

Energy basis via decoherence

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Abstract. The question of the emergence of a preferred basis which is generally understood as that basis in which the reduced density matrix is driven to a diagonal (classically interpretable) form via environment induced decoherence is addressed. The exact solutions of the Caldeira–Leggett Master Equation are analyzed for a free particle and a harmonic oscillator system. In both cases, we see that the reduced density matrix is driven diagonal in the energy basis, which is momentum for the free particle and the number states for the harmonic oscillator. This seems to single out the energy basis as the preferred basis which is contrary to the general notion that it is the position basis which is selected since the coupling to the environment is via the position coordinates.

Keywords. quantum-classical transition; decoherence; preferred basis.

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1. Introduction

Quantum Brownian motion models [1–3] are fundamental to the study of quantum dissipative systems in many areas of physics. More recently, the role of interaction between a quantum system and its environment has been exploited to explain the emergence of classical behaviour from underlying quantum dynamics, especially in the context of quantum measurements [3–6]. Quantum coherence is shown to be dynamically suppressed by decoherence. Decoherence is now widely accepted as the mechanism that prevents the existence of systems in superpositions of macroscopically distinct states, thus explaining the emergence of classicality. More specifically, the decoherence mechanism takes a pure state density matrix to a statistical mixture thus bringing about the quantum-classical transition through this reduction. Advances in technology have increased the prospects of experimentally producing superpositions of quantum states in macroscopic devices [8]. It is well known that all such systems are inevitably weakly coupled to many degrees of freedom which constitute their environments and hence it is natural to expect decoherence in such systems. Recently there have been several proposals to exploit purely quantum mechanical features like the linear superposition principle ('quantum parallelism') to build high speed quantum computers [9]. Quantum cryptography [10] and quantum teleportation [11]

schemes also rely on these unique quantum features. For these schemes to work successfully, the quantum nature of the superpositions (quantum coherence) must be maintained all the time. Since environmental influence is often unavoidable, the accompanying phenomenon of decoherence can ruin the functioning of such systems. It becomes necessary, therefore, to have a clearer understanding of the behaviour of quantum coherences in such dissipative environments.

The concept of a preferred basis has been interpreted and understood in many different ways in the literature. If the interaction between the system and its environment is described by a density matrix, decoherence causes a suppression of the off-diagonal elements of the reduced density matrix of the system making all information on the system classically interpretable. Our experience of the classical world suggests that unlike quantum systems, which are allowed to exist in all possible states, classical systems exist in only a few select states which are singled out by environmental influence from the larger quantum menu. However, in the models studied so far, the question of the basis in which the density matrix becomes diagonal or the basis 'preferred' by the environment [5] is not completely understood. For simplified models where the self Hamiltonian of the system has either been ignored or considered co-diagonal with the interaction Hamiltonian, the 'pointer' variable or the preferred basis has been shown to be the one that commutes with the interaction Hamiltonian [5]. The final reduced density matrix of the system of interest then ends up becoming diagonal in this same basis. Since in most of the models studied the coupling between the system of interest and the oscillators constituting the environment is through the coordinate basis, it is often expected that the preferred basis will always be position. We address this question here and show that in terms of the usual definition of the preferred basis as the one that eventually diagonalizes the reduced density matrix for the system of interest, it is the *energy basis* for both the free particle and the harmonic oscillator.

2. Preferred basis

The concept of a preferred basis is crucial in the environment induced decoherence approach to the problem of the emergence of classicality from quantum dynamics and that of quantum measurement. Consider any measurement scheme that involves the system-apparatus interaction between a two-state system and a two-state detector, which leaves the combined setup in the following state:

$$\psi(S + D) = a|S\uparrow\rangle|D\uparrow\rangle + |S\downarrow\rangle|D\downarrow\rangle. \quad (1)$$

There is a one-to-one correlation between the states of the system and that of the detector and the above expression seems to imply that the observable, S , of the system that is being measured corresponds to the basis states: $|S\uparrow\rangle$ and $|S\downarrow\rangle$. Quantum mechanics allows us to re-express the apparatus states in any other basis and one can write the detector states also as:

$$\begin{aligned} |D^+\rangle &= \frac{|D\uparrow\rangle + |D\downarrow\rangle}{\sqrt{2}}, \\ |D^-\rangle &= \frac{|D\uparrow\rangle - |D\downarrow\rangle}{\sqrt{2}} \end{aligned} \quad (2)$$

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The state $\psi(S + D)$ now looks like:

$$\psi(S + D) = \frac{1}{\sqrt{2}} \left([a|S\uparrow\rangle + b|S\downarrow\rangle] |D^+\rangle + [a|S\uparrow\rangle - b|S\downarrow\rangle] |D^-\rangle \right) \quad (3)$$

The system observable S' correlated with the apparatus states here now corresponds to the basis states: $a|S\uparrow\rangle + b|S\downarrow\rangle$, and $a|S\uparrow\rangle - b|S\downarrow\rangle$! This ambiguity of basis seems to leave us the choice of deciding the system variable that is measured long after the system-apparatus interaction has ceased. This, of course, clashes with our classical intuition and understanding of the measurement process where the apparatus is left in a definite state and the choice of the basis is not left to us long after the system-apparatus interaction. Measurement apparatus obviously exist in a definite state ('pointer position'), superpositions of which are not allowed. It becomes necessary, therefore, to ask what process determines this pointer basis which corresponds to such classical observables as the pointer position. The environment induced decoherence approach to the problem of quantum measurement was first demonstrated by Zurek [5] via a simplified model where the self Hamiltonian of the system was ignored. He considered a spin-1/2 system interacting with a spin-1/2 apparatus which in turn couples to an environment of many spin-1/2 systems. In the measurement scheme considered by Zurek [5], the 'pointer' variable or the preferred basis in which the reduced density matrix diagonalized and system-apparatus correlations were established was then shown to be the one that commutes with the interaction Hamiltonian [5]. There was, thus, no ambiguity in the nature of the basis that the environment selects out and in which the density matrix appears classical. It obviously corresponded to the basis of the observable that commuted with the interaction Hamiltonian. However, in more general and realistic situations (e.g. when we are dealing with continuous variables like in the Caldeira-Leggett model) where all terms are included in the total Hamiltonian and its various parts may not commute, it is not obvious what decides the emergent preferred basis. Even in such situations, Zurek *et al* [5] have emphasised the role of the observable that commutes with the interaction Hamiltonian in deciding the preferred basis.

In the literature, the preferred basis has been described in many ways: as the one in which the final state density matrix becomes diagonal (and hence classically interpretable); the basis in which macroscopic superpositions disappear (explaining the absence of 'Schrödinger's Cat' in the classical world). Savage and Walls [6] have shown that decoherence causes the macroscopic superpositions of coherent states of an oscillator to vanish, reducing the density matrix of the system to a statistical mixture. It is not obvious that this makes the coherent states the preferred basis for a harmonic oscillator since there is no notion of diagonalization in any particular basis.

Preferred states have also been termed as the set of basis states which are characterized by maximum stability or a minimum increase in linear or statistical entropy [12]:

$$S = \text{Tr}[\rho - \rho^2] \quad (4)$$

Zurek et al derived an approximate expression for the 'predictability sieve' which is a measure of this quantity for a harmonic oscillator system undergoing Quantum Brownian Motion in the limit of weak coupling and high temperatures [12]. Their starting point is the high temperature Caldeira-Leggett Master equation,

$$\frac{\partial \rho(x, x', t)}{\partial t} = \frac{\hbar}{2im} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x'^2} \right) \rho(x, x', t) + i \frac{m\omega^2}{2\hbar} (x^2 - x'^2) \rho(x, x', t)$$

$$-\gamma(x-x')\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)\rho(x,x',t) - \frac{D}{4\hbar^2}(x-x')^2\rho(x,x',t). \quad (5)$$

where x and x' denote the initial and final positions respectively. In the high temperature limit, $KT \gg \hbar\omega$, $D = 8m\gamma KT$. Zurek *et al* assume that if Planck's constant is small compared to the actions involved and if the object in question is massive, the last term, $\frac{2m\gamma KT}{\hbar^2}(x-x')^2\rho(x,x',t)$ dominates. They then show that if the linear entropy is given by (4), in the limit of weak coupling and assuming that the state remains approximately pure,

$$\frac{dS}{dt} \sim 4D\Delta x^2. \quad (6)$$

They then integrate (6) in the weak coupling limit over an oscillator period, replacing the free Heisenberg equations for the oscillator operators. They show that:

$$S(\tau) = 2D \left[\Delta x^2 + \frac{\Delta p^2}{m^2\omega^2} \right]. \quad (7)$$

Here Δx and Δp are the *initial* dispersions in x and p . The above expression is minimum if $\Delta x\Delta p = \hbar/2$ and, $\Delta x^2 = \hbar/2m\omega$. This is just the spread in position of the ground state or of a coherent state of a oscillator. On this basis Zurek *et al* claim that coherent states are the preferred basis for a harmonic oscillator. However, as is obvious, this result is approximate. It is not clear that such a criteria firmly establishes the role of coherent states as the preferred basis for a harmonic oscillator.

In a measurement like scenario, the preferred states have also been understood as those states of the apparatus with which permanent and stable correlations are established by the desired system variables. Tegmark and Shapiro [13] have shown that harmonic heat-bath models lead to the production of generalized coherent states, suggesting that this is a consequence of the decoherence mechanism. It is not obvious from the examples studied in the literature whether the same set of states satisfy all these criteria or whether for a given model a unique preferred basis exists.

3. General solutions to the Caldeira–Leggett master equation

We have recently shown that the general solution to the Caldeira–Leggett Master equation can be obtained in a nice factorizable form for the free particle and the harmonic oscillator systems [14]. These solutions are valid for arbitrary initial conditions. The solution to the Master equation for the case of a free particle for arbitrary initial conditions in the partial Fourier transform representation is [14]:

$$\begin{aligned} \bar{\rho}(Q, r, t) = & \bar{\rho}(Q, r', 0) \exp\left(-\frac{DQ^2t}{16\gamma^2m^2}\right) \\ & \exp\left(-\frac{D}{16\hbar^2\gamma}\left[\left(r - \frac{\hbar Q}{2\gamma m}\right)^2(1 - e^{-4\gamma t})\right.\right. \\ & \left.\left.+ \frac{2\hbar Q}{\gamma m}\left(r - \frac{\hbar Q}{2\gamma m}\right)(1 - e^{-2\gamma t})\right]\right) \end{aligned} \quad (8)$$

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where

$$r' = \left(r - \frac{\hbar Q}{2\gamma m} \right) e^{-2\gamma t} + \frac{\hbar Q}{2\gamma m}. \quad (9)$$

The solution is in the changed coordinates:

$$\begin{aligned} r &= x - x' \quad ; \quad R = \frac{x + x'}{2}, \\ Q &= u - v \quad ; \quad q = \frac{u + v}{2}, \end{aligned} \quad (10)$$

where u and v are conjugate to x and x' . If one looks at the solution for large times ($\gamma t \gg 1$), then it is clearly driven to a diagonal distribution in momentum space irrespective of the initial condition. This diagonalization in momentum space occurs over a time scale:

$$\tau_m = \left(\frac{DQ^2}{16\gamma^2 m^2} \right)^{-1}, \quad (11)$$

This diagonalization in momentum space is independent of the form of the initial condition. The reduced density matrix for the system of interest has, thus, ended up diagonal in the *energy* eigenstates of the system Hamiltonian. If the preferred basis is indeed interpreted as the one in which the final density matrix is diagonal, then it is obviously the momentum (energy) eigenstates for the free particle.

The solution for the harmonic oscillator for arbitrary initial conditions in the partial Fourier transform representation is [14]:

$$\tilde{\rho}(Q, r, t) = \tilde{\rho}(Q', r', 0) \exp(\alpha Z), \quad (12)$$

where

$$\alpha = \frac{D}{16m^2(\gamma^2 - \omega^2)}, \quad (13)$$

$$\begin{aligned} Z &= \frac{1}{\gamma} \left(Q - \frac{r}{\lambda_+} \right) \left(Q - \frac{r}{\lambda_-} \right) (1 - e^{-2\gamma t}) \\ &\quad - \frac{m\lambda_+}{2\hbar} \left(Q - \frac{r}{\lambda_+} \right)^2 \left(1 - e^{-\frac{2\hbar t}{m\lambda_+}} \right) \\ &\quad - \frac{m\lambda_-}{2\hbar} \left(Q - \frac{r}{\lambda_-} \right)^2 \left(1 - e^{-\frac{2\hbar t}{m\lambda_-}} \right), \end{aligned} \quad (14)$$

$$\lambda_{\pm} = \frac{\hbar}{m\omega^2} \left(\gamma \pm \sqrt{\gamma^2 - \omega^2} \right), \quad (15)$$

$$C_{\pm} = \left(Q - \frac{r}{\lambda_{\pm}} \right) e^{-\frac{\hbar t}{m\lambda_{\pm}}}, \quad (16)$$

$$Q' = \frac{C_+ \lambda_+ - C_- \lambda_-}{\lambda_+ - \lambda_-}, \quad (17)$$

$$r' = \lambda_+ \lambda_- \frac{C_+ - C_-}{\lambda_+ - \lambda_-}. \quad (18)$$

If we look at the general solution (12) at large time scales such that $\gamma t \gg 1$ and $\gamma > \omega$, (the overdamped regime) then it is clear that at $t \rightarrow \infty$, the solution reduces to:

$$\tilde{\rho}(Q, r, t) = \tilde{\rho}(0, 0, 0) \exp\left(\frac{-D}{16m^2\omega^2\gamma} \left[Q^2 + \frac{m^2\omega^2 r^2}{\hbar^2}\right]\right). \quad (19)$$

The density matrix is obviously not diagonal either in position or momentum. Consider the final solution, (19) in the position representation:

$$\rho(x, x', t) = A \exp(-c_1 x^2 - c_1 x'^2 - 2c_2 x x') \quad (20)$$

where,

$$A = \sqrt{\frac{\gamma}{2D}} \tilde{\rho}(0, 0, 0) 4m\omega, \quad (21)$$

$$c_1 = \frac{D}{16\hbar^2\gamma} - \frac{m^2\omega^2\gamma}{D}, \quad (22)$$

$$c_2 = \frac{2m^2\omega^2\gamma}{D} - \frac{D}{16\hbar^2\gamma}. \quad (23)$$

The set of basis states in which $\rho(x, x', t)$ is diagonal should satisfy the following eigenvalue equation:

$$\int_{-\infty}^{\infty} \rho(x, x') \psi(x') dx' = \lambda_n \psi_n(x') \quad (24)$$

It can be seen that the energy eigenstates of the harmonic oscillator:

$$\psi_n(x) = N \exp\left(\frac{-m\omega x^2}{2\hbar}\right) H_n\left(\sqrt{m\omega/\hbar} x\right), \quad (25)$$

where N is the normalization constant and $H_n(\sqrt{m\omega/\hbar} x)$ is the n th order Hermite polynomial, are the solutions to (24) with the eigenvalues

$$\lambda_n = \lambda_0 \left(\frac{c_1 - m\omega/2\hbar}{c_1 + m\omega/2\hbar}\right)^{n/2} \quad (26)$$

where,

$$\lambda_0 = \sqrt{\frac{\gamma}{2D}} \tilde{\rho}(0, 0, 0) 4m\omega \sqrt{\frac{\pi}{c_1 + m\omega/2\hbar}}. \quad (27)$$

Thus, the final density matrix is diagonal in the energy eigenstates of the oscillator.

4. Conclusions

Our analysis of the general solutions to the Caldeira–Leggett Master equation for the free particle and for the harmonic oscillator show that the solutions at large times diagonalize completely in the energy eigenstates. This is the momentum basis for the free particle and

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the number state basis for the harmonic oscillator. By the naive definition of the preferred basis as being the one in which the density matrix diagonalizes, it is the energy eigenstates that are obviously special for these systems. This is contrary to the general expectation that the position basis should be preferred [5] since the system-environment coupling here is through the position. The emergence of coherent states as special states for the oscillator too is not obvious. We examine this question further in a forthcoming paper [14].

References

- [1] H Dekker, *Phys. Rep.* **80**, 1 (1981); *Phys. Rev.* **A16**, 2116 (1977)
- [2] G S Agarwal, *Phys. Rev.* **A4**, 739 (1971); *Phys. Rev.* **A178**, 2025 (1969)
- [3] A O Caldeira and A J Leggett, *Physics* **A121**, 587 (1983); *Phys. Rev.* **A31**, 1059 (1985) **37**, 4419 (1988)
- [4] H D Zeh, *Found. Phys.* **1**, 69 (1970)
- [5] W H Zurek, *Phys. Rev.* **D24**, 1516 (1981); In *Prog. Theor. Phys.* **89**, 281 (1993); *Physics Today* **44** (10), 36 (1991); In *Quantum optics, experimental gravitation, and measurement theory*, edited by P Meystre and M O Scully (Plenum, New York, 1986)
- [6] C M Savage and D F Walls, *Phys. Rev.* **A32**, 2316 (1985)
- [7] A Venugopalan, *Phys. Rev.* **A50**, 2742 (1994); A Venugopalan, D Kumar and R Ghosh, *Physica* **A220**, 563 (1995)
- [8] A J Leggett, *Prog. Theor. Phys.* Supplement No. **69**, 80 (1980)
- [9] A Ekert, in *Proc. ICAP '94*, edited by S Smith, C Wieman and D Wineland (to be published)
- [10] P W Shor, in *Proceedings of the 35th Annual symposium on the foundations of computers science, Los Alamitos, CA* (IEEE Computer Society Press, New York, 1994) p 124
- [11] C H Bennett, G Brassard, C Crépeau, R Jozsa, A Peres, W K Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993)
- [12] W H Zurek, S Habib, J P Paz, *Phys. Rev. Lett.* **70**, 1187 (1993)
- [13] M Tegmark and H S Shapiro, *Phys. Rev.* **E50**, 2538 (1994)
- [14] S M Roy and A Venugopalan (in preparation)

