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An analysis of 900 optical rotation curves: Dark matter in a corner?

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Abstract. One of the largest H_α rotation curve data bases of spiral galaxies currently available is that provided by Persic and Salucci (PS) [1], which has been derived by them from unreduced rotation curve data of 965 southern sky spirals obtained by Mathewson, Ford and Buchhorn (MFB) [2]. Of the original sample of 965 galaxies, the observations on 900 were considered by PS 1995 to be good enough for rotation curve studies, and the present analysis concerns itself with these 900 rotation curves.

The present analysis shows that the rotation velocity, V, at any radial displacement, R, in the optical disc of any given spiral galaxy satsifies the law $V/R^\alpha = V_0/R_0^\alpha$, where R_0 and V_0 are given as approximate functions of the galaxy's absolute magnitude and surface brightness whilst α is an unidentified function of other galaxy parameters – of which the most significant ones will be the relative proportions of the disc, bulge and halo mass-components. It is this latter function which provides the opportunity for a dark-matter modelling process which is independent of any particular dynamical theory.

Keywords. Galaxies; dark matter.

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1. Introduction

A first analysis of the data considered here was given by Persic *et al* [3], hereafter PSS, in which they concluded that there exists a 'universal rotation curve' for spiral galaxies which is determined primarily by the total luminosity of the galaxy concerned, and the essence of this conclusion is confirmed here.

However, the present analysis of the same data goes much further and shows, finally, that the rotational velocity in the optical disc of a spiral galaxy with absolute magnitude M and surface brightness S behaves according to

$$\begin{aligned}\frac{V}{R^\alpha} &= \frac{V_0}{R_0^\alpha},\\ \log V_0 &\approx -0.584 - 0.133\,M - 0.000243\,S,\\ \log R_0 &\approx -3.291 - 0.208\,M - 0.00292\,S,\end{aligned} \tag{1}$$

Since this model is shown to account for over 90% of the variation in the pivotal diagram, it can be considered as, at the very least, an extremely good approximation to the statistical reality, as judged for a large number of spiral discs.

2. The data

The data given by PS is obtained from the raw $H\alpha$ data of MFB by deprojection, folding and cosmological redshift correction. For any given galaxy, the data is presented in the form of estimated rotational velocities plotted against angular displacement from the galaxy's centre; estimated linear scales are not given and no data-smoothing is performed.

The analysis proposed here requires the linear scales of the galaxies in the sample to be defined which, in turn, requires distance estimates of the sample galaxies from our own locality. This information is given in the original MFB paper in the form of Tully–Fisher distances.

3. A basic correlation imposed by power-law rotation curves

The starting point for the presented analysis was the hypothesis that $V = AR^{\alpha}$ for rotation curves. For each of the 900 rotation curves in the sample, we then regressed $\log V$ on $\log R$ to obtain estimates of $\log A$ and α; the 900 pairs $(\alpha\,, \log A)$ were then plotted on a single diagram, shown in figure 1.

This figure shows that there exists an extremely strong negative $(\alpha, \log A)$ correlation.

4. The model

A detailed analysis of figure 1 shows that it imposes very strong order on the luminosity characteristics of the galaxies in the sample. Specifically, when we partition the whole sample into magnitude-limited quartiles, we find that figure 1 decomposes into the diagrams of figure 2. We see a clear evolution of the $(\alpha, \ln A)$ plots as total luminosity decreases and, within each plot, we see that surface-brightness varies systematically in the sense of low surface-brightness on the bottom boundary of each plot to high surface-brightness on the upper boundary of each plot.

Table 1.

$\log A = b_0 + b_1\,M + b_2\,S - c_0\,\alpha - c_1\,\alpha\,M - c_2\,\alpha\,S$

Pred.	Coeff. $\times 10^4$	Std. Dev. $\times 10^4$	t-ratio	p
Const.	−5840	890	−7	0.00
M	−1330	44	−30	0.00
S	−2.434	0.77	−3	0.00
α	32910	1600	21	0.00
$\alpha\,M$	2080	84	25	0.00
$\alpha\,S$	29.23	2	14	0.00

$R^2 = 90.3\%$

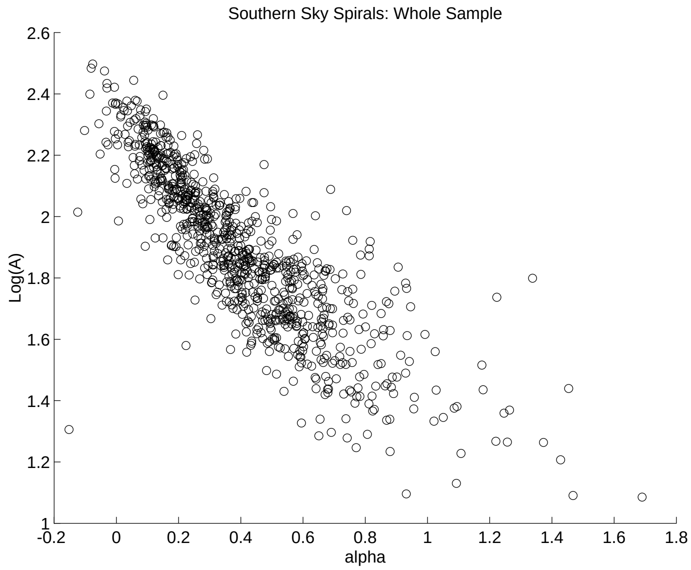

Figure 1. $(\alpha, \log A)$ plotted for each of 900 rotation curves.

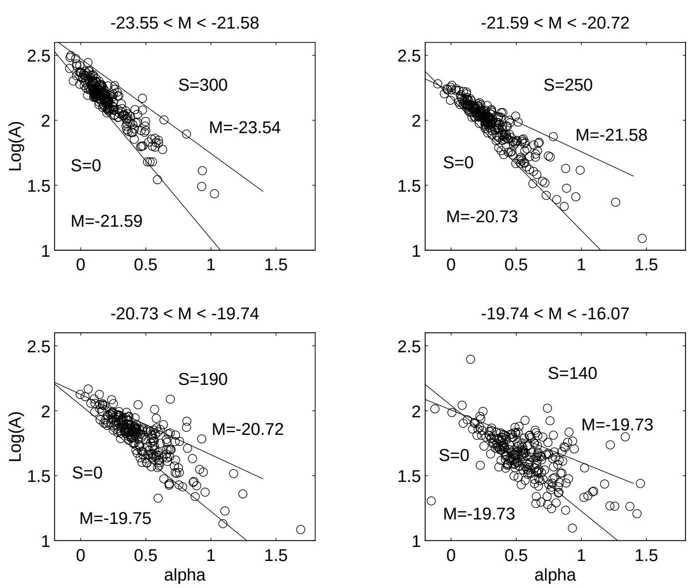

Figure 2. Surface-brightness boundaries shown for each magnitude-limited quartile.

Defining

$$\log A = \log V_0 - \alpha \log R_0,$$
$$\log V_0 = b_0 + b_1 M + b_2 S,$$
$$\log R_0 = c_0 + c_1 M + c_2 S,$$

where M is the absolute magnitude, and S is the surface brightness in solar luminosities per square parsec we arrive, finally, at the model

$$\frac{V}{R^\alpha} = \frac{V_0}{R_0^\alpha},$$
$$\log V_0 = -0.584 - 0.133\,M - 0.000243\,S,$$
$$\log R_0 = -3.291 - 0.208\,M - 0.00292\,S. \tag{2}$$

Reference to table 1 shows that all the predictors of this model are supported by very strong statistics.

5. Theory-independent dark-matter models

Of potentially greatest significance, it is to be noted that halo mass makes no *explicit* appearance in the model (2); given the quality of fit of the model to the data (over 90% of the variation in the pivotal figure 1), this implies that the only place that halo-mass can enter the system is through the undetermined function α. It is this latter fact which provides the possibility of a dark-matter modelling process which is independent of any particular dynamical theory. Specifically, since, for any given galaxy, reasonable estimates for d_m and b_m can be obtained, then it becomes possible to model α in terms of these two quantities. Analysis of any systematic differences arising between α-values calculated from the rotation curves, and the modelled α-values, will then, in principle, allow systematic estimates of any given galaxy's dark-matter content to be given in terms of the galaxy's disc-mass, bulge-mass and surface brightness.

6. Conclusions

By demonstrating how completely the power-law model for rotation curves resolves rotation curve data, magnitude data and surface brightness data over a large data-base, this paper:

1. substantially refines the well-known correlations that exist between the rotational kinematics of spiral galaxies and various of their non-kinematic properties, such as absolute magnitude;
2. shows how the power-law rotation curve is, almost certainly, exact for idealized discs (that is, for discs without the irregularities inevitably present in real optical discs).

The detailed nature of the present analysis also provides a means for the generation of statistical dark-matter models which are *independent* of any particular dynamical theory.

Apart from the objective desirability of such models, they have an obvious potential in distinguishing between competing dynamical theories.

References

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