cosmological models.

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