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ΛN Space-exchange effects in the s-shell hypernuclei and ${}^{9}_{\Lambda}$Be[†]

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Abstract. Variational Monte Carlo calculations of the ground state separation energies B_Λ of the s-shell hypernuclei and also of ${}^{9}_{\Lambda}$Be have been made for an Urbana-type central space-exchange ΛN potential consistent with Λp scattering, and also including three-body ΛNN forces. The s-shell hypernuclei are treated as A-body systems (A = baryon number), and ${}^{9}_{\Lambda}$Be is analysed as a partially nine-body problem in the $\Lambda - 2\alpha$ model. The reduction of B_Λ due to the space-exchange ΛN potential has been calculated for the s-shell hypernuclei for a range of interactions: both ΛN and $\Lambda N + \Lambda NN$ forces. For $A = 3, 4, 5$ the exchange energy is approximately, 0.04, 0.15 and 0.50 MeV, respectively. For ${}^{9}_{\Lambda}$Be a much more limited study gives $\simeq$ 1.3 MeV. These values are much larger than that for 'soft' $\Lambda N + NN$ potentials when the correlations are weak.

Keywords. Hypernuclei; space-exchange ΛN potential; variational Monte Carlo; ΛN scattering.

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1. Introduction

The reduction of the Λ separation energy B_Λ of ${}^{5}_{\Lambda}$He due to the space-exchange ΛN potential was studied earlier [1, 2] and found to be quite small $\simeq$ 0.03 MeV. The effect of exchange potentials for ${}^{9}_{\Lambda}$Be in a three-body $\Lambda - 2\alpha$ model was also found to be moderate [2, 3] $\simeq$ 0.4 MeV. In the light of these results [1–3], later analyses [4–7] did not include the space-exchange ΛN potential. However, with repulsive-core potentials and appropriate correlations the ΛN space-exchange energy may be much larger. In this paper we study the effects of exchange with the variational Monte Carlo (VMC) method for realistic potentials. We have considered the s-shell hypernuclei ${}^{3}_{\Lambda}$H, ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$H*, ${}^{5}_{\Lambda}$He, and also to a limited extent ${}^{9}_{\Lambda}$Be which has a p-shell spinless core. The s-shell hypernuclei are treated as A-body systems (A-baryon number), and ${}^{9}_{\Lambda}$Be is treated as a partially nine-body problem within the $\Lambda - 2\alpha$ model. The VMC calculation of B_Λ for these hypernulei is described in refs. 6, 7; we have made suitable modifications to include ΛN exchange. The potentials and the hamiltonians are discussed in § 2. The wave functions and correlation

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functions are given in § 3 and in the appendix. In § 4 the results with a space-exchange potential are discussed. Our results and their implications are summarized in § 5.

2. Potentials

The potentials we use have been extensively used in previous work [3–7]. These potentials [8–11] are phenomenological but are consistent with general expectations of meson-exchange models [12].

A. ΛN potential

An Urbana-type central ΛN potential with space-exchange and consistent with Λp scattering has been used. This potential has a theoretically reasonable attractive tail. It has the form [3–7]

$$V_{\Lambda N}(r_{i\Lambda}) = V(r_{i\Lambda})[1-\varepsilon+\varepsilon P_x(i\Lambda)] \equiv V(r_{i\Lambda}) + V_x(r_{i\Lambda}), \tag{1}$$

$$V_x(r_{i\Lambda}) = -\varepsilon V(r_{i\Lambda})[1-P_x(i\Lambda)] \tag{2}$$

where P_x is the space-exchange operator for the ith nucleon and Λ particle and $V_x(r_{i\Lambda})$ is the space-exchange potential with ε determining its relative strength. The direct potential is

$$V(r) = W_0/[1+\exp\{(r-R)/a\}] - V_0 T_\pi^2(r), \tag{3}$$

where $W_0 = 2137\,\text{MeV}$, $R = 0.5\,\text{fm}$, $a = 0.2\,\text{fm}$. The strength of the spin-average ΛN potential which is consistent with Λp scattering is $V_0 = 6.15 \pm 0.05\,\text{MeV}$. In terms of the singlet and triplet strengths $V_0 = (V_s + 3V_t)/4$. For $A = 3, 4, 4^*, 5$ the appropriate spin averages to be used in eq. 3 are: $V_3 = V_0 + V_\sigma/2$, $V_4 = V_0 + V_\sigma/4$, $V_4^* = V_0 - V_\sigma/12$, $V_5 = V_0$ where $V_\sigma = V_3 - V_t$. $T_\pi(r)$ is the one-pion exchange tensor potential shape modified with a cut-off:

$$T_\pi(r) = (1 + 3/x + 3/x^2)(e^{-x}/x)(1 - e^{-cr^2})^2 \tag{4}$$

with $x = 0.7r$ and $c = 2.0\,\text{fm}^{-2}$. The exchange parameter ε is quite poorly determined from the Λp forward–backward asymmetry to be $\simeq 0.1$–0.38. We also made some calculations for a Gaussian potential without hard core

$$V_{\Lambda N}(r) = -V_g e^{-r^2/b^2}/(\sqrt{\pi}b)^3 \tag{5}$$

where b is the range and V_g its strength.

B. Three-body ΛNN potentials

We use two types of three-body ΛNN potentials, the details of which are given in earlier work [9, 10]. One is a dispersive ΛNN potential [4–7]. This is a representation of the suppression of the two-pion exchange (TPE) ΛN potential arising from the modification ('dispersion') of the intermediate $\Sigma, N^*, \Delta \ldots$ components by the medium (a '2nd nucleon'). We consider the following dispersive ΛNN potential

$$V_{\Lambda NN}^D = W T_\pi^2(r_{1\Lambda}) T_\pi^2(r_{2\Lambda}). \tag{6}$$

A strength $W = 0.01\,\text{MeV}$ gives a repulsive contribution which is consistent with the suppression of TPE ΛN potential in a coupled channel reaction–matrix approach [4, 8]. The other type of three-body ΛNN force arises from TPE, appropriate to a p-wave pion interaction of the Λ with two nucleons (1 and 2), and has the form [10]

$$V_{\Lambda N}^{2\pi} = -(C_P/6)(\tau_1 \cdot \tau_2)\{[(\sigma_1 \cdot \sigma_\Lambda)Y(r_{1\Lambda}) + S_{1\Lambda}T(r_{1\Lambda})], \times [(\sigma_2 \cdot \sigma_\Lambda)Y(r_{2\Lambda}) + S_{2\Lambda}T(r_{2\Lambda})]\}, \tag{7}$$

where $r_{i\Lambda} = |\mathbf{r}_i - \mathbf{r}_\Lambda|$ with $i = 1$ and 2, $\{A, B\} = AB + BA$,

$$Y(x) = \exp(-x)(1 - \exp(-\hat{c}r^2))/x,$$

$T(x)$ is given by eq. 4 with c replaced by $\hat{c}$ and $x = \hat{\mu}r$. The other symbols in the equation have their usual meaning. Theoretical estimates give $C_P = 1$–$2\,\text{MeV}$ and two values 2 and $3\,\text{fm}^{-2}$ were used for cut-off $\hat{c}$ in ref. [7]. In this work we consider only $C_P = 2\,\text{MeV}$, $\hat{c} = 2\,\text{fm}^{-2}$ and $\hat{\mu} = 0.7\,\text{fm}^{-1}$.

For the s-shell hypernuclei:

$$V_{\Lambda NN}^{2\pi} = C_P[1 + (3\cos^2\theta - 1)T_\pi(r_{1\Lambda})T_\pi(r_{2\Lambda})]Y_\pi(r_{1\Lambda})Y_\pi(r_{2\Lambda}). \tag{8}$$

C. NN Potential

For the NN potential we use the central Mafliet–Tjon potential [11]

$$V_{NN}(r) = \hbar c[7.39\exp(-3.11r) - 2.39\exp(-1.55r)]/r. \tag{9}$$

This gives reasonable ground state energies and rms radii for ^{3}H and ^{4}He with the VMC approach. For $^3_\Lambda$H, for which the core nucleus is ^{2}H, the coefficient of the attractive part is 3.20 appropriate to $S = 1$, $l = 0$ and gives $E(^2H) = -2.23\,\text{MeV}$.

D. α-α potential

Since $^9_\Lambda$Be is treated as a partially nine-body system in the $\Lambda + 2\alpha$ model, we also need the α-α potential $V_{\alpha\alpha}$. The α-α potential of Chien and Brown [13] for relative angular momentum $l = 0$ is used. This potential fits the α-α scattering data up to $\simeq 15.0\,\text{MeV}$ c.m. energy. Earlier work [6] which considered a range of α-α potentials consistent with scattering, showed that the results for $^9_\Lambda$Be and $^{10}_{\Lambda\Lambda}$Be were essentially the same for all the α-α potentials considered.

3. Hamiltonian

The Hamiltonian for the s-shell hypernuclei is

$$H^{(A)} = H_N^{(A-1)} + T_\Lambda + \sum_{i=1}^{A-1} V_{\Lambda N}(i\Lambda) + \sum_{1=i<j}^{A-1} V_{\Lambda NN}(ij\Lambda), \tag{10}$$

where

$$H_N^{(A-1)} = \sum_{i=1}^{A-1} T_N(i) + \sum_{1=i<j}^{A-1} V_{NN}(ij) \tag{11}$$

is the Hamiltonian for $A-1$ nucleons of the core nucleus. Also

$$V_{\Lambda NN} = V^{D}_{\Lambda NN} + V^{2\pi}_{\Lambda NN}. \tag{12}$$

The Hamiltonian for ${}^{9}_{\Lambda}$Be is

$$H^{(A)} = H_N^{(A-1)} + T_\Lambda(r_\Lambda) + \left[\sum_{i=1}^{4} V_{\Lambda N}(r_{i\Lambda}) + \sum_{1=i<j}^{4} V_{\Lambda NN}(r_{ij\Lambda})\right] + \left\{\sum_{i=5}^{A-1} V_{\Lambda N}(r_{i\Lambda}) + \sum_{5=i<j}^{A-1} V_{\Lambda NN}(r_{ij\Lambda}) + \sum_{\alpha_1=1}^{4}\sum_{\alpha_2=5}^{A-1} V_{\Lambda NN}(r_{\alpha_1\alpha_2\Lambda})\right\}. \tag{13}$$

The Hamiltonian for the core nucleus ^{8}Be is

$$H_N^{(A-1)} = \sum_{i=1}^{8} T_N(r_i) + \sum_{i<j=4}^{4} V_{NN}(r_{ij}) + \sum_{i<j=5}^{8} V_{NN}(r_{ij}) + V_{\alpha\alpha}(r_{\alpha\alpha}). \tag{14}$$

4. Variational Monte Carlo calculations of B_Λ

Our trial function for the s-shell hypernuclei are:

$$\Psi^{(A)} = \left\{\prod_{i=1}^{A-1} f_{\Lambda N}(i\Lambda) \prod_{i<j}^{A-1} f_{NN}(ij) \prod_{i<j}^{A-1} f_{\Lambda NN}(ij\Lambda)\right\}\chi^{(A)}, \tag{15a}$$

$$\Psi^{(A-1)} = \left\{\prod_{i<j}^{A-1} f_{NN}(ij)\right\}\eta^{(A-1)}. \tag{15b}$$

We treat the ${}^{9}_{\Lambda}$Be as a partially nine-body problem in the $\Lambda + 2\alpha$ model. We thus write its wave function as:

$$\Psi^{(9)} = \left\{\prod_{i=1}^{4} f_{\Lambda N}(i\Lambda) \prod_{i<j}^{4} f_{NN}(ij)\right\} \times \left\{\prod_{i=5}^{8} f_{\Lambda N}(i\Lambda) \prod_{i=5<j}^{8} f_{NN}(ij)\right\} \times \prod_{i<j}^{8} f_{\Lambda NN}(ij\Lambda) f_{\alpha_1\alpha_2}\chi^{(9)}. \tag{16}$$

In (15) and (16) χ and η are the appropriate spin functions. The two-body correlation functions f_{NN}, $f_{\Lambda N}$ and $f_{\alpha_1\alpha_2}$ are assumed to be spin-independent and are obtained with procedures developed by the Urbana group. Three-body ΛNN correlations $f_{\Lambda NN}$ are included and were found to be important [7]. More details may be found in ref. 7 and in the appendix.

To summarize:
A number of variational parameters determine $f_{\Lambda N}$ and f_{NN}; in particular κ_{NN} and $\kappa_{\Lambda N}$ determine the asymptotic behaviour. A strength parameter s reduces the strength of V_A in determining $f_{\Lambda N}$. Reducing s below one was found to have most effect when $V^{D}_{\Lambda NN}$ is present. Other variational parameters for f_{NN} and $f_{\Lambda N}$ were fixed at the values of ref. [7] (also given in footnotes to tables 1 and 2), since their variation from these values was

found to have little effect. The three-body ΛNN correlations $f_{\Lambda NN} = f^D_{\Lambda NN} f^{2\pi}_{\Lambda NN}$ involve the variational parameters α for $f^D_{\Lambda NN}$ and β for $f^{2\pi}_{\Lambda NN}$.

The separation energy B_Λ is obtained from

$$B_\Lambda = \frac{(\Psi^{(A)}|H^{(A)}|\Psi^{(A)})}{(\Psi^{(A)}, \Psi^{(A)})} - \frac{(\Psi^{(A-1)}|H^{(A-1)}|\Psi^{(A-1)})}{(\Psi^{(A-1)}, \Psi^{(A-1)})} \tag{17}$$

by separately minimizing both terms. The space-exchange energy is calculated by exchanging the coordinates of the Λ with each nucleon in turn in the wave function $\Psi^{(A)}$ of the hypernucleus:

$$\left\langle \Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_{A-1}; \mathbf{r}_\Lambda) \left| \sum_{i=1}^{A-1} V(\mathbf{r}_{i\Lambda}) P_x(i\Lambda) \right| \Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_i, \ldots, \mathbf{r}_{A-1}; \mathbf{r}_\Lambda) \right\rangle$$
$$= \left\langle \Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_{A-1}; \mathbf{r}_\Lambda) \left| \sum_{i=1}^{A-1} V(\mathbf{r}_{i\Lambda}) \right| \Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_\Lambda, \ldots, \mathbf{r}_{A-1}; \mathbf{r}_i) \right\rangle. \tag{18}$$

The multi-dimensional integrations in eq. (17) are performed using the standard Monte Carlo technique with a random walk. For the s-shell hypernuclei typically 100,000–150,000 configurations were generated for the integrations involving the optimum parameter set. Whereas for ${}^9_\Lambda$Be, one million configurations were generated to bring the statistical error within the reasonable limits.

5. The s-shell hypernuclei and ${}^9_\Lambda$Be with ΛN space-exchange

A. *Core energies*

To obtain B_Λ the appropriate core energy is required. As described is § 2 these were obtained for $A \leq 5$ with Mafliet–Tjon potentials. For ^{3}H the optimum parameters are $\kappa_{NN} = 0.28\,\text{fm}^{-1}$, $c_{NN} = 2.5\,\text{fm}^{-1}$, $a_{NN} = 0.6\,\text{fm}$, $R_{NN} = 1.3\,\text{fm}$ giving $E = -8.25 \pm 0.1\,\text{MeV}$, $\langle r^2 \rangle^{1/2} = 1.66\,\text{fm}$, and for ^{4}He: $\kappa_{NN} = 0.304\,\text{fm}^{-1}$, $c_{NN} = 1.0\,\text{fm}^{-1}$, $a_{NN} = 0.5\,\text{fm}$, $R_{NN} = 1.0\,\text{fm}$ giving $E = -31.20 \pm 0.03\,\text{MeV}$, $\langle r^2 \rangle^{1/2} \doteq 1.42\,\text{fm}$. For ${}^2H$, the appropriately modified potential gives -2.23 MeV. The ground state of ^{8}Be is under bound by 0.1 MeV with respect to break-up into $2\alpha s$, and its energy is thus $2E({}^4\text{He})-0.1\,\text{MeV} \cong -62.3\,\text{MeV}$ for the Mafliet–Tjon potential.

B. *s-Shell hypernuclei*

The contribution of the space-exchange potential V_x (eq. 2) is obtained using eq. (18). The potential energy due to exchange is then

$$\langle V_x \rangle = \left\langle \Psi^A_\varepsilon \left| \sum_{i=1}^{A-1} V_x(r_{i\Lambda}) \right| \Psi^A_\varepsilon \right\rangle$$
$$= -\varepsilon \left\langle \Psi^A_\varepsilon \left| \sum_{i=1}^{A-1} V(r_{i\Lambda})[1 - P_x(i\Lambda)] \right| \Psi^A_\varepsilon \right\rangle, \tag{19}$$

where $\Psi_\varepsilon^{(A)}$ is the (normalized) hypernuclear wave function for ε. The total contribution due to exchange is

$$E_x = {}^A E(\varepsilon) - {}^A E(0), \tag{20}$$

where ${}^A E(\varepsilon)$ and ${}^A E(0)$ are respectively, the (optimized) energies for ε and for $\varepsilon = 0$ (no space exchange). If the contribution due to V_x is small, it may be treated as a perturbation and Ψ_ε may be taken as independent of ε; then

$$E_x \cong \langle \Psi_\varepsilon | V_x | \Psi_\varepsilon \rangle \cong \langle \Psi_{\varepsilon=0} | V_x | \Psi_{\varepsilon=0} \rangle. \tag{21}$$

We made the following checks:

1. From eq. (19): $\langle \Psi | V_x | \Psi \rangle \equiv 0$ if $f_{\Lambda NN} \equiv 1$ and $f_{\Lambda N} = f_{NN}$, since then the Λ and nucleon wave functions are identical. This was confirmed within the statistical errors.
2. $\langle \Psi | V_x | \Psi \rangle \equiv 0$ if $V(r)$ is of very short range. This was checked for the Gaussian potential eq. 5, for a small range $b = 0.1$ fm.

Results are shown in tables 1–4. Table 1 ($A = 3, 4$) and table 2 ($A = 5$) show results for selected sets of variational parameters, mostly those for the lowest E and with the smallest statistical error ΔE. For table 3 ($A = 3, 4$) and table 4 ($A = 5$) we have averaged over nearby variational parameter sets which give comparable values of E and ΔE, resulting in reduced errors. We found that for ${}^5_\Lambda$He even with only $V_{\Lambda N}$, the results with V_x are frequently considerably improved if three-body dispersive correlations $f^D_{\Lambda NN}$ are included

Table 1. Variational results for ${}^3_\Lambda$H and ${}^4_\Lambda$H.

V_3	W	C_p	ε	κ_{NN}	$\kappa_{\Lambda N}$	s	α	β	$E \pm \Delta E$	$\langle V_x \rangle$
Results for ${}^3_\Lambda$H										
6.28	0.00	0	0.0	0.24	0.08	1.00	0.00	0.00	2.44 ± 0.01	—
6.28	0.00	0	0.3	0.28	0.08	1.00	0.00	0.00	2.41 ± 0.01	0.039 ± 0.004
6.31	0.01	0	0.0	0.26	0.08	1.00	0.00	0.00	2.39 ± 0.01	—
6.31	0.01	0	0.3	0.26	0.08	1.00	0.00	0.00	2.35 ± 0.01	0.034 ± 0.003
6.40	0.02	2	0.0	0.24	0.08	1.00	0.00	0.20	2.59 ± 0.01	—
6.40	0.02	2	0.3	0.24	0.08	1.00	0.00	0.20	2.56 ± 0.01	0.028 ± 0.002
Results for ${}^4_\Lambda$H										
6.05	0.00	0	0.3	0.31	0.100	0.95	0.00	0.00	9.14 ± 0.02	0.12 ± 0.01
6.20	0.00	0	0.0	0.31	0.120	1.00	0.00	0.00	10.73 ± 0.03	—
6.20	0.00	0	0.3	0.31	0.120	1.00	0.05	0.00	10.61 ± 0.02	0.11 ± 0.01
6.15	0.01	0	0.0	0.31	0.120	0.95	0.15	0.00	9.54 ± 0.03	—
6.15	0.01	0	0.3	0.31	0.115	0.95	0.15	0.00	9.35 ± 0.03	0.16 ± 0.02
6.25	0.01	0	0.3	0.31	0.120	1.00	0.20	0.00	10.29 ± 0.02	0.15 ± 0.02
6.25	0.02	0	0.0	0.31	0.120	0.95	0.15	0.00	9.88 ± 0.03	—
6.25	0.02	0	0.3	0.31	0.120	0.95	0.15	0.00	9.69 ± 0.02	0.15 ± 0.02
6.25	0.02	0	0.0	0.31	0.115	0.95	0.00	0.15	9.65 ± 0.03	—
6.25	0.02	2	0.3	0.31	0.115	0.95	0.00	0.15	9.54 ± 0.03	0.12 ± 0.01

All results are for a cut-off $\hat{c} = 2\,\mathrm{fm}^{-2}$ and with $\bar{c} = \hat{c}$, and for $\bar{\mu} = \hat{\mu} = 0.7$ fm. For ${}^3_\Lambda$H for all cases $c_{NN} = 3.7\,\mathrm{fm}^{-1}$, $a_{NN} = 5.0$ fm, $R_{NN} = 3.0$ fm, $c_{\Lambda N} = 3.0\,\mathrm{fm}^{-1}$, $a_{\Lambda N} = 3.0$ fm, $R_{\Lambda N} = 1.0$ fm. For ${}^4_\Lambda$H for all cases: $\kappa_{NN} = 0.31\,\mathrm{fm}^{-1}$, $c_{NN} = 2\,\mathrm{fm}^{-1}$, $a_{NN} = 0.6$ fm, $R_{NN} = 1.3$ fm, $c_{\Lambda N} = 2.0\,\mathrm{fm}^{-2}$, $a_{\Lambda N} = 0.8$ fm, $R_{\Lambda N} = 1.0$ fm. All energies and potential strengths are in MeV.

Table 2. Variational results for $^5_\Lambda$He.

#	V_0	W	C_P	ε	$\kappa_{\Lambda N}$	s	α	β	$E \pm \Delta E$	$\langle V_x \rangle$
01	6.00	0.000	0	0.25	0.105	1.00	0.00	0.00	34.10 ± 0.01	0.27 ± 0.02
02	6.15	0.000	0	0.30	0.125	0.95	0.05	0.00	36.83 ± 0.10	0.40 ± 0.05
03	6.20	0.000	0	0.00	0.132	0.95	0.00	0.00	38.42 ± 0.04	—
04	6.20	0.000	0	0.30	0.125	1.00	0.05	0.00	38.17 ± 0.04	0.30 ± 0.04
05	6.20	0.000	0	0.60	0.105	0.95	0.05	0.00	37.06 ± 0.04	0.81 ± 0.15
06	6.20	0.000	0	0.60	0.125	0.95	0.00	0.00	37.41 ± 0.19	0.70 ± 0.17
07	6.20	0.000	0	1.00	0.125	1.00	0.05	0.00	37.44 ± 0.05	1.03 ± 0.40
08	6.20	0.000	0	−0.30	0.125	0.95	0.00	0.00	38.47 ± 0.18	0.35 ± 0.07
09	6.10	0.007	0	0.30	0.105	0.95	0.10	0.00	34.46 ± 0.05	0.47 ± 0.04
10	6.16	0.010	0	0.30	0.125	0.90	0.05	0.00	34.69 ± 0.05	0.50 ± 0.06
11	6.20	0.020	0	0.00	0.125	0.85	−0.05	0.00	34.12 ± 0.05	—
12	6.20	0.017	0	0.30	0.105	0.90	0.05	0.00	34.16 ± 0.04	0.50 ± 0.05
13	6.20	0.020	0	0.25	0.105	0.90	0.15	0.00	33.83 ± 0.04	0.44 ± 0.04
14	6.10	0.006	2	0.30	0.125	0.95	0.10	0.20	34.37 ± 0.05	0.40 ± 0.03
15	6.16	0.010	2	0.00	0.125	0.95	0.15	0.15	35.05 ± 0.02	—
16	6.15	0.011	0	0.60	0.145	0.90	0.05	0.00	33.74 ± 0.08	0.99 ± 0.16
17	6.10	0.006	2	0.30	0.125	0.95	0.15	0.20	34.37 ± 0.05	0.40 ± 0.03
18	6.16	0.010	2	0.25	0.145	0.90	0.05	0.10	34.45 ± 0.06	0.38 ± 0.04
19	6.16	0.010	2	0.60	0.145	0.95	0.05	0.10	34.04 ± 0.04	0.70 ± 0.11
20	6.20	0.013	2	0.00	0.105	0.95	0.10	0.20	34.92 ± 0.04	—
21	6.20	0.013	2	0.30	0.125	0.90	0.10	0.20	34.45 ± 0.05	0.40 ± 0.04

As for table 1 but with $\kappa_{NN} = 0.304\,\text{fm}^{-1}$, $c_{NN} = 1.0\,\text{fm}^{-1}$, $a_{NN} = 0.5\,\text{fm}$, $R_{NN} = 1.0\,\text{fm}$, $c_{\Lambda N} = 2.0\,\text{fm}^{-1}$, $a_{\Lambda N} = 0.8\,\text{fm}$, $R_{\Lambda N} = 1.0\,\text{fm}$.

(c.f. cases 5 and 6 in table 2). This may perhaps partly allow for exchange correlations which are of course not explicitly included in our calculations.

Comparison of $\langle V_x \rangle$ and E_x for $\varepsilon \leq 0.6$ for $^5_\Lambda$He (table 4) shows no evidence, within the errors, of any non-linearity with ε. The variational principle implies that for large ε, the total exchange contribution E_x should increase less rapidly than linearly with ε. Comparison of $\varepsilon = 0.6$ with $\varepsilon = 1$ (for $V_0 = 6.2$, $V_{\Lambda NN} = 0$) indicates perhaps some levelling off of E_x for large $\varepsilon \cong 1$. The fluctuations of $\langle V_x \rangle$ become quite large for larger ε ($= 0.6$ and 1), and the results for E_x which depend on the difference of two variationally obtained, and not very different energies may have quite large errors, and will depend critically on the value of $E(\varepsilon = 0)$. The values marked with * were taken from ref. [7] or interpolated from the results of this reference, and may have fairly large errors. Some improved results for $\varepsilon = 0$ (consistent with the earlier ones) are shown in tables 1 and 2. The values of $\langle V_x \rangle$ and E_x for $^5_\Lambda$He, scaled to ε (for $\varepsilon \leq 0.6$), show no significant dependence on the interaction (e.g. on V_A, W, C_P) or on B_Λ, within the errors. Also $\langle V_x \rangle$ and E_x are consistent with each other, again within the errors. In fact for $A = 5$, the interactions $V_{\Lambda N} + V_{NN}$ were mostly chosen to give $E \cong 34.7$–35.0 MeV ($B_\Lambda = 3.5$–3.8 MeV), so as to give the experimental $B_\Lambda \cong 3.1$ MeV using $E_x \cong 0.5$ MeV. Also for $A = 3$ and 4, for given ε, there is no apparent significant dependence on the interaction and on B_Λ. Thus $V_4 = 6.05$ and 6.20 MeV (with $V_{\Lambda NN} \equiv 0$) for $\varepsilon = 0$ give respectively $B_\Lambda = 1.0$ and 2.4 MeV, corresponding to $A = 4^*$ (excited $S = 1$ state) and $A = 4$ (ground $S = 0$ state). Both values of V_4 give the same $\langle V_x \rangle$ and E_x within the errors, with perhaps slightly larger values preferred for $V_4 = 6.05$ MeV. However $\langle V_x \rangle$ and E_x depend strongly on A. Thus for $\varepsilon = 0.3$ the values

Table 3. Exchange energies for ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H.

V_3	W	C_P	ε	$E \pm \Delta E$	$\langle V_x \rangle$	E_x
6.28	0.00	0	0.0	2.440 ± 0.007	—	—
6.28	0.00	0	0.3	2.393 ± 0.008	0.038 ± 0.002	0.047 ± 0.010
6.31	0.01	0	0.0	2.390 ± 0.006	—	—
6.31	0.01	0	0.3	2.350 ± 0.010	0.036 ± 0.003	0.040 ± 0.010
6.40	0.02	2	0.0	2.590 ± 0.010	—	—
6.40	0.02	2	0.3	2.560 ± 0.010	0.031 ± 0.020	0.030 ± 0.015
				Results for ${}^{4}_{\Lambda}$H		
6.05	0.00	0	0.0	$9.26 \pm 0.02^*$	—	—
6.05	0.00	0	0.3	9.15 ± 0.01	0.123 ± 0.020	0.11 ± 0.02
6.20	0.00	0	0.0	10.73 ± 0.03	—	—
6.20	0.00	0	0.3	10.60 ± 0.02	0.106 ± 0.005	0.13 ± 0.04
6.15	0.01	0	0.0	9.55 ± 0.03	—	—
6.15	0.01	0	0.3	9.35 ± 0.02	0.160 ± 0.010	0.20 ± 0.04
6.25	0.01	0	0.0	$10.44 \pm 0.05^*$	—	—
6.25	0.01	0	0.3	10.30 ± 0.01	0.130 ± 0.010	0.15 ± 0.15
6.25	0.02	0	0.0	9.88 ± 0.03	—	—
6.25	0.02	0	0.3	9.70 ± 0.02	0.150 ± 0.010	0.18 ± 0.03
6.25	0.02	2	0.0	9.69 ± 0.04	—	—
6.25	0.02	2	0.3	9.51 ± 0.03	0.130 ± 0.010	0.18 ± 0.05

The results are averages over runs for neighbouring values of the variational parameters for which the values of E and ΔE are close. The variational parameters given in the footnote to table 1 were fixed at those values. Energies E for $\varepsilon = 0$ marked with an * are taken from ref. 7 or obtained by interpolation of these results. All energies and potentials strengths are in MeV.

of $\langle V_x \rangle$, E_x are $\cong 0.03$–0.04 MeV for $A = 3$, $\cong 0.1$–0.2 MeV for $A = 4$, 4^* and $\cong 0.4$–0.5 MeV for $A = 5$. For $\varepsilon \leq 0.6$ the values may be obtained from those for $\varepsilon = 0.30$ by linear scaling.

We obtained results in the case of ${}^{5}_{\Lambda}$He for the Gaussian potential, eq. (5), for $b = 1.04$ fm and $\varepsilon = 0.6$; $E = -34.20 \pm 0.5$ MeV, $\langle V_x \rangle \cong 0.27 \pm 0.03$ MeV. Scaled to $\varepsilon = 0.3$ this gives $\langle V_x \rangle \cong 0.15$ MeV, rather less than half the value for our hard-core ΛN potentials (with $V_{\Lambda NN} \equiv 0$). Even these values, obtained with a hard-core V_{NN} are very much larger than those previously obtained for soft ΛN and soft NN potentials for which there are only very weak correlations ($f_{\Lambda N} \cong f_{NN} \cong 1$).

C. *p-Shell hypernucleus* ${}^{9}_{\Lambda}$*Be*

The separation energy of ${}^{9}_{\Lambda}$Be which has a spinless p-shell core, is analysed as a partially nine-body problem in $\Lambda - 2\alpha$ model to a much more limited extent. We have considered only the interaction:

$$V_0 = 6.16\,\text{MeV}, \quad W = 0.01\,\text{MeV} \quad \text{and} \quad C_P = 2.0\,\text{MeV}$$

for $\varepsilon = 0.3$. The optimum parameters are as follows:

$$\kappa_{NN} = 0.260\,\text{fm}^{-1}, \quad c_{NN} = 1.0\,\text{fm}^{-1}, \quad a_{NN} = 0.5\,\text{fm}, \quad R_{NN} = 1.0\,\text{fm},$$
$$\kappa_{\Lambda N} = 0.065\,\text{fm}^{-1}, \quad c_{\Lambda N} = 2.0\,\text{fm}^{-1}, \quad a_{\Lambda N} = 1.0\,\text{fm}, \quad R_{\Lambda N} = 1.0\,\text{fm},$$

Table 4. Exchange energies for ${}^{5}_{\Lambda}$He.

V_0	W	C_P	ε	$E \pm \Delta E$	$\langle V_x \rangle$	E_x
6.00	0.000	0	0.00	$34.57 \pm 0.07^*$	—	—
6.00	0.000	0	0.25	34.06 ± 0.05	0.29 ± 0.02	0.51 ± 0.09
6.15	0.000	0	0.00	$34.73 \pm 0.07^*$	—	—
6.15	0.000	0	0.30	36.79 ± 0.09	0.38 ± 0.02	0.51 ± 0.10
6.20	0.000	0	0.00	38.42 ± 0.04	—	—
6.20	0.000	0	0.10	37.92 ± 0.07	0.13 ± 0.07	0.50 ± 0.08
6.20	0.000	0	0.30	37.97 ± 0.03	0.36 ± 0.04	0.45 ± 0.06
6.20	0.000	0	0.60	37.22 ± 0.04	0.82 ± 0.12	1.20 ± 0.06
6.20	0.000	0	1.00	37.43 ± 0.05	0.99 ± 0.20	0.99 ± 0.07
6.10	0.007	0	0.00	$34.80 \pm 0.09^*$	—	—
6.10	0.007	0	0.30	34.29 ± 0.03	0.49 ± 0.03	0.51 ± 0.09
6.16	0.010	0	0.00	$35.20 \pm 0.10^*$	—	—
6.16	0.010	0	0.30	34.57 ± 0.05	0.47 ± 0.03	0.63 ± 0.10
6.15	0.011	0	0.00	$33.90 \pm 0.09^*$	—	—
6.15	0.011	0	0.60	34.74 ± 0.05	1.02 ± 0.10	1.16 ± 0.10
6.20	0.017	0	0.00	$34.70 \pm 0.04^*$	—	—
6.20	0.017	0	0.30	34.13 ± 0.04	0.50 ± 0.03	0.57 ± 0.05
6.20	0.020	0	0.00	33.12 ± 0.05	—	—
6.20	0.020	0	0.25	33.75 ± 0.05	0.45 ± 0.02	0.37 ± 0.07
6.10	0.006	2	0.00	34.80^*	—	—
6.10	0.006	2	0.30	34.39 ± 0.04	0.39 ± 0.02	0.41 ± 0.10
6.16	0.010	2	0.00	35.05 ± 0.02	—	—
6.16	0.010	2	0.25	34.48 ± 0.05	0.39 ± 0.02	0.57 ± 0.05
6.16	0.010	2	0.60	34.05 ± 0.05	0.76 ± 0.08	1.00 ± 0.05
6.20	0.013	2	0.00	34.92 ± 0.04	—	—
6.20	0.013	2	0.30	34.48 ± 0.06	0.38 ± 0.02	0.44 ± 0.07

As for table 3 but for the fixed variational parameters given in the footnote to table 2.

$$\kappa_{\alpha\alpha} = 0.400\,\text{fm}^{-1}, \quad c_{\alpha\alpha} = 5.0\,\text{fm}^{-1}, \quad a_{\alpha\alpha} = 1.5\,\text{fm}, \quad R_{\alpha\alpha} = 0.0,$$
$$\alpha = 0.040, \quad \beta = 0.0, \quad \mu = 0.7\,\text{fm}^{-1}, \quad c = 2.0\,\text{fm}^{-2}$$

and $s = 0.94$.

The total energy of the hypernucleus is $E(0.3) = -68.90 \pm 0.04$ MeV corresponding to $B_\Lambda = 6.5 \pm 0.04$ MeV which is well within the experimental value of 6.7 ± 0.1 MeV. We find $\langle V_x \rangle = 1.25 \pm 0.005$ MeV. This is much larger than the values previously obtained [1–3] with only soft $\Lambda N + NN$ potentials and is consistent with the large A dependence obtained for the s-shell. The repulsive three-body ΛNN contributions $\langle V^D_{\Lambda NN} \rangle = 2.07$ MeV and $\langle V^D_{\Lambda NN} \rangle_{\alpha\alpha} = 0.638$ MeV are much weaker compared to the values required to fit ${}^{9}_{\Lambda}$Be in the earlier analyses [4, 7].

We may remark that the overbinding problem in ${}^{9}_{\Lambda}$Be in the previous analyses [4, 7] was resolved as a result of strongly repulsive ($W = 0.02$ MeV) three-body force without space-exchange ΛN potential. While in the present work repulsive ΛNN force of reduced strength ($W = 0.01$ MeV) along with space-exchange ($\varepsilon = 0.3$) does help in reducing overbinding. The combination of $\Lambda N + \Lambda NN$ forces employed here, may be said to be constituting a better interaction compared to the one used earlier (ref. [4] and [7]) because it explains data over a wide range of mass number as discussed in the next section.

6. Conclusion

The (repulsive) exchange energies we obtain are much larger than those previously obtained [1–3] for 'soft' $\Lambda N + NN$ potentials. However, we find no significant dependence on the interaction for the family of $\Lambda N + \Lambda NN$ potentials we have considered. The exchange energy was found, within the errors, to scale linearly with the exchange parameter ε for $\varepsilon \leq 0.6$. The exchange energy does depend strongly on A: for $\varepsilon = 0.3$ it is $\cong 0.03, 0.15$ and 0.50 MeV for $A = 3, 4, 5$ respectively. Our results for ${}^9_\Lambda$Be give an exchange contribution $\cong 1.3$ MeV which is again much larger than that obtained with soft potentials.

The relatively small value for ${}^9_\Lambda$Be which is not much larger than twice that for ${}^5_\Lambda$He is a result of the two-α structure of ^{8}Be which implies a small overlap of the two loosely-bound αs in ${}^9_\Lambda$Be. For ${}^{17}_\Lambda$O a VMC calculation [14] gives $E_x \cong 8$ MeV (for $\varepsilon = 0.3$), a value which is also obtained from a calculation using the local density approximation. Exchange correlations will be expected to somewhat reduce the repulsive exchange contribution, especially for large ε, according to the variational principle.

Since the effects of exchange are quite small for $A = 3$ and 4, the modifications for the interactions arise mainly through ${}^5_\Lambda$He. The exchange contribution of $\cong 0.5$ MeV for $\varepsilon \cong 0.3$ will then reduce somewhat the strength W of $V^D_{\Lambda NN}$ required to give the experimental $B_\Lambda({}^5_\Lambda\text{He})$. In fact for those interactions $V_{\Lambda N} + V_{\Lambda NN}$ in table 4 which give $E(\varepsilon = 0) \cong -35$ MeV, the strengths of $V^D_{\Lambda NN}$ were adjusted to achieve just this. The results of ref. [7] then show that their interaction SD(2) is consistent with all the s-shell B_Λ as well with Λp scattering.

More significant perhaps, are the implications for the Λ single-particle (s.p.) energies [15] for hypernuclei having mass number A up to $\simeq 90$ which have been analyzed by using effective interaction within the local density approximation [16] to obtain the Λ-nucleus potential and from this the s.p. B_Λ. With only a dispersive ΛNN potential, the reduction in W required by ${}^5_\Lambda$He due to exchange has the result that $V^D_{\Lambda NN}$ gives insufficient repulsion for large A to allow a fit to the s.p. data for values of $\varepsilon \cong 0.3$–0.34 which are also needed for a fit. (If the repulsion due to exchange were increased by increasing ε, then the resulting small effective masses m^*_Λ would lead to a very poor fit to the s.p. B_Λ for $l > 0$). This implies that purely dispersive ΛNN forces are excluded by the s.p. data. However suitable combinations of $V^D_{\Lambda NN} + V^{2\pi}_{\Lambda NN}$ are acceptable since for nuclear matter (i.e. large A), $V^{2\pi}_{\Lambda NN}$ gives a substantial repulsive contribution, in contrast to ${}^5_\Lambda$He.

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Appendix A

Our trial wave function is given by eq. (15) and (16). The two-body correlation developed by the Urbana group, from Schrödinger equations which contain effective potentials through which the variational parameters enter; f_{NN} is allowed to be different for $\Psi^{(A-1)}$. Thus

$$\left[-\frac{\hbar^2}{2\mu_{BN}}\nabla^2 + V_{BN} + \Lambda_{BN}\right] f_{BN} = 0 \tag{A1}$$

where $B = \Lambda$ or N : μ_{BN} is the reduced mass for the pair B, N. For the hypernucleus of mass number A we use for calculating $f_{\Lambda N}$

$$V_{\Lambda N} = V_c - sV_A T_\pi^2 \tag{A2}$$

where V_A is the effective spin-averaged strength of $V_{\Lambda N}$ for the mass number A given in terms of V_0 and V_σ in § 2. The variational parameter s is mostly used only when $V_{\Lambda NN}$ is included and is a convenient way of allowing for the effect of ΛNN forces on the two-body correlation functions $f_{\Lambda N}$; however sometimes it provides convenient additional flexibility to that provided by the parameters in Λ_{BN}.

The auxiliary potential Λ_{BN} is given by

$$\begin{aligned}\Lambda_{BN} = &-\frac{\hbar^2}{2\mu_{BN}}\left[\kappa_{BN}^2 - \frac{2x_{BN}(\nu_{BN}-\hbar)}{r} - \frac{\nu_{BN}(\nu_{BN}-\hbar)}{r^2}\right](1-e^{-r^2/c_{BN}^2}) \\ &+ \nu_{BN}[1+\exp\{(r-R_{BN})/a_{BN}\}]^{-1}\end{aligned} \tag{A3}$$

where κ_{BN}, c_{BN}, R_{BN} are variational parameters. ν_{BN} is an eigenvalue parameter determined by matching logarithmic derivatives at some suitable r. The form of Λ_{BN} is such that f_{NN}, $f_{\Lambda N}$ have the form of asymptotic behaviour required by the full A-body Schrödinger equation, namely

$$f_{BN} \sim r_{BN}^{\nu} \exp(-x_{BN} r) \tag{A4}$$

with the appropriate products of the fs then having the asymptotic behaviour $\sim r^{-1}\exp(-x_B r)$ if the ν_{BN} are chosen appropriately ($A=3$: $\nu_{BN}=1/2$; $A=4$: $\nu=1/3$; $A=5$; $\nu_{BN}=1/4$). For reasons of convenience we actually used for ${}^4_\Lambda$H: $\nu_{NN}=1/3$, $\nu\Lambda_N = 1/4$ and for ${}^5_\Lambda$He: $\nu_{NN}=1/3$, $\nu_{\Lambda N}=1/4$. In fact the results are quite insensitive to the precise values of the ν_{BN} if the variational parameters are optimized for any given choice. The parameter x_B is related by the asymptotic behaviour to the x_{BN} (e.g. for ^{3}H: in the usual way to the separation energy of baryon B from the remaining baryons; x_B is however not constrained to the empirical values.

For the three-body ΛNN correlations we use

$$f_{\Lambda NN} = f_{\Lambda NN}^D f_{\Lambda NN}^{2\pi}, \tag{A5}$$

$$f_{\Lambda NN}^D = 1 - \alpha\tilde{Y}(r_{1\Lambda})\tilde{Y}(r_{2\Lambda}), \tag{A6}$$

$$f_{\Lambda NN}^{2\pi} = 1 - \beta(3\cos^2\theta - 1)(\tilde{Y})(r_{1\Lambda})\tilde{Y}(r_{2\Lambda}), \tag{A7}$$

where $f_{\Lambda NN}^D$ is appropriate for $V_{\Lambda NN}^D$, and $f_{\Lambda NN}^{2\pi}$ for $V_{\Lambda NN}^{2\pi}$. $\tilde{Y}(r)$ are Yukawa functions as defined in the text with the range and cut-off parameters, $\tilde{\mu}$ and $\tilde{c}$, respectively. The $\tilde{\mu}$, $\tilde{c}$ and the correlation strengths α and β are variational parameters. In the present work $\tilde{\mu}$, $\tilde{c}$ are kept fixed (see tables 1 and 2). The introduction of $f_{\Lambda NN}^{2\pi}$ turns out to be essential and qualitatively changes the contribution of $V_{\Lambda NN}^{2\pi}$, especially for ${}^5_\Lambda$He.

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