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String dynamics near a black hole

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Abstract. The dynamics of a string near a Kaluza–Klein black hole are studied. Solutions to the geodesic equations are obtained using the world sheet velocity of light as an expansion parameter. For a string falling into a magnetically charged black hole, it is shown that the compact dimension decreases with the world-sheet coordinate τ.

Keywords. Kaluza–Klein theory; black hole; string dynamics; compactification.

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1. Introduction

String theory [1] has recently become important in the context of studying black hole and cosmological backgrounds. Since a string possesses internal degrees of freedom, the dynamics are fundamentally different from the point particle case. A string traces out a world-sheet as opposed to a point particle which traces a world-line, therefore the equations of motion of a string are parametrized by two variables σ and τ. The string world-sheet action is given by

$$S = -T_0 \int \mathrm{d}\tau \mathrm{d}\sigma \sqrt{-\mathrm{det} g_{ab}} \tag{1}$$

where $g_{ab} = G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu$ is the two dimensional world-sheet metric. We can use reparametrization invariance of the world sheet and take the conformal gauge $g_{ab} = \rho(\sigma,\tau)\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the two-dimensional Minkowskian metric.

The classical equations of motion in this gauge are given by

$$\partial_\tau^2 X^\mu - c^2\partial_\sigma^2 X^\mu + \Gamma^\mu_{\nu\rho}\left[\partial_\tau X^\nu \partial_\tau X^\rho - c^2 \partial_\sigma X^\nu \partial_\sigma X^\rho\right] = 0. \tag{2}$$

with the constraints

$$\begin{aligned} \partial_\tau X^\mu \partial_\sigma X^\nu G_{\mu\nu} &= 0 \\ [\partial_\tau X^\mu \partial_\tau X^\nu + c^2 \partial_\sigma X^\mu \partial_\sigma X^\nu] G_{\mu\nu} &= 0, \end{aligned} \tag{3}$$

where c is the world-sheet velocity of light.

The system of equations being highly non-linear, various simplifying ansatze exist for solving these equations. We follow the approach as suggested in [3], where the string

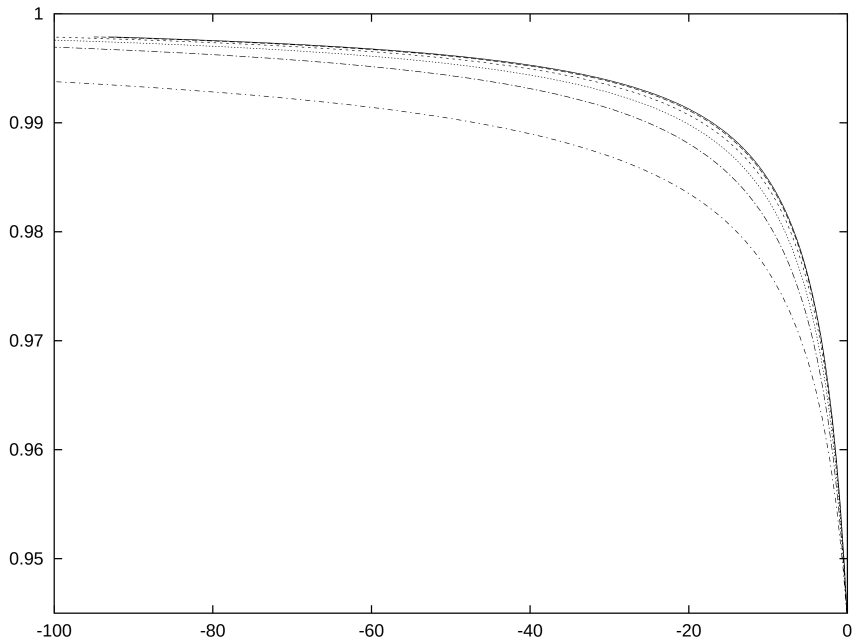

Figure 1. Kaluza–Klein radius as a function of τ.

coordinates are expanded using the world-sheet velocity of light as an expansion parameter. In the small c limit, the derivatives with respect to τ dominate over the σ derivatives. As opposed to the limit of large c, this gives a dynamical picture. If $c << 1$, the coordinate expansion is suitable to describe strings in a strong gravitational background (see [4,5]). This limit corresponds to the vanishing string tension limit, as the world-sheet velocity of light is proportional to the string tension.

2. Kaluza-Klein black holes and string dynamics

We restrict ourselves to the case where the world-sheet velocity of light is small. The string coordinates are expressed as

$$X^\mu(\sigma,\tau) = A^\mu(\sigma,\tau) + c^2 B^\mu(\sigma,\tau) + c^4 C^\mu(\sigma,\tau) + \cdots, \tag{4}$$

and solve the equation of motion for the zeroth order $A^\mu(\sigma,\tau)$. These equations describe the high energy behaviour of strings corresponding to the limit of vanishing string tension.

The metric background to study string propagation [6] is

$$\mathrm{d}s^2 = -\mathrm{e}^{4k\frac{\varphi}{\sqrt{3}}}(\mathrm{d}x_5 + A_\alpha \mathrm{d}x^\alpha)^2 + \mathrm{e}^{-2k\frac{\varphi}{\sqrt{3}}} g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta, \tag{5}$$

where $k^2 = 4\pi G$, x_5 is the extra dimension and should be identified modulo $2\pi R_0$. Here $g_{\alpha\beta}$ is the four-dimensional spacetime. The spherically symmetric, time-independent solutions in the Kaluza–Klein black holes can be characterized by mass M, total electric charge Q and total magnetic charge P (for details see [6]). For simplicity, we discuss only the electrically neutral black hole, i.e $Q = 0$, as a test case.

The equations of motion in the purely magnetic case can be reduced to quadratures if we restrict the motion of the string in the equatorial plane, i.e. $\theta = \pi/2$. The integrals can then be evaluated numerically to obtain the string coordinates in terms of the world-sheet coordinate τ. We work in the region where $r >> \Sigma$, Σ is the total scalar charge, and $r > M$ i.e., the region just outside the horizon. as a function of τ.

The physically interesting quantity to study how the extra compact dimension unfold is the Kaluza–Klein radius which is defined as

$$R(r) = R_0 \mathrm{e}^{2k\varphi/\sqrt{3}} \tag{6}$$

$R(r)$ is a dynamical quantity as it depends implicitly on τ as $R(\tau) = R(r(\tau))$. The effect of the magnetic field is to shrink the extra dimension (as already mentioned in [6]) i.e., as the string approaches the black hole, the value of the Kaluza–Klein radius which it sees becomes smaller than its asymptotic value; as illustrated in figure 1.

3. Conclusions

We have studied propagation of a null string in a five-dimensional magnetically charged, Kaluza–Klein black hole background. Here, we have tried to explore the behaviour of the extra *fifth* dimension as the string approaches the black hole horizon. The solutions we have obtained are valid in the region outside the horizon but not asymptotically far from the horizon.

The solutions, in the limit $\Sigma \longrightarrow 0$, match with the ones given in [5]. The essential difference lies in the presence of the extra dimension. Another paper that follows a complementary approach [7] finds out string corrections to the five-dimensional Kaluza–Klein black hole metric, while our approach is to study the dynamics of a string probe in a classical background.

The above considerations clearly show that even in the classical regime we can explore the behaviour of the extra dimensions. It would be interesting to explore higher dimensional cosmologies; to understand how four-dimensional cosmological solutions arise dynamically from a higher dimensional theory.

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