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Cosmological parameters

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Abstract. There is some consensus emerging on the values of the basic parameters of classical cosmology. The baryon number density estimated from the light element abundance or X-ray gas in galaxy clusters tends towards 5% of closure density; the dark matter content based on a number of independent methods appears to be somewhat less than half the closure density; Hubble constant obtained from local measurements, gravitational lens or Sunyaev Zeldovich method are all probably centred around 60 km/sec/Mpc and the age of the Universe is generally agreed to be around 14 Gyr - all specified with bearable error bars. The supernova projects and CMBR anisotropy together favour a finite cosmological constant, and gravitational lens statistics support the same conclusion.

Keywords. Cosmology; Hubble constant; cosmological constant; critical density.

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1. Introduction

General relativity provides a framework to study the global features of the Universe. Some of the well established constraints which any successful model of the large scale structures and geometry of the Universe should satisfy are:

1. Cosmic microwave background radiation with a temperature of $T_0 = 2.728 \pm 0.002\,\mathrm{K}$
2. Observation of light elements with abundance decreasing as function of time:
   (a) deuterium $\sim 6 \times 10^{-5}$ by mass fraction in hydrogen clouds at high redshifts,
   (b) lithium^7 $\sim 10^{-9}$ by mass fraction in the atmospheres of metal poor stars,
   (c) observed helium $\sim$ 24% of baryons, with a slow increase as a function of metal abundance.
3. Expansion of the space-time.

The above constraints together point towards a hot past phase of the Universe. The remarkable isotropy of

1. microwave and other background radiations,
2. distribution of faint galaxies in the sky and

cosmological principle indicate spatial homogeneity and isotropy of the Universe at least during the hot phase since baryogenesis and possibly upto when the large scale structures started forming.

Probes to large scale geometry of the Universe:

1. Classical cosmology: Direct measurement of basic parameters (Hubble constant, baryon abundance, mean mass density of the Universe, deceleration parameter etc.)
2. Background radiation fields (e.g. cosmic microwave radiation; if a relic gravitational wave background is detected, it could start a new pathbreaking field of cosmology).
3. Dynamics and evolution of structures (e.g. galaxies in formation, clusters or AGNs; distribution of galaxies as function of redshift).
4. Windows to view the structures (e.g. gravitational lens, Lyα systems).

Various steps involved in the above analysis fall broadly into two types:

Geometrical method: Determination of certain distances as a function of redshift and inversion to obtain the cosmological model (e.g. standard candle to measure distance or ratio of distances). Relative distance measurement over a limited range of redshift cannot reliably distinguish between various cosmological models, and in view of the continuing debate on the Hubble constant it is most desirable to use two complementary methods which measure distances at low redshift as well as very high redshift. (e.g. supernova Ia could trace relative distances at redshifts of around 1 while the cosmic microwave background anisotropy can relibaly indicate the horizon scale at the last scattering surface.)

Physical model: Detection of the imprint of cosmological evolution on an observable. (e.g. Light element abundance vs. the matter density; CMBR anisotropy). The model necessarily requires knowledge of all the physical processes operative. Consequently, most often the method reduces to testing specific model. It should be noted that a single method generally cannot provide good limits on the cosmological parameters. For example, the distance vs. redshift relation has very similar shape for $\Omega_M = 0.2$ open Universe and $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ flat Universe upto redshift of 1, and taking into account the uncertainty in the Hubble constant, even wider range of redshift and cosmological parameter space behave the same. Consequently, the ideal technique to probe the large scale geometry of the Universe involves one low redshift method like the supernova and another operative at very high redshift like the CMBR fingerprints at the last scattering surface.

Only recently, cosmologists feel they have realistic error bars in the basic parameters of cosmology and some hope that a physically consistent picture has started emerging. In this talk, an idea of the accepted values for the various cosmological parameters will be given and two of the methods being now used extensively to study the global geometry of the Universe, namely, the supernova Ia and gravitational lens will be discussed.

2. Governing equations

The FRW metric is the natural choice to describe a spatially homogeneous, isotropic Universe, in which the expansion of the space time is manifested through the scale factor

$a(t)$, which relates the co-ordinate distance to the true physical distance. For a Universe filled with baryonic and dark matter as well as a vacuum energy represented through the cosmological constant, the time evolution of the scale factor is given by

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G\left(\rho_m + 3P_m\right) + \frac{\Lambda}{3}. \tag{1}$$

The matter density evolves according to the equation for energy conservation,

$$\dot{\rho}_m = -3\left(\rho_m + P_m\right)\frac{\dot{a}}{a}, \tag{2}$$

ρ_m is the contribution to density and P_m to pressure from radiation and ordinary matter; P_m is believed to be negligible at present. However, if the dark matter, which constitues most of the gravitating mass has substantially different equation of state, P_m could decide the geometry as well as fate of the Universe. Λ is the cosmological constant, which is treated by the astroparticle physicists as vaccuum energy, following an equation of state of $P_v = -\rho_v$ so that ρ_v is constant with time.

The boundary conditions to be specified at the present epoch:

1. Λ, the cosmological constant.
2. The present density of the Universe, expressed as a ratio to the closure density, $\Omega_m = \rho_M/\rho_c$.
3. The logarithmic rate of change of the scale factor at present, $\dot{a}/a$, known as the Hubble constant. The absolute value of a is irrelevant.
4. The time interval from the big bang to the present, i.e. the age of the Universe.

3. Supernova Ia as a standard candle

Supernova Ia is among the brightest transient events and occurs in all galaxies with old population of stars. CO or Ne White Dwarf at Chandrasekhar limit detonates/ deflagrates/ slowly detonates due to slow/ fast accretion resulting in a fast/ normal/ slow supernova producing 1/ 0.7/ 0.4 $M_\odot$ Ni which radioactive decays with characteristic time scale of 77 days into Co and Fe. For a fixed rate of decay of the light curve, the B band magnitude M_B of local SN Ia has intrinsic scatter of less than 0.2 magnitude and the light curve is fairly the same, which makes it an ideal standard candle. However, due to possible heavy extinction in a gas rich host galaxy, SN Ia events are preferentially searched in monitoring programmes in elliptical galaxies for cosmological studies. Two hallmarks of SN Ia are

1. Characteristic decay light curve powered by the radio activity of Ni and Co.
2. Si and S absorption lines at the initial stage turning to emission later.

The controversy still remains on the peak luminosity value; Tamman relates this discrepency to the ongoing battle on the value of Hubble constant, through the relation between the peak B magnitude and the Hubble constant:

$$M_B(\max) \sim -18.2 + 5\log\frac{H_0}{100\,\mathrm{km\,s^{-1}}} \tag{3}$$

for normal SN Ia [1,2]. In spite of this debate, SN Ia is a powerful probe tracing the distance scale from less than 10 Mpc to upwards of redshift 1. However, a few questions should be considered seriously, while using SN Ia to make definitive conclusions on the global geometry of the Universe:

1. Can the peak be determined from the observed light curve? This depends on the timing, extinction correction and sampling with multiple bands to take into account K correction.
2. Can we identify SN Ia? (if we observe Si absorption line, yes).
3. Will a white dwarf typically 10 billion years old and a 1 billion year old one show the same intrinsic light curve? Notice that some of the very bright SN II have light curves similar to SN Ia though spectroscopically they are not the same.

There had been many SN Ia monitors over the last few years. Indeed, Narasimha and Chitre had been arguing that SN Ia in giant arcs in the field of rich clusters could become powerful probes of cosmology in many ways [3].

3.1 *Hubble constant from SN Ia*

Sandage and colleagues calibrated a few galaxies where ~ 20 SN Ia were observed over the last 70 years [4], from the Cepheid period–luminosity relation using the HST and derived H_0 ranging from 42 to 60 km/sec/Mpc assuming that

1. $M_{B\,\max} \sim -19.6 \pm 0.2$ mag and $M_{\mathrm{Ni}^{56}} \sim 0.6$ M$\odot$ for normal SN Ia,
2. $M_{B\,\max} \sim -20.2$ mag for fast SN Ia.

van den Bergh defined a color corrected V band maxima for SN Ia [5]

$$M_V^*(\max) = M_V(\max) - 3.1(B-V)_{\max} = -19.6 \pm 0.05 \tag{4}$$

with a scatter of 0.29 mag. Using the Calan/Tololo data [6] he estimated $H_0 = 60 \pm 3$ kms^{-1} Mpc^{-1}. However, Reiss *et al* [7], using the shape of light curves arrived at a value of $H_0 = 67 \pm 7$ kms^{-1} Mpc^{-1}.

3.2 *Large scale geometry of the Universe from SN Ia*

Currently, two long term projects involving multiple telescopes including the HST and Keck (supernova cosmology project [8], high-z supernova search team [9] are in progress; the preliminary results from the first is given in figure 1. Upto March 1998, the total number of SN Ia detected is 64, though it is non-trivial to combine the results from these two projects. The salient features of the analysis are:

1. K correction using multiwavelength observations.
2. Extinction correction using multi-wavelength band. (In our galaxy, stellar extinction computed this way generally agrees with estimates from spectroscopy of hot stars; but occasionally wide variation is seen depending on the source of extinction [10].

3. Stretch factor depending upon decay time corrected for redshift.
4. Malmquist bias estimations to correct for systematic loss of fainter SN Ia at larger distances.

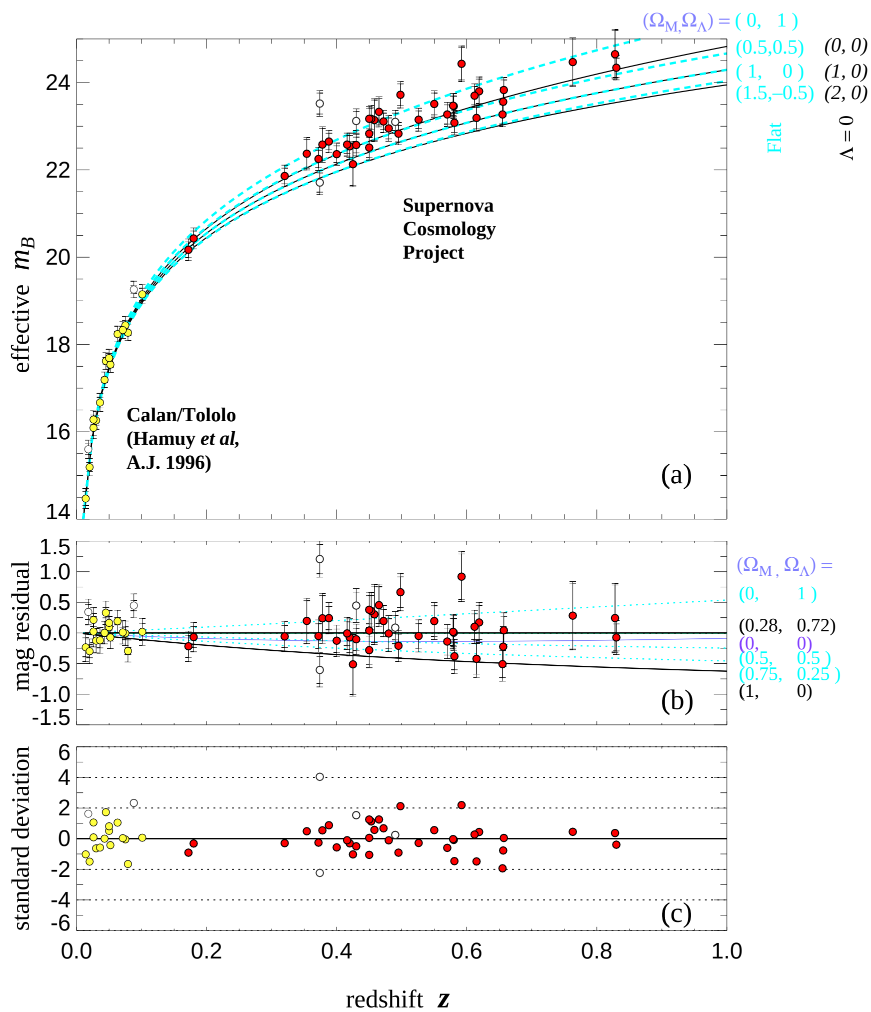

Figure 1. Hubble diagram for the SN Ia from Calan/Tololo [23] and supernova cosmology project [8], after corrections described in the text. The continuous curves correspond to the theoretical expectation for various cosmological parameters with $\Lambda = 0$ while the dashed one are for flat Universe. The low redshift points have error due to peculiar velocity as well as intrinsic scatter. The points marked with open circles are not included in the final analysis either due to excessive extinction or deviation from the mean. The scatter of the points about the mean values is shown in absolute magnitude as well as in units of the standard deviation in the lower two graphs. The figure is taken from Perlmutter *et al* [8].

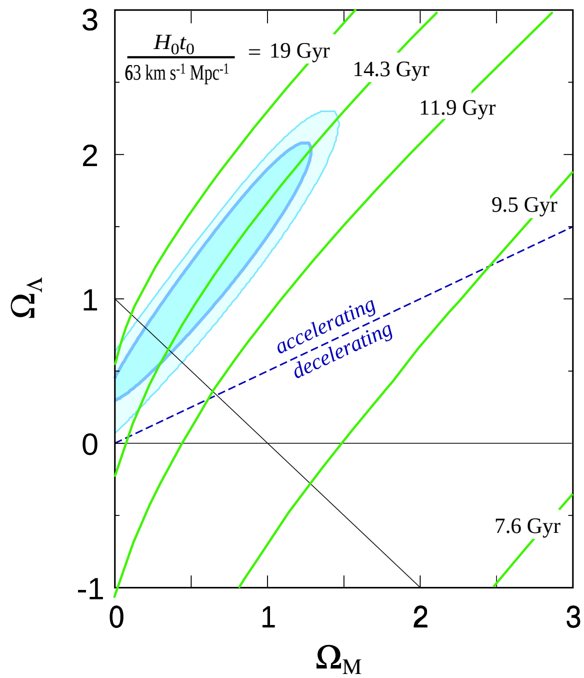

Figure 2. The best fit confidence regions in the $\Omega_M - \Lambda$ plane obtained by the supernova cosmology group [8]. The m_B of figure 1 is a direct measure of the effective luminosity distance and the redshift of the SN can be converted into a theoretical luminosity distance for a set of (Ω_M, Ω_Λ) and the best fit comparison provides the confidence limits. The 68% and 95% confidence limits are bounded by shaded region. (Notice that essentially there is a lower limit on the cosmological constant, if SN alone are used.) Also shown in solid curves are isochrones of constant $H_0 t_0$ (where t_0 is the age of the Universe) for $H_0 = 63$ km s^{-1} Mpc^{-1} (an average of the values in this review based on SN Ia and gravitational lens), which also agrees with our results on Cepheid distance to Virgo [24,25]. (However, see HST key project reports, for instance the review [26] for discussions favouring higher H_0.) The straight line is the flat Universe. The acceleration/deceleration partition (dashed line) corresponds to the present epoch only; as discussed in the text, the equation of state of the dark matter is essential to determine the fate of the Universe. This diagram is based on [8].

Analysis carried out together with low redshift SN survey of Calan/Tololo [6] indicate that the SN Ia at low redshift are similar to the high redshift ones and the data give strong support for a finite positive cosmological constant cf: figure 2.

The result can be summarized as follows:

1. In isolation, either $\Omega \sim 0.2$ open Universe or $\Omega \sim 0.3$ and $\Lambda \sim 0.7$ flat Universe. In effect, $0.8\Omega_M - 0.6\Lambda = -0.2 \pm 0.1$. Efsthathiou *et al* [11] suggest, $0.78\Omega_M - 0.62\Lambda \sim -0.25 \pm 0.13$. If some of the SN were subject to magnification bias due to lensing, the best fit result remains the same, though the confidence limit worsens.

2. Along with CMBR anisotropy (estimation of the power spectrum and specifically the first position of the acoustic peak), $\Lambda \sim 0.7$ model strongly suggestive [12], as can be seen from figure 3.

Using these analyses, $H_0 \times t_0 \sim 0.90^{+0.07}_{-0.05}$ has been argued, where t_0 is the age of the Universe. Consequently, if the age of the Universe is greater than 14 Gyr and Hubble constant is larger than 60 km/sec/Mpc, cosmological constant should be non-zero! In effect, the SN Ia projects provide a lower limit and the CMBR anisotropy, an upper one for the cosmological constant as evident in figure 3.

From the two figures 2 and 3 notice that there exists a narrow intersection corresponding to a finite cosmological constant and the estimated matter density, allowed value of the age of the Universe etc agree well with other independent measurements of the cosmological parameters.

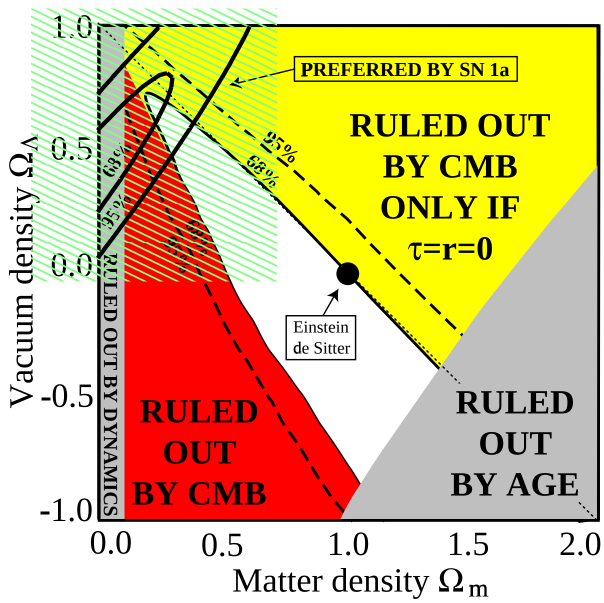

Figure 3. The various likelihood regions in the $\Omega_M - \Lambda$ plane obtained from the CMBR power spectrum [27], SN Ia projects, age of the Universe and constraints relating to the observed power spectrum of matter density [12]. As noted earlier, SN Ia projects measure luminosity distance over limited redshift range and provide constraints on certain difference between Ω_M and Λ, thereby giving possible lower limit on Λ. The position of the first peak in CMBR spectrum varies inversely as $\sqrt{\Omega_M + \Omega_\Lambda}$, thereby indicating an upper limit to Λ. Further constraints are provided by the product of the age of the Universe and Hubble constant (in the figure, $H_0 t_0 < 0.6$ region is ruled out), the infrared matter density at various scale lengths (usually expressed in terms of σ_8, the rms dimensionless density fluctuation when smoothed over a sphere of radius $8\,h^{-1}$ Mpc) and peculiar velocity. Further limits are inferred for specific models of structure formation (e.g. τ in the figure refers to optical depth for reionization and r, contribution from relic background gravitational waves). This diagram is taken from [12].

Table 1. Estimates of Hubble constant from time delay in gravitational lenses for two combinations of the parameters Ω and Λ (given in brackets).

System	Redshift (z_S, z_L)	Time delay days	D_{eff} (Mpc)	Hubble constant (0.2, 0)	Hubble constant (0.3, 0.7)
0957+561	(1.4, 0.36)	423	1870	57± 10	66 ± 11
1115+080	(1.72, 0.31) (time delay	$25^{+3.3}_{-3.8}$ ratio $\frac{\tau_{AC}}{\tau_{BA}}$	1550 revised	56± (?) from 0.7	64 ±(?) to $1.13^{+0.18}_{-0.17}$
1830−211	(0.89, 2.5)	26	3300	67	80
	old value:	46 to 54	6000	38	45
0218+357	(0.68, 0.95)	11.7 ±0.9	9900	45± 9	53±10

4. Gravitational lens

Gravitational lens offers an unbiased estimate of the mass distribution in the Universe since the source and the lens locations are independent. There are considerable amount of analyses of the image separation and redshift distribution of the sources or lenses from which case has been made for non-zero cosmological constant. However, the systematics relating to (1) magnification bias, (2) cross section for multicomponent lenses (3) dark mass should be worked out before we have a reliable handle on the large scale geometry of the Universe based on gravitational lens. It took half a decade to detect signatures of the lens for 2345+007 and 2016+112 after the dark lens was predicted [13]. What is more bothersome is that, among the galaxy scale lenses, as many as five show strong influence of proximity near a cusp caustic and consequent large magnification or arc/ring formation. The galaxy clusters offer better prospects; the method, present status and what we expect in the near future is detailed by Mellier [14]. The bottom line is: Cluster lens can tell us the geometry, but will take some time. Possibly, the estimation of Hubble constant from time delay measurements could be made more robust.

4.1 *Time delay in gravitational lens systems*

Multiple images cover different paths and see different gravitational potential. Consequently, intrinsic variability is seen at varied time in the images. The time interval between the occurences of the variability in the images is the *time delay* between the images.

Time delay has been measured in 4 systems so far, for which there is also considerable amount of observational results on the images and the lens. Our models of these systems suggest the following effective distances: Unfortunately, all these systems have one or other drawback, either observational (like the value of the time delay) or modeling.

1. 0957+561: Only two images and one or more galaxy cluster. Narasimha *et al* [15] pointed out after detailed models incorporating the VLBI features that, depending upon the position of the cluster centre the product of time delay and Hubble constant will vary considerably. Detailed analysis of a possible giant arc in the field will be required to narrow the limits.

2. 1115+080: Observed that certain ratio of time delays is incompatible with single component lens; the problem was circumvented by a reanalysis of the data. The magnification ratio between the bright images was redetermined recently [16].
3. 1830−211: Two values have been reported for the time delay in the last three years.
4. 0218+357: Two images – cannot fix the bulge component of the lens spiral. Since the bright image is near the minor axis of the lens, a small time delay is expected and will crucially depend on the bulge mass.

In spite of these shortcomings, the trend appears to be definitive, namely,

1. Open Universe with zero cosmological constant and $\Omega_M = 0.2$: Hubble constant is $H_0 = 56 \pm 6$ km s^{-1} Mpc^{-1}
2. At the present epoch a flat Universe with $\Omega_M = 0.3$: $H_0 = 66 \pm 6$ km s^{-1} Mpc^{-1}.

4.2 *Large scale geometry*

Gravitational lens can probe the large scale geometry of the Universe in three ways:

1. Mass distribution at scales of galaxy and galaxy clusters can be probed effectively, if the biases mentioned earlier are adequately answered. It is possible to trace even superclusters [17], and evidence is mounting that by redshift of 0.5, groups of clusters existed. The mass reconstruction is based on the image distortion and modification of source number density due to gravitational lensing. The background sources in the field of a cluster lens are distorted and the ellipticity of the sources as well as their orientation is a function of the shear due to lens. Consequently, a map of the shear can be used with appropriate closure conditions to determine the mass distribution in the lens cluster, if the redshift information of the sources is available through multi-band observations. This will be a major project in the coming years, with cameras capable of imaging square degree regions becoming operational.
2. The number density of background sources as a function of redshift and position in the field of a rich galaxy cluster at intermediate redshift provides a useful method to determine the large scale geometry of the Universe. The observed spatial number density of background images at a known redshift interval is inversely proportional to the magnification. Consequently, the expected number density of background sources in a fixed field of a lens cluster as a function of redshift shows a characteristic peak, if the unlensed number density is not significantly modified due to evolution. The peak redshift and the shape of the function is an indicator of the geometry of the Universe.
3. Time delay as function of redshift combinations. This is a geometrical method similar to the SN Ia. However, due to small number statistics, it might be desirable to combine lens with the Sunyaev–Zeldovich effect to derive cosmological parameters.

One of the major efforts is towards getting the σ_8 from cluster lenses. The recent consensus is that various measures of the mass or gravitational potential of galaxy–clusters match. The results are given by Singh [18]. In summary, the mass to light ratio of clusters, determined from lensing as well as other methods generally suggest a value in the range of 200 to 400 $h\mathrm{M}_\odot/\mathrm{L}_\odot$ for the central parts. Using the typical value of 250 h, compared to 1200 h for closure of the Universe, it is argued that $\Omega_M \sim 0.2$. Only some 10% of the galaxies are associated with galaxy clusters, but since the clusters have gathered mass from a comoving volume of ~ 10 Mpc size, it is believed that the observed cluster mass to light ratio can be used for estimation of the power.

Though there is no universal agreement on the age of the Universe, two developments in the last few years have made the stellar models more robust:

1. Hipparcos distance to calibrate luminosity of stars in globular clusters.
2. OPAL opacities and equation of state for better reliability of stellar evolution computations.

Result: moderates say the age of the Universe is 14 Gyr [19].

5. Where is matter distributed?

The estimation of the mass of hot gas from the X-ray flux measurements favours baryon fraction of $\sim$ 15% of the total mass of the cluster. If this ratio holds at larger scales too, it would support $\Omega_b \sim 0.05$, within 50% error limits. The deuterium abundance in Lymanα systems at high redshift in front of quasars is accepted to be $D/H \sim 3.4 \times 10^{-5}$ [20] and lithium^7 in the atmospheres of old stars, $\sim 210^{-10}$ [21]; hence the primordial nucleosynthesis implies $\Omega_b h^2 \sim 0.019$, or $\Omega_b \sim 0.05$. The luminous matter in field galaxies, including dead stars constitute 1% of the closure density and $\sim$ similar amount is present in galaxy clusters.

Dark matter: Non-baryonic matter provides at least 20%, (possibly more than 40%) of the mass in the Universe. It is present in:

1. Galaxies (possibly dark halo) as seen from (a) Rotation curve of spiral galaxies, (b) X-ray emission from giant elliptical galaxies.
2. Galaxy clusters: It is traced by the gravitational potential estimated from (a) dynamics of member galaxies, (b) temperature of the X-ray emitting gas and (c) gravitational lens mass reconstruction.

The mass estimates at the cluster scale indicate that there is more mass than what baryons can contribute. At larger scales, arguments have been given for even more dark matter [22]. *What is this matter*? *Its equation of state? Equation for its mass conservation*? If the equation of state for the bulk of the gravitating, dark mass of the Universe follows $P < -\rho/3$, (violation of strong energy condition) *large scale geometry of the Universe* will be *time-dependent*. Without the equation of state, the present day mass density and Hubble constant alone cannot determine the spatial curvature or the ultimate fate of the Universe. So should we worry that the observed evolution of galaxies at redshift of the order of 1 to 2 through starburst activities, and change in their number density due to vanishing of faint blue objects by redshift of 0.3, which are not adequately explained by our models show up as an equivalent mass component important at redshift of upto 1 or 2?

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