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# String cosmology

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**Abstract.** In this review, we discuss various cosmological issues related to our Universe from a string theoretic perspective. We analyse the pre-big bang cosmological scenario which appears naturally in this context due to the existence of scale factor duality symmetry in string theory. We then discuss some of the attractive and problematic features of this scenario. Finally, we introduce a method which is powerful enough to search for cosmological solutions in various low energy limits of string theories.

**Keywords.** String theory; cosmology

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## 1. Introduction

Quantum gravitational interactions play an insignificant role at low energies. However, since the gravitational coupling increases with energy, at higher energies gravitational interactions become significant. In particular, at an energy $E > M_{\rm planck}$, the gravitational perturbation theory breaks down – leading to the problem of nonrenormalizability. Unfortunately, this is precisely the region which needs to be better understood in order to answer various cosmological questions. The experimental observations of the red shift of the cosmic microwave background leads us to the conclusion that as we go back in time, we encounter epochs of high temperature, energy and curvature. It is thus clear that in order to understand this region, we need to have a better understanding of the gravitational interaction at high energies or equivalently at short distance scale. One possible way is modify the theory by smearing out the interaction in spacetime which controls the short distance divergences. Until now, there is only one way to achieve such behaviour which is string theory [1]. In string theory, there is a natural short distance scale which is set by the length of the string itself. This theory cuts off all ultraviolet divergences without spoiling consistency. We will have more to say on this in the cosmological context later in this note. Before passing over to string cosmology, let us first mention briefly a few aspects of the standard model of cosmology which is based on general relativity.

The standard model of cosmology assumes that the Universe starts from a singularity (big bang) when it is very hot, highly curved and initial conditions are rather random. As we go forward in time, we encounter a period of superluminal expansion which is commonly known as inflation. A certain period of inflation is absolutely required in order to solve the so called horizon and flatness problem among others. These issues are discussed in detail elsewhere [2] and we will have nothing much to say on these. However, let us mention that in all the models of inflation, the basic structure is as follows. Here, one *invokes* a certain scalar field called the inflation. After the big bang phase, this scalar finds itself away from the minimum of the potential. Thus, while rolling down to the minimum of the potential *slowly*, it releases energy. This energy, in turn, drives the Universe to expand quasi-exponentially.

Several authors, however, have put forward criticisms about the implementation of inflation. This is mainly due to the fact that there is a lack of a convincing model for what the inflaton ought to be. It is also hard to justify the initial conditions that can provide a sufficiently long inflationary period. The condition for the onset of inflation thus has to come from the physics of very early times. However, at this time, general relativity certainly does not make sense. Hence, we hardly have any handle on the theory.

A natural question thus arises: How can string theory help us in this context? This is a difficult question to answer at this stage. The reason is mainly due to the fact that we do not yet understand the non-perturbative nature of string theory and therefore, we do not know of a reliable action in the strong coupling regime. However, it is well known that, given the perturbative knowledge of string theory, it is extremely hard to get a working inflationary model out of string theory. In the low energy string effective action, there are various scalars (dilaton, axion, string moduli etc.). However, they are always coupled with other degrees of freedom in the theory and hence evolve in time. The evolution of these scalars thus generically slows down the growth of the horizon. This makes it very difficult to get out of the horizon and flatness problems. Thus it was clear that if inflation had to take place in string theory, it had to come in a very different way. A step forward in this direction was made by Veneziano [3] and is further developed in [4]. We will discuss certain aspects of it in this review. A complete over all understanding of the subject can be found in [5].

Before we go to the discussion of pre-big bang cosmology let us first list few important properties of string theory. Each of these properties have no analogue in general relativity and will play crucial roles in our discussion.

1. There is a natural length scale in the theory, as discussed before. This is the size of a string and it works as an ultraviolet cut off in the theory. We will denote it as $l_s$ in the following.
2. Rather than being pre-determined, all the couplings are dynamically determined. These couplings are controlled by the expectation value of a scalar (dilaton) and is present in all versions of string theory.
3. String theory has various perturbative symmetries. One of them which will be of special interest to us is known as scale factor duality (SFD). This, in its simplest form, inverts the radius of the Universe and shifts the dilaton. We will have occasion to discuss this symmetry in detail in the next section.
4. There are strong evidences that string theory is endowed with certain non-perturbative symmetries. A special class of such symmetry transformation inverts the string coupling – leading to strong-weak coupling duality; this is part of the S-duality group $SL(2, Z)$. We will discuss some aspects of it in the last section.

In the next section, we first discuss Veneziano's proposal in the simplest setting. We then review, in brief, some of the interesting features of the proposal. We also discuss a major unsolved problem with this scenario. In the last section, we concentrate on an algorithm to find cosmological solutions in a large class models which are particularly interesting after the recent developments on non-perturbative symmetries in string theory.

## 2. The pre-big bang cosmology

The pre-big bang scenario *postulates* that the Universe at early times is *empty, cold, flat and free* and hence described by perturbative string theory. Let us thus start with the simplest and the *universal* part of the string effective action which contains all the basic flavours of pre-big bang cosmology.

$$S = \int \mathrm{d}^4 x \sqrt{-g} \frac{e^{-\phi}}{l_s^2} [R + \partial_\mu \phi \partial_\nu \phi]. \tag{1}$$

Here, $R$ is the curvature, $\phi$ is the dilaton and, as mentioned before, $g_{\rm st} = e^{\phi/2}$ is the string coupling. This action is only valid description of the model when $g_{\rm st} << 1$ or ,in turn, when dilaton is very large and negative. We will be interested, in the following, when the Universe is governed by Friedmann–Robertson–Walker (FRW) metric. This is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \Big[ \frac{\mathrm{d}r^2}{1 - kr^2} + r^2 \mathrm{d}\Omega^2 \Big], \tag{2}$$

where $k = 0, 1, -1$ for flat, closed or open Universe. In this section, we will concentrate on $k = 0$ case. However, $k = -1$ metric and its cosmological behaviour will be discussed in the last section in a more general setting.

Using the metric (2) in (1), it is easy to write down the field equations for $a(t)$ and $\phi(t)$. We do not display them here except mentioning a crucial property that they are symmetric under the following transformations:

$$a(t) \to a^{-1}(-t), \quad \phi(t) \to \phi(-t) - 6 \log a(-t). \tag{3}$$

This is the so called SFD transformations as we mentioned earlier. This symmetry of the equations of motion leads to various interesting cosmological consequences. Moreover, we define Hubble parameter and the shifted dilaton respectively as, the Hubble parameter $H = \dot{a}/a$ and $\bar{\phi} = \phi - 3 \log a(t)$, (3). One can see from the equation of motion for the scale parameter and the dilaton that there are four solutions: two in the regime $t < 0$ and other two in the positive $t$ side. In the negative $t$ range there is a solution for which $H > 0$ and $\dot{H} > 0$ such that the Universe starts from zero size with zero coupling and undergoes accelerated expansion. This solution is called the pre-big bang (PBB) solution. On the other hand, for $t > 0$, there is a solution which corresponds to expanding Universe with deceleration, i.e. $\dot{H} < 0$. This could be identified with the FRW solution. One can show that PBB solution gets related to the FRW type solution through SFD and time reversal transformations. This is a special attribute of string theory. Thus it is evident that due to the presence SFD, inflation appears naturally in string theory. Furthermore, the inflation is driven by the kinetic term of the dilaton. Therefore, in this scenario, there is no need to invoke a fine–tuned potential for inflation.

However a question naturally arises: Can we put together the $t > 0$ and the $t < 0$ regions as a single evolution? This will lead to a dilaton driven pre-big bang inflation for

$t < 0$ and the standard FRW cosmology at later time i.e. $t > 0$. In this scenario, since the Hubble parameter has a maximum at $t = 0$, this instant can therefore be identified with the emergence of the big bang of standard cosmology. However, it turns out that as we approach $t = 0$, the Hubble parameter $H$ becomes large and we proceed towards the high curvature regime. Much before we reach the singularity at $t = 0$, when the Hubble parameter, $H \sim l_s^{-1}$, the tree level effective equations (1) do not provide an adequate description of cosmology, since we already reach high curvature and strong coupling (note that $\phi$ approaches infinity as $t \to 0$). Therefore, one must take into account the effects of higher powers of curvature and higher order contributions in string perturbation theory. Due to these problems, there is as yet no concrete answer to the question that is posed in the beginning of this paragraph.

Thus one of the central issues in PBB string cosmology is to understand the transition from the accelerated expanding solution to the decelerating and expanding solution; known as the graceful exit problem. In other words, we would like to understand the mechanism for transition of the Universe from PBB to FRW branch. Of course, graceful exit problem needs to be resolved in any cosmological model which incorporates inflation. In the PBB context the evolution equations have been studied in the presence of a dilatonic potential. Although, it is not possible to derive the dilaton potential in the string perturbation theory, however, one expects that eventually a dilaton potential will appear since there is no evidence for a long range weak force (comparable to gravitational force). It has been shown that [6] that such a branch change from PBB (inflationary) to post big bang (FRW) is not possible in the presence of any realistic dilaton potential if we deal with the tree level string effective action. It is quite possible that a satisfactory answer to the question needs better understanding of non-perturbative string theory. One of the way to understand the graceful exit problem is to consider higher order string effective action taking into account higher powers of curvature and analyse the possibility of overcoming the no go theorems. Another proposal is to invoke ideas of quantum cosmology in order to resolve the problem of graceful exit [7,8]. Let us discuss, qualitatively, the proposal of Gasperini, Maharana and Veneziano to understand graceful exit in quantum string cosmology. Recall that the classical string cosmological equations, when analysed in the presence of a dilaton potential do not allow transition from the PBB phase to the FRW regime. There is an impenetrable regime in the phase space where the Hubble parameter become unphysical and the trajectories representing classical solutions must end on a surface characterized by the Hubble parameter and the dilaton on a two dimensional plane. These conditions are known as the egg equations. If one starts with quantum evolution equations, i.e. Wheeler de Witt equation for the graviton-dilaton system, then it might be possible to circumvent the classical no go theorem. Indeed this approach was followed [7] to illustrate how graceful exit could occur in string cosmology through a simple toy model. In this model, in the presence of a shifted dilatonic potential, one finds that there is a wave function which in the remote past corresponds to the background configurations of accelerating, expanding Universe. Then one finds that the transition probability for finding the Universe corresponding to the FRW configuration of the background goes like $\exp -1/g_{\rm str}^2$. This result is quite encouraging although obtained for a toy model. Subsequently, the issue of graceful exit in quantum string cosmology was studied for the graviton, dilaton and axion system and the amplitudes were calculated [8].

## 3. Various cosmological solutions

As it is mentioned in the introduction, investigation of explicit cosmological solutions always help us as a guidepost. With this in mind, in this section we give a general analysis constructing various solutions that follows from string theory. In particular, we first discuss a model which appears in various string theories in their low energy limits. This model becomes particularly interesting when we take into account various non-perturbative duality symmetries of string theory – a direction which activated much of interests in the recent past. Various other complicated models can be analysed along the similar line. For that we refer the readers to the original papers. We will then go on to discuss the solutions to WDW equations for our model as, we expect, the solution of WDW equations (governing the quantum description of our Universe) is likely to give important hints for the resolution of graceful exit problem in pre-big bang cosmology.

### 3.1 *An illustrative example*

The simplest cosmological model in $D$ dimension involves the metric, a dilaton and an $n$-rank field strength $F_n$. The action is given by

$$S = \int \mathrm{d}^D x \sqrt{-G} \left[ R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!} e^{a\phi} F_n^2 \right] , \tag{4}$$

where the constant $a$ can be parametrised as

$$a^2 = \Delta - \frac{2(n-1)(D-n-1)}{D-2} . \tag{5}$$

Here, the action (4) is written in Einstein frame. The string metric $g_{\mu\nu}$ of previous section is related to the Einstein metric $G_{\mu\nu}$ by

$$G_{\mu\nu} = e^{\frac{-2\phi}{D-2}} g_{\mu\nu}. \tag{6}$$

$\Delta$ in (5) is a dimension dependent constant that can be found in [9]. We will assume that all the fields depend only on time. The background metric is assumed to have the form

$$\mathrm{d}s^2 = -e^{2U}\mathrm{d}t^2 + e^{2A}\mathrm{d}\bar{s}^2 + e^{2B}\mathrm{d}y^m \mathrm{d}y^m, \tag{7}$$

where $\mathrm{d}\bar{s}^2$ represents the $p$-dimensional metric on the spatial section of a $d$-dimensional spacetime, with $\mathrm{d} = p+1$. We shall consider spatial metrics of the maximally-symmetric form

$$\mathrm{d}\bar{s}^2 = \frac{\mathrm{d}r^2}{1-kr^2} + r^2 \mathrm{d}\Omega^2 , \tag{8}$$

where $d\Omega^2$ is the metric on a unit $(p-1)$-sphere and the constant $k$ has been defined earlier. In (7), $m$ runs over $q$ dimensions so that $D = 1 + p + q$.

In the gauge $U = pA + qB$, the action (4) reduces to

$$S = \int \mathrm{d}t \Bigg[ (\dot{\Phi})^2 + \frac{2q(D-2)a^2}{p-1}\dot{Y}^2 - \frac{2p\Delta}{p-1}\dot{X}^2 - \Delta\lambda^2 e^{\Phi} + 2kp\Delta(p-1)e^{2X} \Bigg]. \tag{9}$$

In writing down this action, one can use either of the two ansätze for the field strength $F_n$ that are compatible with the symmetries of the metric (7), giving rise to electric or magnetic cosmological solutions. In the electric solutions, the ansatz for the antisymmetric tensor is given in terms of its potential, and in a coordinate frame, takes the form

$$A_{m_1 m_2 \ldots m_q} = f \epsilon_{m_1 m_2 \ldots m_q}\ , \tag{10}$$

and hence

$$F_{0 m_1 m_2 \ldots m_q} = \dot{f} \epsilon_{m_1 m_2 \ldots m_q}\ , \tag{11}$$

where $f$ depends on $t$ only. For the electric solutions, we have $p = D - n, q = n - 1$. For the magnetic cosmological solutions, the ansatz for the tangent-space components for the antisymmetric tensor is

$$F_{a_1 a_2 \ldots a_p} = \lambda e^{-pA}\, \epsilon_{a_1 a_2 \ldots a_p}\ , \tag{12}$$

where $\lambda$ is a constant. Thus we have $p = n, q = D - n - 1$. In the action (9), $X, Y$ and $\Phi$ are related to $A, B$ and $\phi$ in the following way:

$$X \equiv qB + (p-1)A, \quad Y \equiv B + \frac{p-1}{\epsilon a(D-2)}\phi, \quad \Phi \equiv -\epsilon a\phi + 2qB\ . \tag{13}$$

Here $\epsilon = 1$ is for electric case and $\epsilon = -1$ for the magnetic case. Note that in the electric case, the constant $\lambda$ arises as the integration constant for the function $f$ in (11).

The equations of motion for $X, \Phi$ and $Y$ are

$$\ddot{X} + k(p-1)^2 e^{2X} = 0, \quad \ddot{\Phi} + \frac{1}{2}\Delta\lambda^2 e^{\Phi} = 0, \quad \ddot{Y} = 0\ . \tag{14}$$

The variation of the action (4) with respect to the lapse function $\sqrt{g_{00}}$ provides the canonical constraint:

$$\dot{\Phi}^2 + \Delta\lambda^2 e^{\Phi} + \frac{2q(D-2)a^2}{p-1}\dot{Y}^2 = \frac{2p\Delta}{p-1}\dot{X}^2 + 2kp\Delta(p-1)e^{2X}. \tag{15}$$

Since $X$ and $\Phi$ both satisfy Liouville equations, it is straightforward to solve these equations directly:

$$e^{-X} = \begin{cases} \frac{p-1}{\tilde{P}_X}\cosh(\tilde{P}_X t)\ , & \text{if } k = 1; \\ \frac{p-1}{\tilde{P}_X}\sinh(\tilde{P}_X t)\ , & \text{if } k = -1; \end{cases}$$
$$X = -\tilde{P}_X t\ , \quad \text{if } k = 0, \tag{16}$$

where $\tilde{P}_X$ is an arbitrary constant. Similarly the solution for $\Phi$ is

$$e^{-\frac{\Phi}{2}} = \frac{\lambda\sqrt{\Delta}}{2\tilde{P}_\Phi} \cosh(\tilde{P}_\Phi t) \,, \tag{17}$$

where $\tilde{P}_\Phi$ is again constant. The solution for $Y$ may be taken to be simply

$$Y = -\tilde{P}_Y t \,. \tag{18}$$

The constraint (15) therefore implies that

$$\tilde{P}_\Phi^2 = \frac{p\Delta\tilde{P}_X^2 - q(D-2)a^2\tilde{P}_Y^2}{2(p-1)} \,. \tag{19}$$

The Hamiltonians for the fields $X$, $\Phi$ and $Y$ are given by

$$\begin{aligned} H_X &= \frac{1}{2}P_X^2 + \frac{1}{2}k(p-1)^2 e^{2X} \,, \\ H_\Phi &= \frac{1}{2}P_\Phi^2 + \frac{1}{2}\Delta\lambda^2 e^{\Phi} \,, \\ H_Y &= \frac{1}{2}P_Y^2 \,, \end{aligned} \tag{20}$$

where $P_X, P_\Phi$ and $P_Y$ correspond to momenta conjugate to $X, \Phi$ and $Y$ coordinates. Notice that the solutions for $X, \Phi$ and $Y$ can be cast in a different form in terms of their phase-space variables. For example, when $k = 1$, the solution for $X$ can be written as

$$e^{-X} = \frac{p-1}{\tilde{P}_X}\cosh\tilde{X}, \quad P_X = -\tilde{P}_X \tanh\tilde{X} \,, \tag{21}$$

where $\tilde{X} = \tilde{P}_X t$. In fact, these equations can be viewed as a canonical transformation from the interacting Liouville system, with phase-space coordinates $(X, P_X)$, to a free system with phase-space coordinates $\tilde{X}, \tilde{P}_X$ with the Hamiltonian $\tilde{H}_X = \frac{1}{2}\tilde{P}_X^2$, by re-writing (21) as

$$P_X = (p-1)e^X \sinh\tilde{X}, \quad \tilde{P}_X = (p-1)e^X \cosh\tilde{X} \,, \tag{22}$$

The generating function $F(X, \tilde{X})$ has the following form

$$F(X, \tilde{X}) = (p-1)e^X \sinh\tilde{X} \,, \tag{23}$$

such that

$$P_X = \frac{\partial F}{\partial X} \,, \quad \tilde{P}_X = \frac{\partial F}{\partial \tilde{X}} \,. \tag{24}$$

These are the same equations as in (22). Obviously, since $H_\Phi$ has also the same structure, a similar set of canonical transformations will also bring it to a free Hamiltonian form. Thus by solving a set of free systems and using the canonical mapping (21), we can generate the solutions of the interacting theory given by the action (4). As we shall discuss in next subsection, these transformations can be implemented in the quantum version of the model. This, in turn, will allow us to solve the corresponding WDW equations in a straightforward manner.

Though, here, we have discussed a simple class of models, we would like to point out that the method employed here is quite powerful. Various complicated cosmological systems lead exact solutions both in the classical and quantum level through this method. For more details, we refer to the original papers [10,11].

### 3.2 *The Wheeler-DeWitt equation*

The canonical transformation between the classical Liouville and free theories that have been discussed above can be implemented at the quantum level. This is done by introducing intertwining operators which generate canonical transformations on the quantum operators and on the wave functions. In order to construct such operators we first focus on the cosmological model discussed in the previous subsection.

Let us first concentrate on $H_X$ given in (20). It is known that there exists an operator $C_X$ which transforms the Liouville Hamiltonian to a free one [12]. In particular,

$$C_X H_X C_X^{-1} = \tilde{H}_X \ . \tag{25}$$

As a result, the wave functions $\psi_X$ and $\tilde{\psi}_X$ of the Liouville and free theories are related by $\psi = C_X^{-1}\tilde{\psi}$. The operator $C_X$ has been constructed in [12], and takes the following form:

$$C_X = \mathcal{P}_{(p-1)\sinh X} P_X^{-1} \mathcal{I} \mathcal{P}_{\ln X} \ , \tag{26}$$

where each of the constituent pieces has the following action:

$$\begin{aligned}
\mathcal{P}_{\ln X} &: X \to \ln X \ , \quad P_X \to X P_X \ , \\
\mathcal{I}_X &: X \to P_X \ , \quad P_X \to -X \ , \\
P_X^{-1} &: X \to P_X^{-1} X P_X \ , \quad P_X \to P_X \ , \\
\mathcal{P}_{(p-1)\sinh X} &: X \to (p-1)\sinh X \ , \\
&\qquad\qquad P_X \to \frac{P_X \cosh^{-1} X}{p-1} \ .
\end{aligned} \tag{27}$$

Taking into account the commutation relation $[P_X, X] = -i$, it is immediate that the combined action of (27) is to map the Liouville Hamiltonian $H_X$ to the free Hamiltonian $\tilde{H}_X = \frac{1}{2}\tilde{P}_X^2$. Similarly, the operator $C_X$ has the following action on the wave function [12]:

$$C_X^{-1} : \quad e^{ikX} \to N_k \, K_{ik}(e^X) \ , \tag{28}$$

where $K_{ik}$ is a modified Bessel function. Owing to the fact that the canonical transformation described by $C_X$ is non-unitary (as it must be, since the Liouville theory is not simply equivalent to the free theory), the normalisation of the transformed wave function is not just the same as the normalisation of the free wave function. It can be determined by calculating the effect of the transformation on the Hilbert-space inner product, leading to the result

$$N_k = \frac{1}{\pi}\sqrt{2k\ \sinh(\pi k)}\ . \tag{29}$$

Now consider the WDW equation, which is simply

$$H\Psi(X,\Phi,Y) = 0. \tag{30}$$

Here the total Hamiltonian of the system is given by

$$H = H_\Phi + \frac{2q(D-2)a^2}{p-1}H_Y - \frac{2p\Delta}{p-1}H_X\ . \tag{31}$$

It is clear now from the structure of the Hamiltonian that the wave function $\Psi(X,\Phi,Y)$ will have the following form:

$$\Psi(X,\Phi,Y) = \Psi_X\Psi_\Phi e^{iP_Y Y}\ , \tag{32}$$

where $\Psi_X$ and $\Psi_\Phi$ depend on $X$ and $\Phi$ respectively. Following our previous discussion, there is an intertwining operator which will convert the interacting Hamiltonian $H$ to a sum of free Hamiltonians. It is given by

$$C = \mathcal{P}_{(p-1)\sinh X}P_X^{-1}\mathcal{I}_X\mathcal{P}_{\ln X}\mathcal{P}_{\sqrt{\Delta}\lambda\sinh\Phi}P_\Phi^{-1}\mathcal{I}_\Phi\mathcal{P}_{\ln\Phi}\ . \tag{33}$$

Its action on the Hamiltonian is

$$CHC^{-1} = \tilde{H}_\Phi + \frac{2q(D-2)a^2}{p-1}H_Y - \frac{2p\Delta}{p-1}\tilde{H}_X\ . \tag{34}$$

It is now easy to read off the action of $C$ on the wave functions:

$$\Psi(X,Y,\Phi) = \frac{1}{\sqrt{2\pi}}N_{k_X}N_{k_\Phi}K_{i(p-1)k_X}(e^X)K_{i\sqrt{\Delta}\lambda k_\Phi}(e^\Phi)e^{ik_Y Y}\ , \tag{35}$$

where $N_{k_X}$ and $N_{k_\Phi}$ are momentum-dependent normalisation constants which can be determined from (29).

So far, we have been discussing the case $k=1$. Following similar arguments, we can also study the WDW wave function for an open universe ($k=-1$). In this case, the analogue of (27) is

$$\begin{aligned}
\mathcal{P}_{\ln X} &: X \to \ln X\ , \quad P_X \to XP_X\ ,\\
\mathcal{I}_X &: X \to P_X\ , \quad P_X \to -X\ ,\\
P_X^{-1} &: X \to P_X^{-1}XP_X\ , \quad P_X \to P_X\ ,\\
\mathcal{P}_{(p-1)\cosh X} &: X \to (p-1)\cosh X\ ,\\
&\qquad P_X \to \frac{P_X \sinh^{-1}X}{p-1}\ .
\end{aligned} \tag{36}$$

The operator $C$ is now

$$C_X = \mathcal{P}_{(p-1)\cosh X}P_X^{-1}\mathcal{I}\mathcal{P}_{\ln X}\ , \tag{37}$$

whose action on the wave functions can be evaluated using methods similar to the ones used above in the $k=1$ case. We shall not discuss the $k=0$ case in detail. Following the above discussion, the structure of the wave function is also easily obtained in this case.

## 4. Conclusion

In this brief review, we have discussed a possible cosmological scenario which appears naturally in string theory. We pointed out several attractive features of the model. However, graceful exit remains a major problem in this context. Though there are very encouraging hints to overcome this problem from quantum string cosmology, there is still lack of progress in this direction. Beside the exit problem, questions are raised about whether the PBB scenario solves the homogeneity and flatness problems [13,14]. We have not discussed these issues in this review. In §3, we analysed certain classical solutions and the corresponding WDW equations of a large class of models, from the pre-big bang prospective. We believe that these solutions provide various clues which can help in implementing the PBB scenario successfully.

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