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$T = 0$ versus $T = 1$ pairing in $O(36)$ limit of IBM-4 for heavy $N = Z$ nuclei

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Abstract. In the $O(36)$ limit of the interacting boson model including spin-isospin degrees of freedom (IBM-4), starting with a group chain that preserves s and d boson spins and isospins together with a simple mixing hamiltonian, it is shown that the model generates, for heavy $N = Z$ nuclei, even-even to odd-odd staggering in the number of $T = 0$ pairs in the ground states for moderate difference in the basic $T = 0$ and $T = 1$ s-boson pair energies; the staggering disappears when the energy difference is large.

Keywords. Interacting boson model; IBM-4; isospin; $T = 0$ pairing; $T = 1$ pairing; dynamical symmetries; drip line nuclei.

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1. Introduction

One of the declared goals of the radioactive ion beam facilities that are going to become available in the near future, is to study proton–neutron (pn) pairing in nuclei near the proton drip-line in the mass range $A \sim 60$–100 [1]. An important question here is $T = 0$ versus $T = 1$ pairing in the ground states of heavy $N \sim Z$ odd-odd nuclei, with the $T = 0$ pairing arising only from pn pairs, and the change in the pairing strength from the neighbouring even-even nuclei. So far, the simple isovector $O(5)$ pairing model with protons and neutrons in a single-j shell [2], Monte Carlo shell model method [3], a cranked mean-field model with $T = 0$ and $T = 1$ pairing interactions [4] and the $U(6) \otimes U(6)$ limits of IBM-4 [5] are used to study $T = 0$ versus $T = 1$ pairing in heavy $N = Z$ nuclei. The purpose of this brief report is to present results of further study of this problem using IBM-4 symmetry limits. The spectrum generating algebra (SGA) for IBM-4, with six spin-isospin degrees of freedom for the s and d bosons is $U_{sdST}(36)$ [6]; note that $(ST) = (10) \oplus (01)$. Recently [7] all the symmetry limits of IBM-4 are classified and at the primary level of the $U(36)$ group-subgroup lattice of the model, there are four symmetry limits: (i) $U(6) \otimes U(6)$; (ii) $U(18) \oplus U(18)$; (iii) $U(6) \oplus U(30)$; (iv) $O(36)$. The structure of the $T = 1$ and $T = 0$ bands in ^{74}Rb (the nucleus ^{74}Rb is the heaviest known $N = Z$ odd-odd nucleus in $A > 60$ region that has been studied experimentally with any spectroscopic detail [8]) is described successfully for the first time using IBM in [7] and here a group chain starting from $O(36)$ is used (see (1) ahead). The same group chain is employed in this report for investigating the problem of $T = 0$ vs $T = 1$ pairing in heavy $N = Z$ nuclei.

2. $O(36) \supset O(6) \oplus O(30)$ group chain and number of $T=0$ and $T=1$ pairs

Let us begin with the $O_{sdST}(36)$ group chain considered in [7],

$$\begin{array}{l} \left| \begin{array}{ccccc} U_{sdST}(36) \supset & O_{sdST}(36) \supset & [O_{sST}(6) \supset & O_{S_s}(3) \oplus & O_{T_s}(3)] \oplus \\ \{N\} & [\omega] & [\omega_s] & S_s & T_s \end{array} \right. \\ \begin{array}{cccccc} [O_{dST}(30) \supset & \{O_d(5) \supset O_L(3)\} \otimes & \{O_{S_dT_d}(6) \supset & O_{S_d}(3) \oplus & O_{T_d}(3)\}] \\ [\omega_d] & [\omega_1\omega_2] \quad L & [\sigma_1\sigma_2\sigma_3] & S_d & T_d \end{array} \\ \left. \begin{array}{ccccc} \supset O_L(3) \otimes & [O_S(3) \oplus & O_T(3)] \supset & O_J(3) & \otimes O_T(3) \\ L & S & T & \vec{J} = \vec{L} + \vec{S} & T \end{array} \right\rangle . \end{array} \tag{1}$$

The generators of all the groups in (1) are given in [7]. As all the N boson states (note that the boson number N is even for $N=Z$ even-even nuclei and N is odd for $N=Z$ odd-odd nuclei; the 'N' used for denoting boson number should not be confused with the 'N' used for neutron number) correspond to the totally symmetric irrep $\{N\}$ of $U_{sdST}(36)$, the $O_{sdST}(36)$ irreps are labelled by the seniority quantum number ω in the total $sdST$ space ($\omega = N, N-2, N-4, \ldots 0$ or 1). The $O_{sST}(6)$ and $O_{dST}(30)$ quantum numbers (ω_s, ω_d) are given by the rule [9] $\omega = 2r_{sd} + \omega_s + \omega_d$ where $r_{sd} = 0, 1, 2, \ldots$. All other quantum numbers in (1) will be specified as and when they are needed. Following the applications of good (T_s, T_d) symmetry limits of IBM-3 [10], it is assumed that for both even-even and odd-odd nuclei $\omega = N$ and $\omega_d = 0$ for the ground state (GS) and the later assumption guarantees that $L=0$ for the GS. In this situation except for the $O_{S_s}(3)$ irreps S_s and $O_{T_s}(3)$ irreps T_s, irrep labels for the remaining groups in (1) need not be specified. Then in the symmetry limit (1),

$$|\mathrm{GS}\rangle = |N;\ \omega = N, (\omega_s, \omega_d = 0), S = S_s, T = T_s, L = 0\rangle. \tag{2}$$

In order to calculate number of $T=0$ pairs and number of $T=1$ pairs in the GS, the state (2) is transformed into a basis with good s and d boson numbers. To this end results of [9] are used where a compact formula for transformation brackets (C's in (3) ahead) between $U(\mathcal{N}) \supset O(\mathcal{N}) \supset O(\mathcal{N}_a) \oplus O(\mathcal{N}_b)$ and $U(\mathcal{N}) \supset U(\mathcal{N}_a) \oplus U(\mathcal{N}_b) \supset O(\mathcal{N}_a) \oplus O(\mathcal{N}_b)$, with $\mathcal{N} = \mathcal{N}_a + \mathcal{N}_b$, for symmetric $U(\mathcal{N})$ irreps $\{n\}$ and for any $\mathcal{N}_a$ and $\mathcal{N}_b$ is derived. Denoting the basis states for these two chains by $|n\omega(\omega_a\omega_b)\alpha\rangle$ and $|n(n_a n_b)(\omega_a\omega_b)\alpha\rangle$ respectively,

$$|n\omega(\omega_a\omega_b)\alpha\rangle = \sum_{n_a} C^{n,\omega,\omega_a,\omega_b}_{n_a,n_b}(\mathcal{N}_a, \mathcal{N}_b)|n(n_a n_b)(\omega_a\omega_b)\alpha\rangle;\ n = n_a + n_b. \tag{3}$$

The results in [9] give for example the formulas $\langle n_a\rangle^{n,n,(\omega_a\omega_b)} = \omega_a + (n - \omega_a + \omega_b + \mathcal{N}_b - 2)(n - \omega_a - \omega_b)/(2n + \mathcal{N} - 4)$ and $\langle n_a\rangle^{n,\omega,(\omega_a,\omega-\omega_a)} = \omega_a + (n-\omega)(2\omega_a + \mathcal{N}_a)/(2\omega + \mathcal{N})$. Using (3), the states (2) are transformed into states with number of $T=0$ and $T=1$ pairs being good quantum numbers,

$$\begin{aligned} &|N; \omega = N, (\omega_s, \omega_d = 0),\ S = S_s,\ T = T_s,\ L = 0\rangle \\ &= \sum C^{N,\omega=N,\omega_s,\omega_d=0}_{n_s,n_d}(6,30) C^{n_s,\omega_s,S_s,T_s}_{n_{s;S},n_{s;T}}(3,3) C^{n_d,\omega_d=0,\omega_{dS}=0,\omega_{dT}=0}_{n_{d;S},n_{d;T}}(15,15) \\ &\times |N; (n_{s;S} n_{s;T})(S_s T_s); (n_{d;S} n_{d;T})(S_d = 0, T_d = 0); \\ &\quad S = S_s, T = T_s, L = 0\rangle; n_s + n_d = N, n_{s;S} + n_{s;T} = n_s, n_{d;S} + n_{d;T} = n_d. \end{aligned} \tag{4}$$

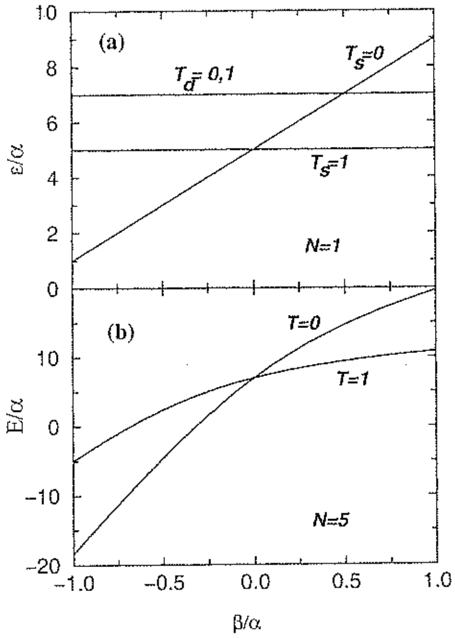

Figure 1. (a) Single s and d boson energies and (b) energies of lowest $T = 0$ $(S = 1)$ and $T = 1$ $(S = 0)$ states for five boson $(N = 5)$ system as a function of β/α; the hamiltonian is defined by (5).

where ω_{dS} and ω_{dT} are the quantum numbers of the groups $O_{dS}(15)$ and $O_{dT}(15)$ respectively and they take trivial values $\omega_{dS} = 0, \omega_{dT} = 0$ as $\omega_d = 0$ for the states (2). Note that in deriving (4) we used $O_{dST}(30) \supset O_{dS}(15) \oplus O_{dT}(15)$ (but not $O_{dST}(30) \supset O_d(5) \otimes O_{S_dT_d}(6)$ as chosen in (1)) as this is more convenient and because the final results do not depend on this choice for $\omega_d = 0$ as in (2). For the basis states on the r.h.s of (4), the number of $T = 0$ pairs is $N_{T=0} = n_S = n_{s;S} + n_{d;S}$ and similarly number of $T = 1$ pairs is $N_{T=1} = n_T = n_{s;T} + n_{d;T}$; $N = N_{T=0} + N_{T=1}$ and the fraction of $T = 0$ pairs is $f(T = 0) = N_{T=0}/N$. Using (4), it is straightforward to calculate $f(T = 0)$ in the states defined by (2) or mixures of them. For the GS of odd-odd $N = Z$ nuclei, the boson number N is odd and $\omega_s = 1$ in (2) giving $(ST) = (10)$ or (01). In the symmetry limit ignoring the $S(S+1)$ and $T(T+1)$ contributions to the energies, the $T = 0$ and $T = 1$ GS energies are degenerate and using the formulas given below (3), it is seen easily that $f(T = 0) = (9N^2 + 162N + 101)/16(N+16)$ for $T = 0$ GS and $f(T = 0) = (7N^2 + 94N - 101)/16(N+16)$ for $T = 1$ GS. For example for $N = 5$, $f(T = 0) = 0.676$ for $T = 0$ GS and $f(T = 0) = 0.324$ for $T = 1$ GS. For the GS of even-even $N = Z$ nuclei $f(T = 0) = 0.5$ as the boson number N is even, $\omega_s = 0$, $S = S_s = 0$ and $T = T_s = 0$. Therefore, in the $O_{sdST}(36)$ symmetry limit (1,2) there is even-even to odd-odd staggering in $f(T = 0)$.

3. Results of mixing calculations

In reality the $T = 0$ and $T = 1$ states for odd-odd nuclei will not be degenerate and a simple hamiltonian (appropriate for the basis defined by (2)) that generates a

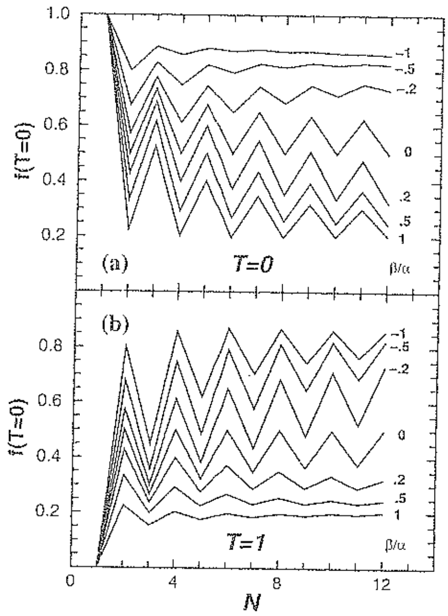

Figure 2. Fractional number of $T = 0$ pairs $f(T = 0)$ as a function of the boson number N for various values of β/α: (a) for $T = 0$ GS (i.e. lowest $S = 1, T = 0$ state) of $N = Z$ odd-odd nuclei (N odd) and $T = 0$ GS (i.e lowest $S = 0, T = 0$ state) of $N = Z$ even-even nuclei (N even); (b) for $T = 1$ GS (i.e. lowest $S = 0, T = 1$ state) of $N = Z$ odd-odd nuclei (N odd) and $T = 0$ GS (i.e lowest $S = 0, T = 0$ state) of $N = Z$ even-even nuclei (N even). See text for further details.

splitting is,

$$H = \alpha C_2(O_{sST}(6)) + \beta C_2(SU_{sS}(3)) + \gamma\left[\frac{1}{31} C_2(SU_{dST}(30))\right]. \tag{5}$$

In (5), C_2's are quadratic Casimir operators and $SU(N)$ instead of $U(N)$ and the factor $1/31$ are used for convenience. The term with α is diagonal in the basis (2) with eigenvalues given by $\alpha\,\omega_s(\omega_s + 4)$. The other two terms are diagonal in the basis defined by the states on the r.h.s of (4) with eigenvalues $\beta n_{s:S}(n_{s:S} + 3)$ and $(\gamma/31)n_d(n_d + 30)$. In the following discussion it is assumed that α is positive. Single boson energies (ϵ's) defined by (5) are, $\epsilon(T_s = 0)/\alpha = 5 + 4(\beta/\alpha)$, $\epsilon(T_s = 1)/\alpha = 5$ and $\epsilon(T_d = 0)/\alpha = \epsilon\,(T_d = 1)/\alpha = 5 + \gamma/\alpha$; see figure 1a. Thus for one boson system, assuming $\gamma/\alpha \geq 0$ and $\alpha > 0$, $T_s = 1$ is GS for $\beta/\alpha > 0$ and $T_s = 0$ is GS for $\beta/\alpha < 0$.

For $N > 1$, the hamiltonian (5) mixes the basis states (2) (i.e. ω_s is mixed). For a given N, the matrix for H is constructed for various values of β/α using (3) and after diagonalizing $f(T = 0)$ is calculated for: (i) lowest $S = 1$, $T = 0$ state (i.e $T = 0$ GS) for N odd (odd-odd nuclei); (ii) lowest $S = 0$, $T = 1$ state (i.e $T = 1$ GS) for N odd (odd-odd nuclei); (iii) lowest $S = 0$, $T = 0$ state (i.e $T = 0$ GS) for N even (even-even nuclei). Numerical calculations showed that the energies $\epsilon(T_d)$ of d-boson states do not significantly alter the behaviour of $f(T = 0)$ and therefore in all the calculations $\gamma/\alpha = 2$ is chosen; this part gives a contribution of about $0.08 - 0.1$ for $f(T = 0)$. Varying β/α (which is a measure of the competition between $T = 0$ and $T = 1$ pairs), it is seen that the GS energy of N boson system is in direct correlation with the single s-boson energies $\epsilon(T_s)$; figure 1b shows this for $N = 5$ case. The results for $f(T = 0)$ as a function of the boson number for various values of β/α are shown in figure 2; results for the cases (i) and (iii) are shown in figure 1a and for the cases (ii) and (iii) in figure 1b. The most significant result that follows from figure 2 is that there is even-even to odd-odd staggering in $f(T = 0)$ in $N = Z$ nuclei for moderate difference in $T = 0$ and $T = 1$ s-boson pair energies (i.e. for $|\beta/\alpha| \lesssim 0.5$) and the staggering disappears when this energy difference is large. Comparing figure 1 with figure 2 it is seen that, in the situation that $T = 0$ and $T = 1$ s-boson pairs compete (i.e. their energies are close) there is staggering and absence of staggering implies dominance of one of them. This result of IBM-4 is consistent with the results obtained from the shell model [2, 3]. More importantly, using the $U(6) \otimes U(6)$ chains of IBM-4 and a hamiltonian similar to (5) without the γ-term but in total six dimensional ST space, $f(T = 0)$ is studied in [5] and this scheme also produces the staggering effect. Thus the feature of even-even to odd-odd staggering in $f(T = 0)$ in the GS of heavy $N = Z$ nuclei is a robust prediction of IBM-4. It should be remarked that the physical sub-spaces chosen in the present work and in ref. [5] are quite different but in both cases the C-coefficients of (3) enter.

4. Conclusion

Results reported in this paper for even-even to odd-odd staggering in the number of $T = 0$ pairs in the ground states of heavy $N = Z$ nuclei together with the description of the observed ground $T = 1$ and excited $T = 0$ bands in ^{74}Rb [7] by the $O(36)$ group chain (1) establish that the $O(36)$ dynamical symmetry limits of IBM-4 are relevant for near proton drip line nuclei.

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