PRAMANA
—journal of
physics

Vol. 51, No. 6
December 1998
pp. 733–742

Vertex function and coupling constant for the virtual decay of ^{7}Li

V K SHARMA
Department of Physics, Institute of Advanced Studies, University of Meerut, Meerut 250 004, India

MS received 12 March 1998; revised 8 June 1998

Abstract. The alpha-triton relative wave function with various nucleon exchange contributions and their asymptotic normalization are considered in the framework of the generator coordinate method (GCM). The asymptotic normalization of relative wave function provide the estimate of the coupling constant. The relative wave function is also used to obtain ^{7}Li–α–t vertex function in the virtual decay of ^{7}Li. The extrapolation of vertex function for negative values of q^2 up to the alpha-triton pole provide the vertex constant, which is compared with the experimentally determined estimates 0.67 FM and 0.72 FM. Our calculated value is 0.656 FM which is in close agreement with the above estimates.

Keywords. Alpha-triton relative wave function; generator coordinate method; asymptotic normalization.

PACS No. 25.10

1. Introduction

The nuclear vertex constants describing the virtual decay of nuclei into two fragments are the fundamental characteristics of the nucleus like, e.g., the mass, spin, parity, binding energy, charge RMS radius etc and they have the same status in nuclear physics as the renormalizable coupling constants in particle physics. The asymptotic normalizations of the relative wave function provide the estimates of the coupling constants. These constants are related to the cross-sections of a number of elastic and inelastic scattering processes through the vertex function [1].

In the past [2–4] considerable progress has been made in the use of the analytic properties of nuclear scattering amplitudes. Partial waves dispersion relations provide simple, explicit but very approximate models which are useful in deciding the origin of the complex scattering effects that occur in reactions involving the few-nucleon system. In contrast, forward dispersion relations have a more limited but more precise role namely the extraction of information on nuclear scattering amplitudes at unphysical energies directly from the experimental data. This information has yielded accurate values for the nuclear coupling constants for few-nucleon system. These nuclear coupling constants can be used to investigate the reliability of the methods and the interactions used in the nuclear structure calculations. Further, they provide the most accurate information available for the normalization of DWBA cross-sections for stripping reactions.

In recent years ^{6}Li–α–d vertex functions and constants are obtained by Lovash *et al* [5] and Blokhintsev *et al* [6]. These vertex functions and constants are of great importance in the theory of nuclear reactions induced by ^{6,7}Li ions especially in view of the future advent of high quality beams of polarized ^{6,7}Li ions or the targets. Very recently a pion scattering experiment on polarized ^{7}Li has been carried out at the Paul Scheerrer Institute [7]. Here we use the ground state wave function of ^{7}Li given in ref. [8] to calculate the ^{7}Li–α–t vertex function and coupling constant. It is found that the experimentally determined estimates by Dolinski *et al* [9] are in good agreement with our calculated values. This justifies the importance of our GCM technique [8, 10] and the present method of the vertex constants. The vertex function and coupling constant will be useful in the future high quality beams of polarized ion ^{7}Li research [7].

In this paper we consider the alpha-triton relative wave function and its asymptotic normalization using the generator coordinate method (GCM) wave function of ^{7}Li which gives better results not only for binding energies but also for other electromagnetic properties [8]. In this GCM method [8, 10] the microscopic structure of ^{7}Li is considered as a superposition of the generator coordinate (GC) basis functions which are defined as the antisymmetrized products of the shell-model wave functions of the two-centres with respect to the generator coordinates with linear variational GC amplitudes.

Recently a similar theory of the so-called antisymmetrized molecular dynamics [11–13] is widely used to study the light-neutron-rich halo nuclei which is just a special case of the Brink's generator coordinate method [14]. In ref. [15] we have already established explicitly that our GCM as the co-existence of the shell-model and cluster aspects – a unique feature in light nuclei [16]. It is also important to note that in our GCM, we have not considered ^{7}Li as made of alpha and triton clusters initially. The alpha-triton cluster structure only appears through the parity projection in GCM [8, 10, 15]. The parity projection before energy variation is very important. For example, if we calculate the structure of the ^{7}Li ground state properly by making the parity projection before energy variation, we find that the ^{7}Li ground state has the $\alpha + t$ clustering structure. But if we make the energy variation before the parity projection, we never get the $\alpha + t$ clustering structure.

The relative wave function is used to calculate the vertex function and coupling constant for the virtual decay of $^7\mathrm{Li} \rightarrow \alpha + t$ via the method of overlap integrals and extrapolation to the corresponding pole is done using the method proposed by Goldfarb *et al* [17]. It is also compared with the available experimental estimate by Dolinksi *et al* [9].

In § 2 we discuss the alpha-triton relative wave function and its asymptotic normalization within the framework of our GCM. The vertex function and vertex constant is estimated and compared with experimental estimates in § 3. Finally in § 4, a brief summary and conclusion are presented.

2. Alpha-triton relative wave function and asymptotic normalization

The relative motion wave function between alpha (^{4}He) and triton (^{3}H) is defined by the method of overlap integrals [1] as,

$$u_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{r}) = \left[\frac{7!}{4!3!}\right]^{\frac{1}{2}} \int^{^7\mathrm{Li}} \Psi^*_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{r}, \vec{\xi}_\alpha, \vec{\xi}_t)\, \Phi^{(\mathrm{in})}_{^4\mathrm{He}}(\vec{\xi}_\alpha)\, \Phi^{(\mathrm{in})}_{^3\mathrm{H}}(\vec{\xi}_t)\, \mathrm{d}\vec{\xi}_\alpha\, \mathrm{d}\vec{\xi}_t \qquad (1)$$

where $\mathbf{r}$ is the relative coordinate between alpha and triton clusters and $\vec{\xi}_\alpha$ and $\vec{\xi}_t$ are the internal Jacobi coordinates for normalized intrinsic wave functions of $\Phi^{(\mathrm{in})}_{^4\mathrm{He}}$ (alpha) and

$\Phi^{(\mathrm{in})}_{^3\mathrm{H}}$ (triton) in the virtual decay of ^{7}Li nucleus. Using the GCM wave function of ^{7}Li [8], the relative wave function (1) can be written in terms of the GC amplitude $f^{\mathrm{JM}\pi}$, which is the weight function and satisfy the Hill–Wheeler equation [8, 14], as

$$u^{JM\pi}_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{r}) = \left[\frac{7!}{4!3!}\right]^{\frac{1}{2}} \sum_{\substack{\mathrm{Spin}\\ \mathrm{Isospin}}} \int \mathrm{d}\mathbf{s}\, \mathrm{d}\vec{\xi}_\alpha\, \mathrm{d}\vec{\xi}_t\, f^{JM\pi}(\mathbf{s}) \Phi_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{r}, \vec{\xi}_\alpha, \vec{\xi}_t, \mathbf{s})$$
$$\otimes\, \chi_{^4\mathrm{He}+^3\mathrm{H}}(\sigma\tau) \Phi^{(\mathrm{in})}_{^4\mathrm{He}}(\vec{\xi}_\alpha) \chi_{^4\mathrm{He}}(\sigma\tau) \Phi^{(\mathrm{in})}_{^3\mathrm{H}}(\vec{\xi}_t) \chi_{^3\mathrm{H}}(\sigma\tau) \tag{2}$$

where superscripts denote the particular states of ^{7}Li ($J^\pi = \frac{3}{2}^-$), χ's are the spin-isospin factors and s is the separation between the two harmonic oscillator wells known as the generator coordinate.

The generator coordinate (GC) function occuring in the integrand of eq. (2) is of the form of $|7 \times 7|$ Slater determinant which is expanded by Laplace expansion [18] into 35 terms of $|4 \times 4| \otimes |3 \times 3|$-type determinants. The nonzero determinant products are only eight terms and describe as follows:

$$\begin{aligned}
\text{no-nucleon exchange}: &\quad (1357)(246),\\
\text{one-nucleon exchange}: &- (2357)(146), -(1367)(245), -(1457)(236),\\
\text{two-nucleon exchange}: &\quad (2457)(136), \quad (2367)(145), \quad (1467)(235),\\
\text{three-nucleon exchange}: &- (2467)(135).
\end{aligned} \tag{3}$$

With the use of the Jacobi coordinates above eight terms can be calculated in terms of the GC function as

$$\begin{aligned}
\Phi_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{x}, \mathbf{s}) &= \Phi_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{r}, \vec{\xi}_\alpha, \vec{\xi}_t, \mathbf{s}) \chi_{^4\mathrm{He}+^3\mathrm{H}}(\sigma\tau),\\
&= \left[\left(\frac{\beta}{\pi}\right)^4\right]^{\frac{3}{4}} \exp\left[-\frac{\beta}{2} \sum_{i=1,\mathrm{odd}}^{\leq 7} (\mathbf{r}_i - \mathbf{S}_1)^2\right]\\
&\quad \otimes \left[\left(\frac{\beta}{\pi}\right)^4\right]^{\frac{3}{4}} \exp\left[-\frac{\beta}{2} \sum_{i=1,\mathrm{even}}^{\leq 7} (\mathbf{r}_i - S_2)^2\right] \chi_{r\sigma\tau}.
\end{aligned} \tag{4}$$

Here $\mathbf{x} \equiv (\mathbf{r}, \sigma, \tau)$ with

$$\begin{aligned}
\chi_{r\sigma\tau} = \frac{1}{\sqrt{7!}} &\det\{\exp[-\beta s^2/14] \exp[-\beta \mathbf{s}.\mathbf{r}_1]\, \chi_{-\frac{1}{2}-\frac{1}{2}}(1),\ \chi_{\frac{1}{2}\frac{1}{2}}(3), \exp[-\beta s^2/14]\\
&\otimes \exp[-\beta \mathbf{s}.\mathbf{r}_5]\, \chi_{-\frac{1}{2}\frac{1}{2}}(5), \chi_{\frac{1}{2}-\frac{1}{2}}(7); \exp[\beta s^2/14], \exp[\beta\mathbf{s}\cdot\mathbf{r}_2]\chi_{-\frac{1}{2}\frac{1}{2}}(2),\\
&\otimes \chi_{\frac{1}{2},\frac{1}{2}}(4), \exp[\beta s^2/14] \exp[\beta\mathbf{s}\cdot\mathbf{r}_5] \otimes \chi_{-\frac{1}{2}-\frac{1}{2}}(6)\}.
\end{aligned} \tag{5}$$

The GC function (4) can easily be rewritten in the following form

$$\Phi_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{x}, \mathbf{s}) = \Phi^{\mathrm{CM}}_{^7\mathrm{Li}}(\mathbf{R}_\mathrm{G}, \mathbf{S}_\mathrm{G}) [\Phi^{\mathrm{rel}}_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{r}, \mathbf{s})\, \Phi^{(\mathrm{in})}_{^4\mathrm{He}}(\vec{\xi}_\alpha) \Phi^{(\mathrm{in})}_{^3\mathrm{H}}(\vec{\xi}_t)] \chi_{\mathbf{r}\sigma\tau}; \tag{6}$$

where

$$\Phi^{\mathrm{GM}}_{^7\mathrm{Li}}(\mathbf{R}_\mathrm{G}, \mathbf{S}_\mathrm{G}) = \left[\frac{7\beta}{\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{7\beta}{2} (\mathbf{R}_\mathrm{G} - \mathbf{S}_\mathrm{G})^2\right], \tag{7}$$

$$\Phi^{\rm rel}_{^4{\rm He}+^3{\rm H}}(\mathbf{r},\mathbf{s}) = \left[\frac{12\beta}{7\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{12\beta}{14}(\mathbf{r}-\mathbf{s})^2\right], \tag{8}$$

$$\Phi^{(\rm in)}_{^4{\rm He}}(\vec{\xi}_\alpha) = \left[\frac{1}{4}\left(\frac{\beta}{\pi}\right)^3\right]^{\frac{3}{4}} \exp\left[-\frac{\beta}{2}\left(\frac{1}{2}\xi^2_{13} + \frac{2}{3}\xi^2_{13,5} + \frac{3}{4}\xi^2_{13,5,7}\right)\right] \tag{9}$$

$$\Phi^{(\rm in)}_{^3{\rm H}}(\vec{\xi}_t) = \left[\frac{1}{3}\left(\frac{\beta}{\pi}\right)^2\right]^{\frac{3}{4}} \exp\left[-\frac{\beta}{2}\left(\frac{1}{3}\,\xi^2_{24} + \frac{2}{3}\xi^2_{24,6}\right)\right] \tag{10}$$

and the spin-isospin function of alpha (or triton) is given by

$$\chi_{^4{\rm He}({\rm or}\ ^3{\rm H})}(\sigma,\tau) = \frac{1}{\sqrt{4!({\rm or}\ 3!)}}\,{\rm det}\left[\sum_{i=1}^{4({\rm or} 3)} \chi_{m_{\sigma_i} m_{\tau_i}}(\sigma_i\tau_i)\right]. \tag{11}$$

Here all the functions are normalized to unity. The c.m. wave function $\Phi^{\rm c.m.}_{^7{\rm Li}}$ is factored out explicitly in the definition of relative wave function $u^{JM\pi}_{^4{\rm He}+^3{\rm H}}(r)$.

Taking all above consideration into account the normalized relative motion P-wave function, using the ground state of $^7{\rm Li}(J^\pi = \frac{3}{2}^-;\ L=1, S=\frac{1}{2})$ after angular momentum projection can be written as

$$u^{JM\pi}_{^4{\rm He}+^3{\rm H}}(\mathbf{r}) = \sum_{M_D} u^{JM\pi}_{L=1}(r)\left[\langle 1M_L\tfrac{1}{2}\mu|JM\rangle Y_{LM_L}(\hat{\mathbf{r}})\right], \tag{12}$$

where the radial alpha-triton relative wave function with various nucleon exchange contributions is given by

$$u^{JM\pi}_{L=1}(r) = {}^{(0)}u^{JM\pi}_{\alpha+t}(r) + {}^{(1)}u^{JM\pi}_{\alpha+t}(r) + {}^{(2)}u^{JM\pi}_{\alpha+t}(r) + {}^{(3)}u^{JM\pi}_{\alpha+t}(r); \tag{13}$$

with $i_{L=1}(z)$ are modified spherical Bessel functions [19],

$$\begin{aligned} {}^{(0)}u^{JM\pi}_{\alpha+t}(r) &= \sqrt{4\pi}\left[\frac{12\beta}{7\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{6\beta}{7}r^2\right] \int_0^\infty {\rm d}s\, s^2 f^{JM\pi}_{L=1}(s) \\ &\otimes \exp\left[-\frac{6\beta}{7}s^2\right] i_{L=1}\left(\frac{12}{7}\beta sr\right), \end{aligned} \tag{14}$$

$$\begin{aligned} {}^{(1)}u^{JM\pi}_{\alpha+t}(r) &= -3\sqrt{4\pi}\left[\frac{12\beta}{7\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{6\beta}{7}r^2\right] \int_0^\infty {\rm d}s\, s^2 f^{JM\pi}_{L=1}(s) \exp\left[-\frac{6\beta}{7}s^2\right] \\ &\otimes \exp\left[\frac{17\beta}{48}s^2\right] i_{L=1}\left(\frac{5\beta}{7}sr\right), \end{aligned} \tag{15}$$

$$\begin{aligned} {}^{(2)}u^{JM\pi}_{\alpha+t}(r) &= -3\sqrt{4\pi}\left[\frac{12\beta}{7\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{6\beta}{7}r^2\right] \int_0^\infty {\rm d}s\, s^2 f^{JM\pi}_{L=1}(s) \exp\left[-\frac{6\beta}{7}s^2\right] \\ &\otimes \exp\left[\frac{20\beta}{48}s^2\right] i_{L=1}\left(\frac{2\beta}{7}sr\right), \end{aligned} \tag{16}$$

$$\begin{aligned} {}^{(3)}u^{JM\pi}_{\alpha+t}(r) &= \sqrt{4\pi}\left[\frac{12\beta}{7\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{6\beta}{7}r^2\right] \int_0^\infty {\rm d}s\, s^2 f^{JM\pi}_{L=1}(s) \exp\left[-\frac{6\beta}{7}s^2\right] \\ &\otimes \exp\left[\frac{9\beta}{48}s^2\right] i_{L=1}\left(\frac{9\beta}{7}sr\right). \end{aligned} \tag{17}$$

The weight function $f_{L=1}^{JM\pi}(s)$ in the numerical form is obtained by considering size $\beta = 0.5\,\text{FM}^{-2}$ which has given right separation energy and all other electromagnetic properties as in ref. [8].

To estimate the coupling constant we consider the asymptotic form of the alpha-triton relative wave function (13) which can also be written in terms of a Whittaker function [1] as,

$$\lim_{r\to\infty} u_{L=1}^{JM\pi}(r) = C\frac{\sqrt{2K}}{r} W_{-\eta,L+\frac{1}{2}}(2Kr) \tag{18}$$

where K is related to the separation energy E, and η the Coulomb parameter;

$$\frac{\hbar^2K^2}{2\mu_{\alpha t}} = E; \quad \eta = \frac{Z_1Z_2e^2\mu_{\alpha t}}{\hbar^2 K}. \tag{19}$$

Here $\mu_{\alpha t}$ is the reduced mass of alpha and triton clusters and Z_1 and Z_2 are the corresponding charges. With the binding energy of triton as -8.48222 MeV, alpha as -28.2969 MeV, and ^{7}Li ground state as -39.2459 MeV with $\eta = 0.2636$ and $K = 0.4516\,\text{FM}^{-1}$ are obtained.

The dimensionless constant C known as the coupling constant can be obtained by comparing (18) and alpha-triton relative wave function (13) in asymptotic range, i.e. for larger value of r. The value of the Whittaker function is approximated by an exponential [20] as

$$W_{-\eta,L}(\rho) \simeq \frac{e^{-\rho}}{(2\rho)^{\eta}}\left[\frac{1+\sum_{m=1}[(L+\frac{1}{2})^2-(\eta+\frac{1}{2})^2]\cdots[(L+\frac{1}{2})^2-(\eta+m-\frac{1}{2})^2]}{m!(2\rho)^m}\right]. \tag{20}$$

The alpha-triton relative motion P-wave function ($L = 1$) with various nucleon exchange terms (13) are plotted in figure 1. The dotted curves are the contributions to the total relative wave function $u(r)$, obtained by separately calculating the various nucleon exchange terms (see eqs 14–17), viz, no-nucleon exchange $^{(0)}u$ one-nucleon exchange $^{(1)}u$, two-nucleon exchange $^{(2)}u$, and three-nucleon exchange $^{(3)}u$ terms. The full solid curve $u(r)$ represents the total alpha-triton relative motion wave function which becomes negative for relative separation r of the two clusters less than 2 FM. This is due to the influence of the Pauli principle and the nature of the nuclear forces. The important facts about the nuclear forces are that they are short range (~ 2 FM), strongly attractive over most of this range, but at very short-range they become strongly repulsive. On the other hand the effect of the Pauli principle in a usual system of nuclear dimensions is to allow low-energy nucleons to move relatively undisturbed throughout the nuclear volume, because these nucleons may not be scattered into other already occupied energy levels.

It is clear from the figure 1 that when the relative distance between alpha and triton is small, the Pauli principle plays an important role as shown by various nucleon exchange contributions to the total alpha-triton relative wave function $u(r)$ (full curve) which becomes negative at less than 2 FM as shown in the figure.

For large values of r, the contribution to total alpha-triton relative wave function $u(r)$ is mostly due to no-nucleon exchange (or direct) term which does not completely reproduce

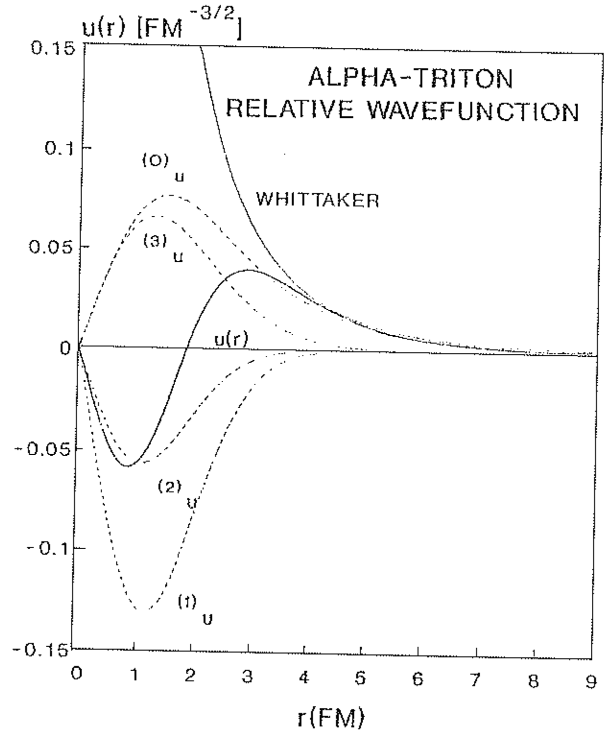

Figure 1. Alpha-triton relative motion P-wave function $u(r)[\mathrm{FM}^{-3/2}]$ ($J^\pi = \frac{3}{2}$; $L = 1$, $S = \frac{1}{2}$) (solid curve) and Whittaker function (solid curve). Various nucleon exchange contributions in $u(r)$ (broken curves), viz., no-nucleon exchange ${}^{(0)}u$, one-nucleon exchange ${}^{(1)}u$, two-nucleon exchange ${}^{(2)}u$, three-nucleon exchange ${}^{(3)}u$.

the proper asymptotic Coulomb behaviour i.e. the Whittaker function (18) (see figure 1). It is obvious that the gaussian approximated Coulomb force was used in the GCM wave function of ^{7}Li which was taken from ref. [8] without any change and used in the present calculations.

By comparison of eqs (13), and (18) for r between 5 FM to 7 FM (see figure 1), we estimate the coupling constant C as

$$C = 0.80 \pm 0.10. \tag{21}$$

3. ^{7}Li$-\alpha+t$ Vertex function and vertex constant

To test the validity of our GCM alpha-triton relative wave function obtained in the previous section, we consider the method of overlap integrals [1] for the evalution of ^{7}Li$-\alpha + t$ vertex function in terms of the alpha-triton relative wave function $u(r)$ (see eq.(1)). We also estimate the vertex constant in the virtual decay of $^7\mathrm{Li} \rightarrow \alpha + t$ by extrapolating at the pole and comparison is made with the independent experimental estimate by Dolinski *et al* [9].

The vertex function is defined as [1],

$$\Gamma(\mathbf{q}) = \left[\frac{7!}{4!3!}\right]^{\frac{1}{2}} \int \exp(i\mathbf{q}\cdot\mathbf{r})\Psi^*_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{r},\vec{\xi}_\alpha,\vec{\xi}_t)V_{\alpha t}\Phi^{(\mathrm{in})}_{^4\mathrm{He}}(\vec{\xi}_\alpha) \otimes \Phi^{(\mathrm{in})}_{^3\mathrm{H}}(\vec{\xi}_t)\mathrm{d}\vec{\xi}_\alpha\,\mathrm{d}\vec{\xi}_t\,\mathrm{d}\mathbf{r}. \quad (22)$$

If the wave function Ψ appearing in (22) is exact eigenfunction, $V_{\alpha t}$ can be eliminated in favour of the separation energy (19) of clusters alpha and triton with reduced mass $\mu_{\alpha t}$ of ^{7}Li nucleus as,

$$\Gamma(\mathbf{q}) = \frac{\hbar^2}{2\mu_{\alpha t}}(q^2+K^2)\left[\frac{7!}{4!3!}\right]^{\frac{1}{2}} \int \exp(i\mathbf{q}\cdot\mathbf{r})\Psi^*_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{r},\vec{\xi}_\alpha,\vec{\xi}_t) \otimes \Phi^{(\mathrm{in})}_{^4\mathrm{He}}(\vec{\xi}_\alpha)\Phi^{(\mathrm{in})}_{^3\mathrm{H}}(\vec{\xi}_t)\mathrm{d}\vec{\xi}_\alpha\,\mathrm{d}\vec{\xi}_t\,\mathrm{d}\mathbf{r}. \quad (23)$$

In terms of the alpha-triton relative wave function (1), the vertex function (23) can be written as

$$\Gamma(\mathbf{q}) = \frac{\hbar^2}{2\mu_{\alpha t}}(q^2+K^2)\left[\frac{7!}{4!3!}\right]^{\frac{1}{2}} \int \exp(i\mathbf{q}\cdot\mathbf{r})u_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{r})\,\mathrm{d}\mathbf{r}. \quad (24)$$

Since the alpha-triton relative wave function in asymptotic region corresponding to the bound-state momentum $\mathbf{K}$ does not have the correct Yukawa tail like in the Whittaker function (18). Therefore, for such wave function we extrapolate the $\Gamma(q^2=-K^2)$ by the method of extrapolation given by Goldfarb *et al* [17] as,

$$\Gamma(q^2) = \left[D_0 + D_1\left(\frac{q}{\mathrm{i}K}\right) + D_2\left(\frac{q}{\mathrm{i}K}\right)^2 + D_3\left(\frac{q}{\mathrm{i}K}\right)^3 + \cdots\right] \quad (25)$$

where

$$D_n = \frac{(\mathrm{i}K)^n}{n!}\left[\frac{\mathrm{d}^n\Gamma(\mathbf{q})}{\mathrm{d}q^n}\right]_{q=0} \quad (26)$$

so that

$$\Gamma(-K^2) = [D_0 + D_1 + D_2 + D_3 + D_4 + D_5 + \cdots], \quad (27)$$

as calculated in § 2, the $K^2 = 0.20039\,\mathrm{FM}^{-2}$.

Substituting the GCM alpha-triton relative wave function (12) into (24) and after angular momentum projection using the expansion of Γ ($\mathbf{q}$) for the ground state of ^{7}Li ($J^\pi = \frac{3}{2}^-$, $L=1$, $S=\frac{1}{2}$) written as

$$\Gamma^{JM\pi}_{^4\mathrm{He}+^3\mathrm{H}}(\mathbf{q}) = \sum_{M_L}\Gamma^{JM\pi}_{L=1}(q)\left[\left\langle 1M_L\frac{1}{2}\mu|JM\right\rangle Y_{1M_L}(\hat{\mathbf{q}})\right], \quad (28)$$

we get

$$\Gamma^{JM\pi}_{L=1}(q) = \sqrt{4\pi}\left[\frac{4\pi}{\mu_{\alpha t}\beta}\right]^{\frac{3}{4}}\frac{\hbar^2}{2\mu_{\alpha t}}(q^2+K^2)\exp[-bq^2]\int_0^\infty \mathrm{d}s\, s^2 f^{JM\pi}_{L=1}(s) \otimes \sum_{\nu=0}^{\nu=\nu_{\max}} {}^{(\nu)}C j_{L=1}(a_\nu qs). \quad (29)$$

Here $b = 1/2\mu_{\alpha t}\beta, \nu_{\max}$ denotes the maximum number of nucleon exchanges i.e. *three* between alpha and triton clusters (see eqs (14–17)), $j_{L=1}(z)$ are the spherical Bessel's functions. For various nucleon exchange terms (3), we have in (29)

$$^{(\nu)}C = (-1)^{(\nu)}\binom{\nu_{\max}}{\nu}\exp\left[-\nu(1-\nu/2\mu_{\alpha t})\frac{\beta}{2}s^2\right]; \tag{30}$$

and

$$a_\nu = [1-\nu/\mu_{\alpha t}], \tag{31}$$

with $\binom{\nu_{\max}}{\nu}$ binomial coefficients [19].

To obtain the vertex constant, $|\Gamma(-K^2)|^2$ value at ^{7}Li–$\alpha + t$ pole using (27), we have calculated the coefficients (26) as

$$\begin{aligned}
D_n &= 0 \quad \text{for all}\, n = \text{even},\\
D_1 &= \frac{(\mathrm{i}K)}{1!}\sqrt{4\pi}\left[\frac{4\pi}{\mu_{\alpha t}\beta}\right]^{\frac{3}{4}}\frac{\hbar^2}{2\mu_{\alpha t}}\int_0^\infty \mathrm{d}s s^2 f_{L=1}^{JM\pi}(s)\sum_{\nu=0}^{\nu=\nu_{\max}} {}^{(\nu)}Ca_\nu K^2 s/3,\\
D_3 &= \frac{(\mathrm{i}K)^3}{3!}\sqrt{4\pi}\left[\frac{4\pi}{\mu_{\alpha t}\beta}\right]^{\frac{3}{4}}\frac{\hbar^2}{2\mu_{\alpha t}}\int_0^\infty \mathrm{d}s s^2 f_{L=1}^{JM\pi}(s)\sum_{\nu=0}^{\nu=\nu_{\max}} {}^{(\nu)}C\\
&\quad \otimes\left[\alpha_\nu^3 K^2 s^3/5 + 6a_\nu s/3 - 2ba_\nu k^2 s\right],\\
D_5 &= \frac{(\mathrm{i}K)^5}{5!}\sqrt{4\pi}\left[\frac{4\pi}{\mu_{\alpha t}\beta}\right]^{\frac{3}{4}}\frac{\hbar^2}{2\mu_{\alpha t}}\int_0^\infty \mathrm{d}s s^2 f_{L=1}^{JM\pi}(s)\sum_{\nu=0}^{\nu=\nu_{\max}} {}^{(\nu)}C\left[a_\nu K^2 s^5/7\right.\\
&\quad \left. - 4a_\nu^3 s^3 - 4ba_\nu^3 K^2 s^3/5 - 40ba_\nu s + 52b^2 a_\nu K^2 s/3\right];
\end{aligned} \tag{32}$$

and, therefore, the vertex constant $\Gamma_{L=1}^{JM\pi}$ is extrapolated for negative values of q^2 up to the pole $q^2 = -K^2$ by eq. (25).

The calculated vertex function $\Gamma(q^2)$ is plotted with q^2 in figure 2. The solid curve is extrapolated up to the ^{7}Li–$\alpha + t$ pole i.e. at $q^2 = -K^2$ and the value of $|\Gamma(-K^2)|^2$ is estimated as

$$|\Gamma(-K^2)|^2 = 0.656\,\mathrm{FM}. \tag{33}$$

The only available experimentally estimated vertex constant $|\Gamma(-K^2)|^2$ for the virtual decay of $^7\mathrm{Li} \to \alpha + t$ is obtained by Dolinski *et al* [9], where the differential cross-sections for $^7\mathrm{Li}(d,t)^6\mathrm{Li}$ reaction at $E_d = 12$ MeV and $^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$ reaction at $E_\mathrm{p} = 45$ MeV are used to estimate the vertex constant $|\Gamma(-K^2)|^2$ as 0.67 FM and 0.72 FM respectively. Our calculated value (33) is 0.656 FM which is in close agreement with the experiments.

4. Summary and conclusion

We have presented an analysis of $^7\mathrm{Li} - \alpha + t$ vertex function in terms of alpha-triton relative wave function within the framework of our generator coordinate method [8, 10, 15]. The technique developed for such analysis can easily be generalizable for any complicated system.

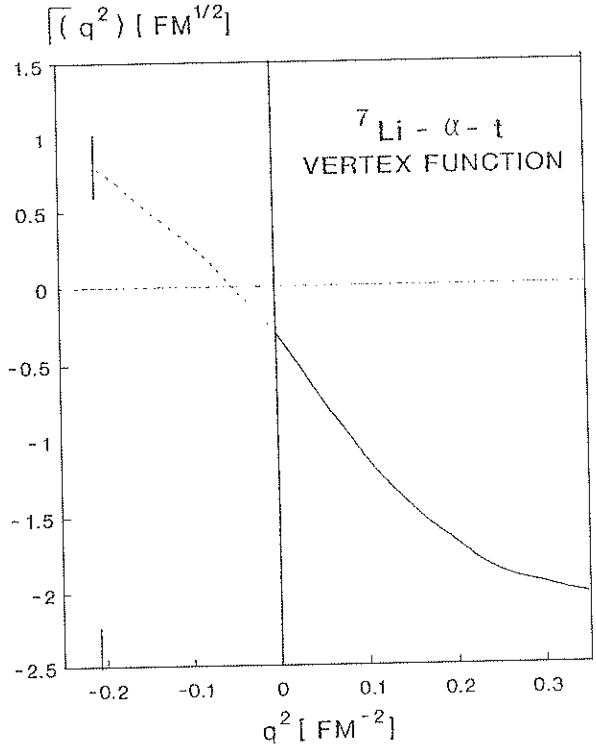

Figure 2. $^7\mathrm{Li} - \alpha + t$ vertex function for positive q^2 (solid curve) and negative q^2 (broken curve) extrapolated to the pole $q^2 = -K^2$.

The success of the model indicates that the alpha-triton relative wave functions with various nucleon exchange contributions plays an important role to understand the mechanism of reactions and nature of nuclear forces. The vertex functions describing the virtual decay of nuclei into two fragments are useful in diverse branches of nuclear physics with various sources of information e.g. the coupling constants in pole graphs, absolute normalization constants for transfer reactions, and reduced widths etc.

References

[1] M P Locher and T Mizutani, *Phys. Rep.* **C46**, 43 (1978)
[2] M P Bornand, G R Plattner, R D Viollier and K Alder, *Nucl. Phys.* **A294**, 492 (1978)
[3] G R Plattner, M P Bornand and R D Viollier, *Phys. Rev. Lett.* **39**, 127 (1977)
[4] S Dubnicka and O Dumbrajs, *Phys. Rep.* **C19**, 141 (1975)
[5] R G Lovas, A T Kruppa, R Beck and F Dickmann, *Nucl. Phys.* **A474**, 451 (1987)
[6] L D Blokhintsev, V L Kukulin, A A Sakharuk, D A Savin, E V Kuznetsova, *Phys. Rev.* **C48**, 2390 (1993)
[7] R Meier *et al*, *Phys. Rev.* **C49**, 320 (1994)
[8] V K Sharma and M A Nagarajan, *J. Phys.* G: *Nucl. Phys.* **10**, 1703 (1984)
[9] E I Dolinski, A M Mukhamedshanov and R Yarmukhamodov, *Bull. Acad. Sci. USSR; Phys. Ser.* (USA) **43**, 140 (1979)

[10] V K Sharma, *Developments of nuclear cluster dynamics* edited by Y Akaishi, K Kato, H Noto and S Okabe (Singapore, World Scientific, 1989) p. 50
[11] A Ono, H Horiuchi, P Maruyama and A Ohnishi, *Phys. Rev. Lett.* **68**, 2898; *Prog. Theor. Phys.* **87**, 1185 (1992); *Phys. Rev.* **C47**, 2652 (1993)
[12] A Ono, H Horiuchi and T Maruyama, *Phys. Rev.* **C48**, 2946 (1993)
[13] H Horiuchi and Y Kanada-En'yo, *Nucl. Phys.* **A588**, 121c (1995)
[14] D M Brink, In *Proc of Int. School of Physics,* "Enrico Fermi" Course **36**, edited by Bloch C. (New York, Academic Press, 1966) p. 247
[15] V K Sharma, Physical Interpretation of Generator Coordinate Method, in *V Int. Conf. on clustering aspects in nuclear and subuclear systems*, Kyoto, Japan, July 28–29, 1988 contributed paper, p. 320
[16] A Arima, *Clustering aspects of nuclear structure*, edited by J S Lilley and M A Nagarajan (Reidel D. Publishing Company, Holland, 1985) p. 394
[17] L J B Goldfarb, J A Gonzales and A C Phillips, *Nucl. Phys.* **A209**, 77 (1973)
[18] H Horiuchi, *Prog. Theor. Phys.* **43**, 375 (1970)
[19] M Abramowitz and I A Stegun, *Handbook of mathematical functions* (Dover, New York, 1965)
[20] D F Hebbard and B A Robson, *Nucl. Phys.* **42**, 563 (1963)