

Symmetries of the standard model without and with a right-handed neutrino

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Abstract. Given the particle content of the standard model without and with a right-handed neutrino, the requirement that all anomalies cancel singles out a set of possible global symmetries which can be gauged. I review this topic and propose a new gauge symmetry $B - 3L_\tau$ in the context of the minimal standard model consisting of the usual three families of quarks and leptons plus just one ν_R . The many interesting phenomenological consequences of this hypothesis are briefly discussed.

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1. Cancellation of anomalies

In the minimal standard model, under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, the quarks and leptons transform as

$$\begin{bmatrix} u \\ d \end{bmatrix}_L \sim (3, 2, 1/6), \quad u_R \sim (3, 1, 2/3), \quad d_R \sim (3, 1, -1/3); \quad (1)$$

$$\begin{bmatrix} \nu \\ l \end{bmatrix}_L \sim (1, 2, -1/2), \quad l_R \sim (1, 1, -1). \quad (2)$$

The sum of the $U(1)$ axial charges is

$$6(1/6) + 2(-1/2) - 3(2/3) - 3(-1/3) - (-1) = 0. \quad (3)$$

This means that the mixed gauge-gravitational anomaly [1] is absent. The number of $SU(2)$ fermion doublets is even. Hence the $SU(2)$ global anomaly [2] is also absent.

Because left-handed and right-handed fermions transform differently, there are potentially several axial-vector triangle anomalies [3] in the standard model. However, they are all canceled as follows:

$$[SU(3)]^2 U(1) : \quad 2(1/6) - (2/3) - (-1/3) = 0, \quad (4)$$

$$[SU(2)]^2 U(1) : \quad 3(1/6) + (-1/2) = 0, \quad (5)$$

$$[U(1)]^3 : \quad 6(1/6)^3 + 2(-1/2)^3 - 3(2/3)^3 - 3(-1/3)^3 - (-1)^3 = 0. \quad (6)$$

2. Global and gaugeable symmetries

There are four global symmetries in the minimal standard model, corresponding to baryon number B and the three lepton numbers L_e , L_μ , and L_τ . Each current [i.e. $J_B^\alpha = \sum_i (\bar{q}_{iL}\gamma^\alpha q_{iL} + \bar{q}_{iR}\gamma^\alpha q_{iR})$, etc.] is conserved at the classical (tree) level. However, at the quantum level [i.e. when loops are included], each current by itself is not conserved. In fact, it was shown long ago [4] that

$$\partial_\alpha J_{B+L}^\alpha = \frac{g^2}{16\pi^2} N_f W_i^{\mu\nu} \tilde{W}_{i\mu\nu}, \quad (7)$$

where $W_i^{\mu\nu}$ is the $SU(2)_L$ field tensor and \tilde{W} its dual, g is the corresponding gauge coupling, and N_f is the number of families of quarks and leptons. Nonperturbative pseudo-particle contributions to the above result in the selection rule

$$\Delta(B+L) = 2N_f\nu, \quad (8)$$

where

$$\nu = \frac{g^2}{32\pi^2} \int d^4x W_i^{\mu\nu} \tilde{W}_{i\mu\nu} = \pm 1, \pm 2, \dots \quad (9)$$

Note that

$$\partial_\alpha J_{B-L}^\alpha \propto 3(3)(1/3) - 1 - 1 - 1 = 0; \quad (10)$$

hence the electroweak phase transition conserves $B-L$. However, the chiral $B-L$ anomaly is nonzero:

$$[U(1)_{B-L}]^3 : 6(1/3)^3 + 2(-1)^3 - 6(1/3)^3 - (-1)^3 \neq 0; \quad (11)$$

Hence $B-L$ itself cannot be gauged without the addition of one ν_R per family.

It is already well-known that if there is one ν_R per family, then the standard model may be extended to become a left-right symmetric model with

$$Q = T_{3L} + Y = T_{3L} + T_{3R} + \frac{1}{2}(B-L). \quad (12)$$

In other words, $SU(2)_L \times U(1)_Y$ naturally becomes $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

Without ν_R , there are still three possible symmetries which may be gauged [5]: $L_e - L_\mu$, $L_e - L_\tau$, or $L_\mu - L_\tau$. The $L_i - L_j$ anomalies cancel because

$$[SU(2)]^2(L_i - L_j) : 1 - 1 = 0, \quad (13)$$

$$[U(1)_Y]^2(L_i - L_j) : [2(-1/2)^2 - (-1)^2](1 - 1) = 0, \quad (14)$$

$$U(1)_Y(L_i - L_j)^2 : 2(-1/2)(1)^2 - (-1)(1)^2 + 2(-1/2)(-1)^2 - (-1)(-1)^2 = 0, \quad (15)$$

$$(L_i - L_j)^3 : 2(1)^3 - (1)^3 + 2(-1)^3 - (-1)^3 = 0. \quad (16)$$

The Z' boson associated with this $U(1)$ may show up in e^+e^- or $\mu^+\mu^-$ collisions as a resonance decaying into charged-lepton pairs.

Without ν_R , all neutrinos are massless, protected respectively by the additive lepton numbers L_e , L_μ , L_τ , unless a Higgs triplet is added [6]. In general, there may be a certain number of ν_R 's, not necessarily equal to the number of ν_L 's, as discussed for example in ref. [7].

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3. Standard model + one ν_R

If there is only one $\nu_R \sim (1, 1, 0)$, then it can couple only to one linear combination of ν_{iL} . Assuming that its Majorana mass is M_R , the 4×4 mass matrix spanning ν_{iL} and ν_R^c is given by

$$\mathcal{M} = \begin{bmatrix} 0 & 0 & 0 & m_1 \\ 0 & 0 & 0 & m_2 \\ 0 & 0 & 0 & m_3 \\ m_1 & m_2 & m_3 & M_R \end{bmatrix} \quad (17)$$

If $m_{1,2,3} \ll M_R$, one linear combination of ν_{iL} gets a see-saw mass [8]

$$m_{\nu_0} \sim \frac{m_1^2 + m_2^2 + m_3^2}{M_R} \quad (18)$$

At this level, there are two massless neutrinos left, but since no lepton number remains conserved, they must pick up finite radiative masses. Specifically, they do so in two loops through the exchange of two W bosons [9]. Their masses are proportional to m_{ν_0} of eq. (18) and are functions of the charged-lepton masses with double GIM suppression [10]. A detailed analytical and numerical study of this mechanism has been made [11].

With one ν_R , the global symmetries left are additive baryon number B and multiplicative lepton number L . This is true also in the canonical scenario where three ν_R 's are added to obtain three see-saw neutrino masses.

4. Gauged $B - 3L_\tau$

As mentioned already, B cannot be gauged by itself without adding to the particle content of the standard model. Nevertheless, the phenomenology of such a possibility has been discussed [12]. On the other hand, if the one ν_R added to the minimal standard model is required to be $\nu_{\tau R}$, then $B - 3L_\tau$ can be gauged [13]. The various axial-vector triangle anomalies are canceled as follows:

$$[SU(2)]^2 U(1)_X : 3(3)(1/3) - 3 = 0, \quad (19)$$

$$[U(1)_X]^2 U(1)_Y : (1/3)^2 [2(1/6) - (2/3) - (-1/3)] \\ + (-3)^2 [2(-1/2) - (-1)] = 0, \quad (20)$$

$$[U(1)_Y]^2 U(1)_X : 3(3)[2(1/6)^2 - (2/3)^2 - (-1/3)^2](1/3) \\ + [2(-1/2)^2 - (-1)^2](-3) = 0. \quad (21)$$

Where does $B - 3L_\tau$ come from? First consider $B - L$ for each family. In that case, the standard model can be extended to $SU(4) \times SU(2)_L \times U(1)'$, under which the quarks and leptons transform as:

$$\begin{bmatrix} u & \nu \\ d & l \end{bmatrix}_L \sim (4, 2, 0), \quad [u \nu]_R \sim (4, 1, 1/2), \quad [d l]_R \sim (4, 1, -1/2), \quad (22)$$

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with the electric charge given by

$$Q = \frac{1}{2}(B - L) + T_{3L} + Y'. \quad (23)$$

Since Y' takes on the values $\pm 1/2$, it is natural to identify $U(1)'$ as $U(1)_R$ and extend [14] the gauge group to $SU(4) \times SU(2)_L \times SU(2)_R$. In the case of $B - 3L_\tau$, consider instead $SU(10) \times SU(2)_L \times U(1)'$, with

$$Q = \frac{1}{3}(B - 3L_\tau) + T_{3L} + Y'. \quad (24)$$

The quarks and leptons are now organized as follows.

$$\begin{bmatrix} u & c & t & \nu_\tau \\ d & s & b & \tau \end{bmatrix}_L \sim (10, 2, 1/10), \quad \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L, \quad \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix}_L \sim (1, 2, -1/2), \quad (25)$$

$$[u c t \nu_\tau]_R \sim (10, 1, 3/5), \quad [d s b \tau]_R \sim (10, 1, -2/5), \quad e_R, \quad \mu_R \sim (1, 1, -1). \quad (26)$$

It is clear that $SU(10)$ breaks down to $SU(9) \times U(1)_{B-3L_\tau}$, and $SU(9)$ to $[SU(3)]^3 \times U(1) \times U(1)$. The possibility of $[SU(3)]^3$ has been discussed [15], but the two extra $U(1)$ symmetries remain unexplored. They may be relevant in understanding whether and how families of quarks could be different. Note also that $L_e - L_\mu$ is still gaugeable together with $B - 3L_\tau$.

The essential prediction of gauged $B - 3L_\tau$ is of course a new gauge boson. Let me call it X . Since it does not couple to e or μ or their corresponding neutrinos, there is no direct phenomenological constraint from the best known high-energy physics experiments, such as e^+e^- annihilation, deep-inelastic scattering of e or μ or ν_μ on nuclei, or the observation of e^+e^- or $\mu^+\mu^-$ pairs in hadronic collisions. Although X does contribute to purely hadronic interactions, its presence is effectively masked by the enormous background due to quantum chromodynamics (QCD). However, unlike the case of a gauge boson coupled only to baryon number [12], X also couples to L_τ . Assuming that $\nu_{\tau R}$ and the t quark are too heavy to be decay products of X , the branching fractions of X are 10/91, 54/91, and 27/91 in the parton approximation for the modes $\bar{q}q$, $\tau^+\tau^-$, and $\bar{\nu}_\tau\nu_\tau$ respectively. There are two possible production mechanisms for X . It can be directly produced in hadronic collisions: $q\bar{q} \rightarrow X$, or indirectly from Z decay if $M_X < M_Z$.

Consider the sequential decays $Z \rightarrow \bar{q}qX$, then $X \rightarrow \bar{\nu}_\tau\nu_\tau$, and $Z \rightarrow \bar{\nu}_\tau\nu_\tau X$, then $X \rightarrow \bar{q}q$. Since the neutrinos are undetected, these processes have the same effective final states as in the search for an invisible Higgs boson in Z decay [16]. Another decay mode to search for is $Z \rightarrow \tau^+\tau^-X$, then $X \rightarrow \tau^+\tau^-$. From the present experimental nonobservation of the above, the coupling g_X has to be smaller than about 0.2 for $M_X < 50$ GeV. From the observed $e - \mu - \tau$ universality in Z decay, a limit of g_X less than 0.2 to 0.4 is also obtained for M_X between 50 and 150 GeV. Details will be presented elsewhere [17].

The global symmetries left up to now are additive B , multiplicative L_τ [from the breaking of gauged $B - 3L_\tau$ with $\chi^0 \sim (1, 1, 0; 6)$], and additive L_e, L_μ . Whereas ν_τ gets a see-saw mass, ν_e and ν_μ remain massless. To allow them nonzero masses, a second Higgs doublet is added:

$$\begin{bmatrix} \eta^+ \\ \eta^0 \end{bmatrix} \sim (1, 2, 1/2; -3), \quad (27)$$

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which induces mixing between τ and the other two leptons. The 3×3 charged-lepton mass matrix is now given by

$$\mathcal{M}_l = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ a_e & a_\mu & m_\tau \end{bmatrix}. \quad (28)$$

There are some additional consequences. (1) The X boson mixes with Z , requiring the vacuum expectation value of η^0 to be much smaller than those of ϕ^0 and χ^0 for it to be consistent with data. (2) Only one multiplicative lepton number is left. (3) Two neutrinos acquire radiative masses through one-loop η^0 and two-loop W exchange, but the values so obtained are too small to be of phenomenological interest. The addition of a charged scalar singlet $\chi^- \sim (1, 1, -1; -3)$ allows it to couple to $\nu_l \tau_L - l_L \nu_\tau$, hence the exchange and mixing of χ^- with the physical linear combination formed by ϕ^- and η^- will be able to generate [18] realistic one-loop neutrino masses. (4) Lepton family number is violated by the parameters a_e and a_μ . The best present limits are from $\mu - e$ conversion in nuclei:

$$\frac{g_X^2}{M_X^2} a_\mu a_e < 1.2 \times 10^{-12}; \quad (29)$$

from the decay of τ to $\mu \pi^+ \pi^-$ over $\nu \pi^- \pi^0$:

$$\frac{g_X^2}{M_X^2} a_\mu m_\tau < 3.8 \times 10^{-7}; \quad (30)$$

and from the decay of τ to $e \pi^+ \pi^-$ over $\nu \pi^- \pi^0$:

$$\frac{g_X^2}{M_X^2} a_e m_\tau < 2.9 \times 10^{-7}. \quad (31)$$

The rare decay $K^+ \rightarrow \pi^+ \nu_\tau \bar{\nu}_\tau$ gets a contribution from X , but since the $X\bar{q}q$ interaction is vectorial, it is like that of a heavy photon or gluon, hence it is very much suppressed relative to the Z contribution which has axial-vector couplings to quarks.

5. Conclusion and outlook

The number of ν_R 's (0, 1, 2, 3, ...) which may accompany the minimal standard model is crucial to understanding the symmetries of the (extended) model and its leptonic properties. Both neutrino physics and high-energy accelerator physics are on the verge of possible major discoveries. The next several years will be decisive in leading us forward in their theoretical understanding, and may even discover radically new physics beyond the standard model.

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