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Topological defects in cosmology

AJIT M SRIVASTAVA
Institute of Physics, Sachivalaya Marg, Bhubaneswar 751 005, India

Abstract. Present status of theories of topological defects in particle theory models of the early Universe is discussed. Various consequences of topological defects in cosmology, such as constraints on particle theory models, structure formation etc. are discussed.

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We present a brief review of topological defects and the role they play in cosmology. Though there is no compelling evidence yet of the existence of these topological defects in the Universe, still, this topic has received much attention recently due to important implications topological defects have for cosmological models [1]. In the following, we first give a physical description of topological defects in detail using simple examples. Then we explain how these defects are thought to arise in the early Universe and various consequences of their existence for the Universe [1,2].

1. General properties of topological defects

It is simple to explain the concept of topological defects using condensed matter examples. Consider a system which can exist in two different phases, (for example steam and water). Generally one defines an *order parameter* which takes different values in different phases. Existence of topological defects crucially relies on the nature of this order parameter depending on which several kinds of topological defects can exist.

1. *Point defects (monopoles)*: These are tiny point like regions of one phase embedded in the other phase. Like a tiny drop of water in steam (but very different in nature as we will explain below).
2. *String defects*: These are thin tube like regions of one phase embedded in the other phase.
3. *Domain walls*: These are sheet like regions of one phase embedded in the other phase.

Strings (domain walls) are either closed loops (surfaces), or they are infinite (for the Universe, in laboratory they can end at boundaries). Their structure shows the origin of the term 'defect'. They are topological because their existence originates from topological considerations. A given property of a system is said to be of topological nature if smooth deformations do not change that property.

Topological defects are stable against smooth deformations (continuous changes) in the system. This is where, for example, a topological point defect differs from the example of a water droplet embedded in steam. Local heating can easily convert the water droplet to steam so that there is no water left anywhere. On the other hand, local heating, or other local deformations can not get rid of a topological point defect.

An example of string defect is vortices in superfluid ^{4}He. The order parameter which describes the superfluid phase is a complex scalar field ψ. Free energy density F is of the form,

$$F = K|\bigtriangledown\psi|^2 - \alpha|\psi|^2 + \beta|\psi|^4 \tag{1}$$

α is negative for temperatures $T > T_c$, and becomes positive for $T < T_c$. Plot of F (for spatially uniform ψ) is shown in figure 1, here $\psi = \psi_1 + i\psi_2$.

We see that for $T > T_c$, F is minimized for $\psi = 0$ while in the superfluid phase ($T < T_c$) F is minimized for $|\psi| \equiv \eta = \sqrt{2\alpha/\beta}$. Clearly this does not fix the phase θ of ψ (which can vary spatially in a phase transition). This remaining degree of freedom spans what is called as the order parameter space V, which is a circle S^1 in this case. Of course any spatial variation of ψ will cost energy due to the gradient term in the expression for F.

Consider now a region of superfluid with the distribution of θ on (and nearby) a closed path L in physical space such that θ changes by 2π as we go around the path L. As $\Delta\theta$ can only change by an integer multiple of 2π around L, its value can not change if we make a small deformation in L. It then follows that one can continuously shrink L down to a point while $\Delta\theta$ around L remains 2π (as long as we do not cross any region of $\psi = 0$ where θ becomes undefined). When L shrinks to a point then ψ must go to zero there to maintain

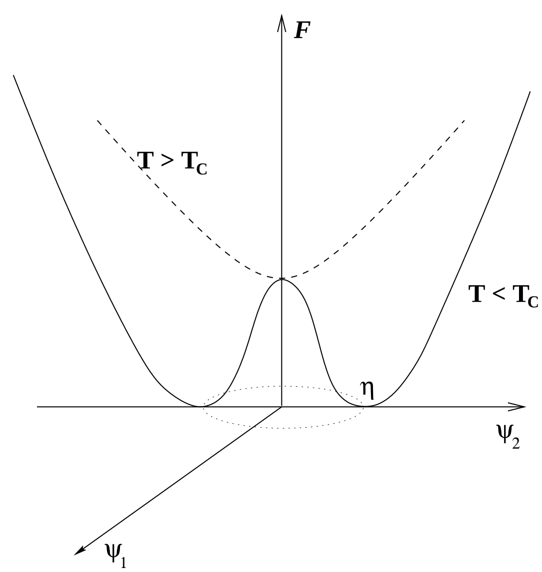


Figure 1. Free energy for superfluid ^{4}He.

finite gradient energy density eq. (1). Thus, we conclude that the original loop L must enclose at least one point where ψ vanishes identically. As L can be shrunk on any surface whose boundary is L, we conclude that L encloses a line like region where $\psi = 0$. Since vanishing of ψ implies normal phase of ^{4}He, one obtains a string like region of normal phase embedded in the superfluid phase of ^{4}He. This is the vortex in superfluid ^{4}He.

Note that in this example, existence of string was related to the fact that there was a closed curve in the order parameter space V (which was S^1 in the above example), which could not be shrunk to a point within V. $\Delta\theta$ can change by $2n\pi$ around L, with each different value of integer n (the winding number of loop in V) corresponding to a topologically distinct string. When there are such non-trivial loops which can not be smoothly shrunk to a point, one says that the 1st homotopy group of V, $\pi_1(V)$ is non-trivial. These ideas are easily generalized to other homotopy groups by considering higher dimensional surfaces in V which can not be smoothly shrunk to point. For example, $\pi_2(V)$ ($\pi_3(V)$) non-trivial means that there are closed 2-surfaces (3-surfaces) in V which can not be smoothly shrunk to a point. $\pi_0(V)$ non-trivial means that V is disconnected.

In general, defects (in 3+1 dimensions) are classified in the following manner. For $\pi_n(V) \neq 1$, with n being either 0,1,2 or 3, one gets a domain wall defect, a string defect, a monopole, or texture (Skyrmion), respectively. (One actually needs to consider what is called as free homotopy as opposed to based homotopy which is used in defining $\pi_n(V)$. We will not go into these details. For details see [3].)

2. Defect formation in cosmology and constraints on particle physics models

It is believed that during expansion and consequent cooling, the Universe may have undergone several phase transitions. For example, when the temperature of the Universe was about 10^{16} GeV (and its age about 10^{-37} sec.) then grand unified theory (GUT) phase transition must have taken place where a GUT symmetry group will spontaneously break to $SU(3) \times SU(2) \times U(1)$ symmetry. As the Universe continued cooling, it will go through the electroweak symmetry breaking phase transition when its temperature is of order 100 GeV. At still lower temperatures of about 200 MeV, the Universe will go through the quark-hadron phase transition.

During any spontaneous symmetry breaking phase transition, topological defects may be produced depending on the symmetry breaking pattern. [e.g., Widyan, in this conference, has discussed strings and domain walls in certain extensions of electroweak model.] First detailed investigations of defect formation during phase transitions in the early Universe were carried out by Kibble [4]. This process of defect formation is generally known as the Kibble mechanism and arises due to a sort of domain formation after the phase transition with defects forming at the junctions of these domains. This mechanism has been experimentally verified in liquid crystal systems. These topics, as well as a new mechanism for defect formation, have been described in detail in the talks by Ray and Sengupta in this conference. Figure 2 shows some stages of defect formation in liquid crystal experiments.

Topological defects with most interesting consequences for cosmology are cosmic strings produced at the GUT scale. Let us first discuss implications of some other defects produced at the GUT scale.

Domain walls: These arise when the vacuum manifold is disconnected. Domain walls are usually disastrous for cosmology. This is because domain walls are extremely massive. For

GUT scale domain walls, the mass per unit area is about 10^{46} tons/cm^2. For comparison, note that mass of a typical galaxy is about 10^{38} tons. So, a micron size domain wall has mass equal to galactic mass. Now, horizon provides an upper limit for any correlated domain size. Then Kibble mechanism predicts [4] of order one domain wall per horizon. For one domain wall in the observed Universe, its energy density will be $\rho_{DW}(t) \sim \eta^3 t^2 t^{-3}$, where η is the scale at which domain wall was produced. Now, the critical energy density $\rho_c \sim m_{pl}^2 t^{-2}$. So, for GUT domain walls ($\eta \sim 10^{16}$ GeV),

$$\frac{\rho_{DW}}{\rho_c}(t_{\mathrm{present}}) \sim (\frac{\eta}{m_{pl}})^2(\eta t_{\mathrm{present}}) \sim 10^{51} \tag{2}$$

Thus GUT domain walls are ruled out. In order to be consistent with observations, the scale of domain wall production must be less than about 1 MeV. Unfortunately there are no natural models for such low energy phase transitions. The domain wall constraint has been most effectively used in constraining axion models.

Magnetic monopoles: These are point like defects with magnetic charge. Important point about monopoles is that these are almost inevitable in any natural GUT model. For example, in $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ symmetry breaking transition monopoles are produced. Main point is that we always need $U(1)$ electromagnetism to remain unbroken. This, along with the choice of any simple GUT group (such as $SU(5)$) necessarily leads to existence of monopole defects [4]. Monopole mass is of order η ($\sim 10^{16}$ GeV for GUT). Using Kibble mechanism, one can estimate that ρ_{mon} $(\mathrm{initial}) \sim \eta/\eta^{-3}$. Since monopoles are superheavy, $\rho_{\mathrm{mon}} \sim R^{-3}$ while $\rho_{\mathrm{radiation}} \sim R^{-4}$ where R is the scale factor of the Universe. Using this one can easily see that monopole mass density will be too large at present to be consistent with the observations. This is known as the *monopole problem*. (Constraints on monopole density also come from inter-galactic magnetic fields.)

This monopole problem was the main motivation which led to the proposal of the inflationary universe where monopoles are diluted away by the exponential expansion. Certain other proposals to avoid monopole problem are:

1. Change symmetry breaking pattern (Langacker–Pi mechanism). Consider, $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ symmetry breaking when monopoles are produced. At the next stage, symmetry is broken to $SU(3)$ leading to production of strings which join monopoles to antimonopoles. This leads to rapid annihilation of monopoles. In the last stage, there is a transition to $SU(3) \times U(1)$ symmetry which generates the $U(1)$ electromagnetism. At this stage the strings melt away.
2. Recently, it has been suggested [5] that if the transition produces monopoles and domain walls, then monopoles melt when they hit domain walls. Ultimately, domain walls decay due to small asymmetry. Thus monopoles are swept away by domain walls.

3. Cosmic strings

These are most interesting topological defects in cosmology. GUT scale strings have mass per unit length of about 10^{16} tons/cm. Since they are very massive, they can lead to structure formation by gravitational accretion. String network formed at the phase transition evolves by coarsening of the network, as shown in the pictures in figure 2. This coarsening happens due to following processes.

1. String intercommutation: This means that two strings, while crossing, can exchange partners at crossing point. Due to this, a string while folding onto itself, can chop of a loop. Intercommutation of strings has been checked in various numerical simulations. It is also observed in liquid crystal experiments.
2. Oscillating loops emit radiation (primarily gravitational radiation for gauge strings) and eventually shrink away.

Because of these processes strings keep decaying. End results is a scaling solution where roughly fixed number of loops and long strings exist in a Hubble volume at any given time. Most early calculations of structure formation by cosmic strings utilize scaling solution. This has become a topic of discussion again recently. The issue is whether sufficient number of long strings are present at the time of formation or not (i.e. is there a scaling regime?).

3.1 *Cosmic strings and structure formation*

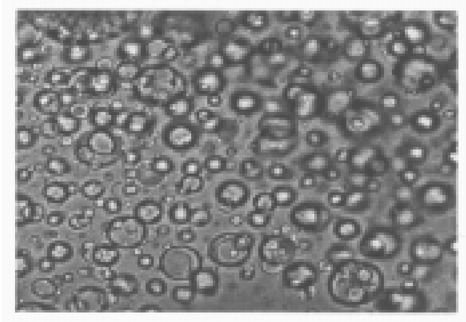
Cosmic strings lead to a component in density fluctuations which is scale invariant. Let us assume that there are about N long strings in a Hubble volume. Then $\rho_{\rm string} \sim N\mu t\, t^{-3} \sim N\mu t^{-2}$ where μ is the mass per unit length of the string. With $\rho_c \sim (Gt^2)^{-1}$, we get $\rho_{\rm string}/\rho_c = N\mu G$ which is independent of horizon scale. Scale invariant density perturbations also arise for certain other topological objects such as for textures and for global monopoles. Strings can lead to structure formation in 3 possible ways.

1. Massive string loops can act as seeds for gravitational accretion of dust and radiation. This contribution was generally believed to be sub dominant. However, there are recent results which show that this contribution may be significant.
2. Filament formation: Strings do not remain straight due to frequent intercommutation etc. and develop wiggly structure. Even though straight strings do not attract matter around them, wiggly strings do attract matter and can lead to formation of filamentary structure.
3. Wake formation: String metric is flat with a deficit angle. Because of this, it leads to planar over densities in regions through which it moves. The metric of straight string along z axis is

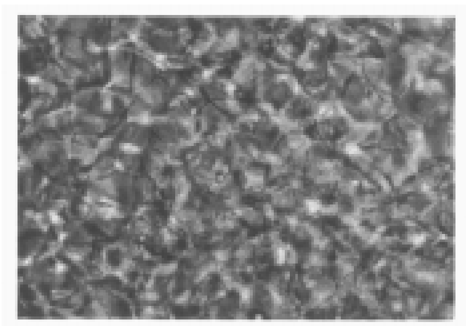
$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}z^2 - dr^2 - (1 - 8G\mu) r^2 \mathrm{d}\phi^2. \tag{3}$$

Define a new angle $\phi' = (1 - 4G\mu)\phi$. Then the last term becomes $r^2 \mathrm{d}\phi'^2$. This shows that the metric is flat with a deficit angle of $8\pi G\mu$. Thus the space-time around a straight cosmic string is locally flat (so there is no gravitational force on test particles) but globally it is like a cone, i.e. there is a deficit angle. This is shown in figure 3.

Due to this, two particles crossing the string (shown by two parallel trajectories in figure 3) will move towards each other. This will lead to wake formation behind a cosmic string moving through matter. Also, presence of a cosmic string along the line of sight will lead to double images of distant objects (galaxies etc.). Typical angular separation of double images is of order of few arc seconds for GUT strings.



(a)



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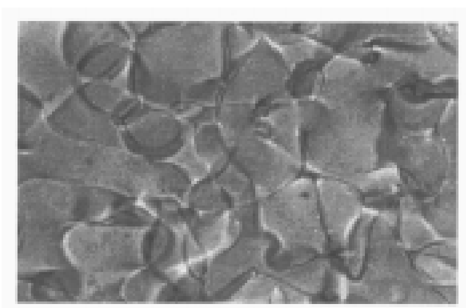

(c)

Figure 2. Various stages of defect formation. Figure (a) shows nucleation of bubbles of broken symmetry phase (nematic phase in the liquid crystals). Figure (b) shows dense network of string defects formed due to coalescence of bubbles. Figure (c) shows how the string network coarsens as strings shrink and string loops decay.

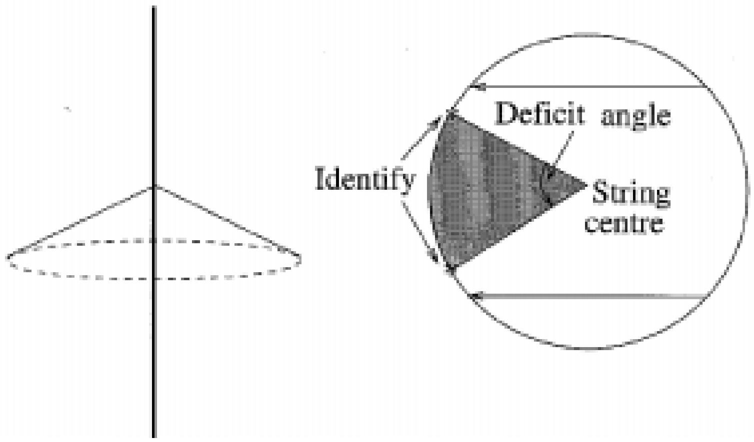

Figure 3. Conical structure of space-time around cosmic string.

3.2 *Cosmic strings and observations*

Cosmic string models predict several observational effects.

1. Cosmic string loops decay by emitting gravitational radiation with the rate $\dot{M} \sim -GM^2R^4\omega^6$ (quadrupole formula). With $\omega \sim R^{-1}$ one gets $\dot{M} \sim -GM^2R^{-2} \sim -G\mu^2$. Numerically one finds $\dot{M} \sim -\gamma G\mu^2$ with $\gamma \sim 60$. This background gravitational radiation will lead to fluctuations in the pulsar timings as observed on the earth. This leads to very strong constraints on cosmic string models. Earlier it was believed that these constraints rule out strings as candidates for structure formation. More refined estimates constrain the value of $G\mu/c^2 < 5 \times 10^{-6}$ which is o.k. for GUT strings and for structure formation [6].
2. Cosmic Microwave Background (CMB) observations. Due to conical nature of space-time around a cosmic string, photons arriving from the two sides of the string will have step like discontinuities in temperature. This is known as Kaiser–Stebbins effect.
3. CMB anisotropy constraints. Recently it has been argued that data of CMB anisotropies at scales of 100 Mpc does not agree with the predictions of scaling defect models [7].

Several ways out of this problem have been proposed. It is proposed that in an open Universe, or with non-zero cosmological constant ($\Omega_\Lambda \sim 0.7$) strings do not scale after curvature domination [8]. Another issue has been raised is that it is incorrect to neglect loops in structure formation. Recent high resolution simulations suggest that contribution of loops may be similar to that of long strings. Also, loops are highly correlated to long strings due to intercommutation events. This correlation seems to persist in time [9].

In summary, topological defects are fascinating possibility from the early Universe. Many of the predictions are tightly constrained by observations. Several aspects of cosmic defects have universal character such as theory of formation of defects via domain structure. This leads to remarkable possibility of checking predictions relating to defects in cosmology in condensed matter systems. Recent observations have raised many questions about theories of defect formation and evolution. These are active topics of research presently.

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