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Theoretical understanding of direct photon production

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Abstract. We review some theoretical developments concerning higher order QCD corrections to the bremsstrahlung cross-section and the problems raised by the definition of isolated cross sections. Then we discuss recent phenomenological analyses of direct photon fixed target data.

Keywords. Particle physics; direct photons.

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1. Introduction

The direct photon production [1–3] is a privileged reaction to test perturbative QCD and to measure the gluon content of the proton and the pion. Indeed we have a direct access to the gluon distribution through the observation of the Compton subprocess (figure 1a). The Born cross section also has a contribution from the annihilation subprocess (figure 1b) which is however not dominant in pp collisions. Moreover these two contributions can be disentangled by taking differences of pp and $p\bar{p}$ cross sections.

Furthermore the pointlike coupling of the direct photon to quarks of the hard subprocess allows us to reduce the uncertainties of the theoretical predictions. Thus the cross section does not rely on not well-determined fragmentation functions of quarks and gluons (as it is the case for inclusive large-$p_\perp$ hadron cross sections), nor on hadronization effects as in the case of jet cross sections. Therefore we expect unambiguous tests of QCD.

The calculations of higher order QCD contributions [4, 5], coming from Feynman diagrams of the type shown in figure 2, opened the way to quantitative comparisons between theory and experiment. Good agreements were observed between QCD predictions [5] and direct photon data [6–12], and constraints on the gluon distribution at large-x were obtained [13] by simultaneous fits to lepton-proton DIS data [14] and direct photon data [8–12, 15]. This method is now commonly used to determine the parton distributions inside the proton [20–22].

Since the end of the 1980s, new fixed target and collider data have been taken [23–26] which challenge the theoretical predictions. The accuracy of the data makes the scale dependence of the QCD cross sections more critical than before; new kinematical ranges are explored in which the bremsstrahlung contribution is important (figure 1c), making the calculation of HO QCD corrections to this term necessary. The introduction of an experimental isolation cone around the photon (which helps to disentangle the signal from the π^0 background) also raises several theoretical problems. In this short review, I

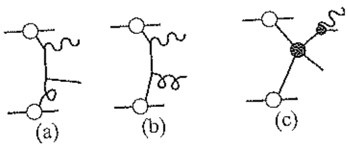

Figure 1. Compton (**a**) and annihilation (**b**) Born contributions. Bremsstrahlung contribution (**c**).

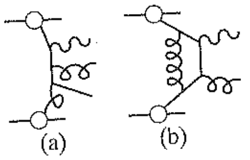

Figure 2. Example of higher order QCD diagrams: real contribution (**a**) and virtual contribution (**b**).

will discuss some theoretical developments related to these points, namely the scale problem, the calculation of HO QCD corrections to the bremsstrahlung contribution and the theoretical treatment of the isolation cone. I then will discuss recent phenomenological analyses.

2. Scale dependence of the cross section and Bresmsstrahlung contribution

A large-$p_\perp$ cross section calculated in QCD depends on two scales μ, the renormalization scale which appears in $\alpha_s(\mu)$ and the factorization scale M in the parton distribution functions. The scale dependence of the Born cross section is compensated by higher order (HO) corrections to the cross section which contain $\log(\mu/p_\perp)$ and $\log(M/p_\perp)$ terms. However the compensation is not complete (when the perturbative series stop at a finite order in α_s) and the theoretical predictions depend on these two scales.

The scale dependence of the direct photon cross section is particularly strong at low energy ($\sqrt{s} \simeq 25$ GeV) corresponding to fixed target experiments. This dependence is shown in figure 3 borrowed from ref. [27]. The optimized scales are defined by the conditions

$$M\frac{\partial\sigma}{\partial M} = \mu\frac{\partial\sigma}{\partial\mu} = 0,$$

where σ represents the inclusive cross section $E\,\mathrm{d}\sigma(h_1h_2 \to \gamma X)/\mathrm{d}\vec{p}$. Thus with the choice $\mu = \mu_{\rm opt}$ and $M = M_{\rm opt}$ the cross section is stable with respect to scale variations around $\mu_{\rm opt}$ and $M_{\rm opt}$ [28, 29]. We can see in figure 3 that the theoretical cross section varies greatly with the scales. The predictions are in good agreement with data when optimized scales are used whereas they are below the data points by some 50% with $\mu = M = p_\perp$.

Also shown in figure 3 is the result of a calculation which takes into account the effect of the primordial transverse momentum $k_\perp$ of the partons in the incident hadrons. A large $\langle k_\perp^2\rangle$ value is needed in order to bring the theoretical curve (with $\mu = M = p_\perp$) in agreement with data.

Such a smearing has been introduced by the authors of ref. [30] in order to bring theory in agreement with a large set of experimental data. In § 4 we will discuss in detail whether

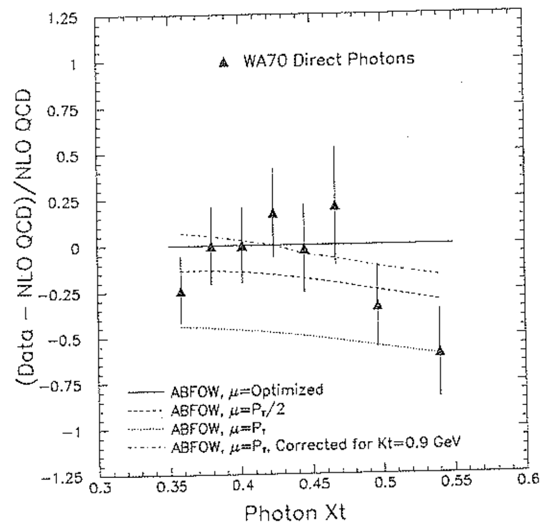

Figure 3. The WA70 [15] direct photon data is compared to NLO QCD calculations using ABFOW [13] parton distributions. Different choices of scale are shown as well as the effect of adding additional k_t broadening to the theory.

the data require the introduction of a primordial $k_\perp$ or not. Here we simply notice that this effect is not required by the WA70 data if optimized scales are used. On the other hand the smearing is evidently not a remedy to the scale dependence, but even more it introduces another parameter in the problem, namely $\langle k_\perp^2 \rangle$.

One could argue that the scale dependence of the cross section prevents the determination of the gluon distribution from direct photon data. However this is not true for experiments which measure direct photons in pp and $p\bar{p}$ collisions, because the cross section difference does not depend on the gluon distribution. This difference allows to test whether the choice of scale leads to a good fit of the data, and then the gluon can be extracted from pp data. Such a method has been used successfully by the UA6 collaboration [24].

In the preceding discussion of the curves of figure 3, HO QCD corrections to the bremsstrahlung contribution were not taken into account. This procedure is justified as long as the contribution is not important, but this is no more the case at large center-of-mass energy. The corresponding corrections have been calculated by two groups [31, 33] who considered the HO corrections to the hard process and to the fragmentation functions of quark and gluon into photons (figure 1c). The corrections to the hard process are estimated from similar corrections calculated by Aversa *et al* [32] for the reaction $h_1 + h_2 \rightarrow h_3 + X$, and the fragmentation functions are calculated by using two-loop kernels for the evolution equations.

In figure 4 which corresponds to Tevatron energy, we can see the importance of the HO corrections (dotted line) to the LO (dashed line) bremsstrahlung contribution. At $p_\perp \sim 10\,\text{GeV}$ the sum of these two contributions clearly dominates the non-bremsstrahlung contribution (full line).

The separation of the prompt photon cross section into a bremsstrahlung and a non-bremsstrahlung part is of course scale dependent. In order to study this scale dependence,

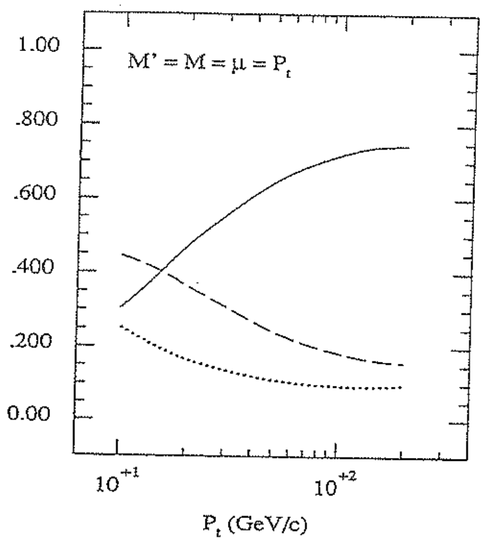

Figure 4. Comparison of the different partial contributions to the inclusive cross-section $d\sigma/dp_t\, dy|_{y=0}$ at the Tevatron, normalized to the total NTLO cross-section. Full line: NTLO (Born + HO) 'internal' contribution. Dashed line: LO bremsstrahlung contribution. Dotted line : HO bremsstrahlung contribution.

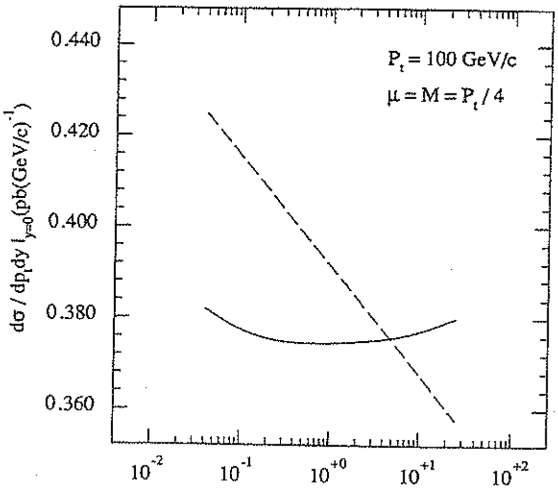

Figure 5. M'-dependence of the inclusive photon cross-section $d\sigma/dp_t\, dy|_{y=0}$, for $M = \mu = p_t/4$. Full line: total cross-section. Dashed line: NTLO (Born + HO) internal + LO bremsstrahlung contribution.

let us fix the renormalization scale μ and the factorization scale M associated with the intial state at $p_\perp/4$, and vary the factorization scale M' of the fragmentation functions. Again for the Tevatron energy, we show in figure 5 the effect of this variation. The totally corrected cross section is much more stable than the cross section with no HO bremsstrahlung corrections and has a stability point around $M'^2 \simeq p_\perp^2$.

Unfortunately the above predictions cannot be directly compared with Tevatron data [24, 26]. Indeed the CDF and DO collaborations measure isolated photon cross sections. An isolated photon is defined by a cone of half-angle $\delta/2$ around the photon containing less hadronic energy than a fraction of the photon energy: $E_h \leq \varepsilon_h E_\gamma$ ($\varepsilon_h \sim 1$). This cone, which is necessary to disentangle the π^0 background from the direct photon signal at large $p_\perp$, deeply changes the theoretical treatment of the direct photon production. The effect of the HO corections to the bremsstrahlung contribution have also been studied by Gordon [33]. An approximate treatment of the photon isolation is used by this author to compare the theoretical predictions with Tevatron data.

3. Isolated cross sections

In order to understand the effect of the isolation cone on the direct photon cross section, first let us consider the lowest order diagram for the final state (figure 6). To calculate the inclusive cross section without isolation we integrate over $\vec{p}\,'$ and obtain the following result in the leading logarithm (LL) approximation (m^2 is a collinear regularization):

$$\sigma_\gamma^{\rm incl} \sim \sigma_p \otimes e_q^2 \frac{\alpha}{2\pi} P_{\gamma q} \log \frac{p_{\perp\gamma}^2}{m^2} = \sigma_p \otimes D_\gamma^{(0)}(p_\perp^2) \tag{1}$$

with $P_{\gamma q}(z) = (1 + (1-z)^2)/z$, σ_p being the large-$p_\perp$ quark cross section. The symbol $\otimes$ indicates a convolution $(g \otimes f)(z) = \int {\rm d}x\, g(x) \int {\rm d}y\, f(y) \delta(xy - z)$. If we require the final quark to be in a cone of half-opening $\delta/2$, we obtain [34]

$$\sigma_\gamma^{\rm cone} \sim \sigma_p \otimes e_q^2 \frac{\alpha}{2\pi} P_{\gamma q} \log \left[\left(\frac{\delta}{2}\right)^2 \frac{p_{\perp\gamma}^2}{m^2} \right]. \tag{2}$$

At this order, the isolated cross section with no quark in the cone (the condition $E_h \leq \varepsilon_h E_\gamma$ will be discussed below) is given by

$$\sigma_\gamma^{\rm isol} = \sigma_\gamma^{\rm incl} - \sigma_\gamma^{\rm cone} \sim \sigma_p \otimes e_q^2 \frac{\alpha}{2\pi} P_{\gamma q} \log \left(\frac{2}{\delta}\right)^2. \tag{3}$$

The effect of the gluon emission by the quark lines of momentum p and p' can be taken into account and resummed to all orders, in the inclusive case (no cone), through the Altarelli–Parisi evolution equations. The lowest order fragmentation function of (1) is then replaced by a fragmentation function $D_\gamma(p_\perp^2, z)$ calculated to all orders in α_s in the LL approximation [35, 36] or beyond LL approximation [31, 37].

The situation is totally different in the presence of an isolation cone. The phase space restriction put by the cone condition prevents an exact cancellation of infrared sensitive terms between the real and virtual contributions; the cross section contains double

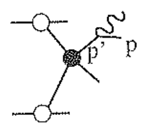

Figure 6. Lowest order bremsstrahlung diagram.

logarithms of the isolation parameters. The effects of the isolation cone have been studied at order $\mathcal{O}(\alpha\alpha_s)$ in e^+e^- annihilation by Berger, Guo and Qiu [38, 39]. These authors concluded that the conventional factorisation of the collinear singularities is not valid. This affirmation has been criticized, in ref. [40], and recently the authors of ref. [41] have demonstrated the factorization of collinear singularities is valid to all orders in α_s.

To understand the problem, let us first consider in e^+e^- annihilation the final state $q(p_1) + \bar{q}(p_2) + \gamma(p_0)$ and the cross section $\mathrm{d}\sigma(e^+e^- \to \gamma X)/\mathrm{d}x_\gamma$ where we define $x_\gamma = 2p_0 \cdot Q/Q^2$. The cross section is obtained by integrating over the available phase space for p_1, p_2 and p_0. The configuration $\vec{p}_0//\vec{p}_1$ produces a collinear singularity (a pole in ε in dimensional regularization) and we have

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}x_\gamma} &= \sigma_0\left(-\frac{1}{\varepsilon}\frac{\alpha}{2\pi}\frac{1+(1-x_\gamma)^2}{x_\gamma}\right) + \text{non singular terms}\\ &= \sigma_0 \cdot D_q^{(0)\gamma}(x_\gamma, \varepsilon) + \text{non singular terms} \qquad (4)\end{aligned}$$

with the order $O(\alpha)$ fragmentation function $D_q^\gamma(x_\gamma, \varepsilon)$. σ_0 is the Born $e^+e^- \to q\bar{q}$ cross-section. The energy condition is written $x_1 \le \varepsilon_h x_\gamma$ on $x_1 + x_\gamma \le (\varepsilon_h + 1)x_\gamma \equiv x_\gamma/x_c$ where we define $x_c = 1/(1+\varepsilon_h)$. In the collinear configuration $x_1 + x_\gamma = 1$ and the isolation condition is verified if $x_\gamma \ge x_c$.

QCD corrections to this Born process come from the emission of a real gluon and from the $\mathcal{O}(\alpha_s)$ virtual contributions. The final state of the real process is $q(p_1) + \bar{q}(p_2) + g(p_3) + \gamma(p_0)$ and we consider the collinear configuration leading to expression (4), namely with $\vec{p}_0//\vec{p}_1$. Therefore we are led to study the subprocess $e^+e^- \to q(p_1') + \bar{q}(p_2) + g(p_3)$ where $p_1' = p_0/x_\gamma$. The energy condition when the gluon is in the cone is given by

$$x_1 + x_3 + x_\gamma \le x_\gamma/x_c \quad \text{or} \quad x_1' + x_3 \le x_\gamma/x_c. \qquad (5)$$

We see that this condition (5) can be verified only if $x_\gamma \ge x_c$ (remember that $x_1' + x_3 + x_2 = 2$ and $x_1' + x_3 \ge 1$). For $x_\gamma < x_c$ the gluon must be outside the cone. Therefore we suddenly change the isolation condition by varying x_γ around x_c and we expect some singular behaviour of the isolated cross-section for $x_\gamma = x_c$.

For $x_\gamma \ge x_c$, we can integrate over the phase space for p_3 except for a small domain within the cone where the condition (5) is violated. (Notice that (5) is always verified in the collinear configuration $\vec{p}_3//\vec{p}\,'_1$). The integral over p_3 in the whole phase space leads to a collinear singularity ($\vec{p}_3//\vec{p}\,'_1$) and to infrared singularities ($\vec{p}_3 \to 0$). The infrared singularities are cancelled by the virtual contributions and the collinear singularity is factorised into the fragmentation function. In this way we build the order $O(\alpha\alpha_s)$ fragmentation function $D_q^{(1)}(x_\gamma, \varepsilon)$. It remains a nonsingular part from which we have to remove the contribution coming from the forbidden small domain. This contribution which is singular when $x_\gamma \to x$ is given by (we define $r_\gamma = x_\gamma/x_c$)

$$\begin{aligned}\mathrm{d}\sigma_{s,i}/\mathrm{d}x_1' = C_F\Bigg\{&\delta(1-x_1')\left[-\ln^2\left(\frac{(1-r_\gamma)}{\tan^2(\delta/2)}\right) - \frac{3}{2}\ln\left(\frac{r_\gamma - 1}{\tan^2(\delta/2)}\right)\right]\\ &+ \left(\frac{1+x_1'^2}{1-x_1'}\right)_+ \ln\left(\frac{r_\gamma - 1}{\tan^2(\delta/2)}\right)\Bigg\} + \mathcal{O}(1). \qquad (6)\end{aligned}$$

For $x_\gamma < x_c$, the gluon must be outside the cone and we obtain (also for $x'_1 \to 1$)

$$\begin{aligned} \mathrm{d}\sigma^{(1)}_{s,0}/\mathrm{d}x'_1 = & C_F\delta(1-x'_1)[\ln^2((1-r_\gamma)\tan^2(\delta/2)) \\ & + \tfrac{3}{2}\ln((1-r_\gamma)\tan^2(\delta/2))] + \mathcal{O}(1). \end{aligned} \tag{7}$$

These two singular contribution must be convoluted with the fragmentation function and the direct photon cross-section is given by

$$\begin{aligned} \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}x_\gamma} = & \int \mathrm{d}x'_1(\delta(1-x'_1) + \mathrm{d}\sigma^{(1)}_s/\mathrm{d}x'_1) \\ & \int \mathrm{d}z(D^{(0)\gamma}_q(z,\varepsilon) + D^{(1)\gamma}_q(z,\varepsilon))\delta(x'_1 z - x_\gamma) + O(\alpha\alpha_s^2), \end{aligned} \tag{8}$$

where we have written $\mathrm{d}\sigma^{(1)}_s = \mathrm{d}\sigma^{(1)}_{s,0} + \mathrm{d}\sigma^{(1)}_{s,i}$. The collinear divergences are factorised in the fragmentation function $D^\gamma_q(z,\varepsilon)$ and the hard subprocess contains logarithmic, integrable singularities when $x_\gamma \to x_c$.

A formal proof of this factorization can be found in ref. [41] with a detailed discussion of the singularites in x_γ. One must notice that they occur inside the physical region avaiable for x_γ: $0 < x_\gamma < 1$; this phenomenon has been studied in general in ref. [42].

In the hadronic production of direct photons, x_γ is not fixed. There is another convolution in the cross-section definition (with the initial parton distributions) which makes smoother the logarithmic divergences.

4. The WA70-E706 puzzle

In this section I shall concentrate on the analysis of fixed targent data, namely the CERN WA70 data [15] obtained at $\sqrt{s} = 23$ GeV and the FNAL E706 data [43] corresponding to $\sqrt{s} = 31$ GeV and 38 GeV. These two fixed target experiments observed direct photons in pp and $\pi^- p$ collisions. There is no isolation cone around the photon. Here I do not mention Tevatron, UA2 and ISR result which have been discussed in ref. [44, 45].

The study of these fixed target data are interesting, because the FNAL experiment is in strong disagreement with the theoretical predictions. This fact leads the authors of ref. [43] to introduce a primordial transverse momentum $k_\perp$ of the partons in the incoming hadrons in order to bring theory in agreement with the experimental results. But such a $k_\perp$-kick is not necessary to explain the WA70 data. Therefore a new theoretical study of fixed target data has been performed [46] in order to clarify the situation. The new predictions take into account HO QCD corrections to the bremsstrahlung contribution (§ 2) and use a beyond leading order parametrization of the parton in photon fragmentation functions [47]. The parton distributions in the incoming proton and pion are those of the ABFOW [13] and ABFKW [48] collaborations; they correspond to $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 230$ MeV.

Because of the disagreement in normalization between the E706 data and theory, we first present results obtained for ratios of cross-sections. Indeed we expect that a part of the theoretical uncertainties cancels in such ratios. The sensitivity to the scales, to the fragmentation functions, to the primordial $k_\perp$ if any should decrease when ratios of cross-sections are considered. The results obtained for the ratio $R_\gamma = E(\mathrm{d}\sigma/\mathrm{d}\vec{p})(\pi^- p \to \gamma X)/E(\mathrm{d}\sigma/\mathrm{d}\vec{p})(pp \to \gamma X)$ are shown in figure 7 for the WA70 and E706 data; we can see that the agreement is good for both experiments.

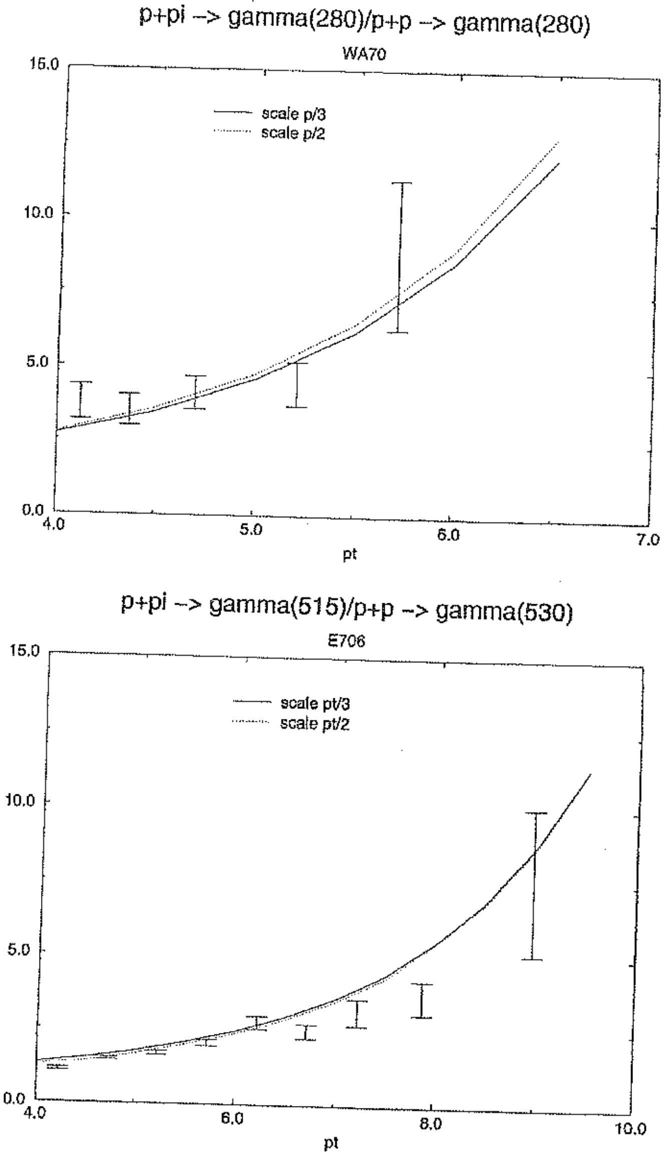

Figure 7. Theoretical predictions and data for the ratio R_γ.

We now turn to the cross sections. In figure 8 we display the results for the $\pi^- p \to \gamma X$ cross sections. Similar results are obtained for the $pp \to \gamma X$ reaction. We immediately notice a difference between the WA70 and the E706 data. Whereas the former are in good agreement with theory, the latter are a factor 2–3 above predictions in the whole $p_\perp$-range. The agreement with WA70 data is expected, because these results have been used to determine the gluon in the proton [13] and pion [48].

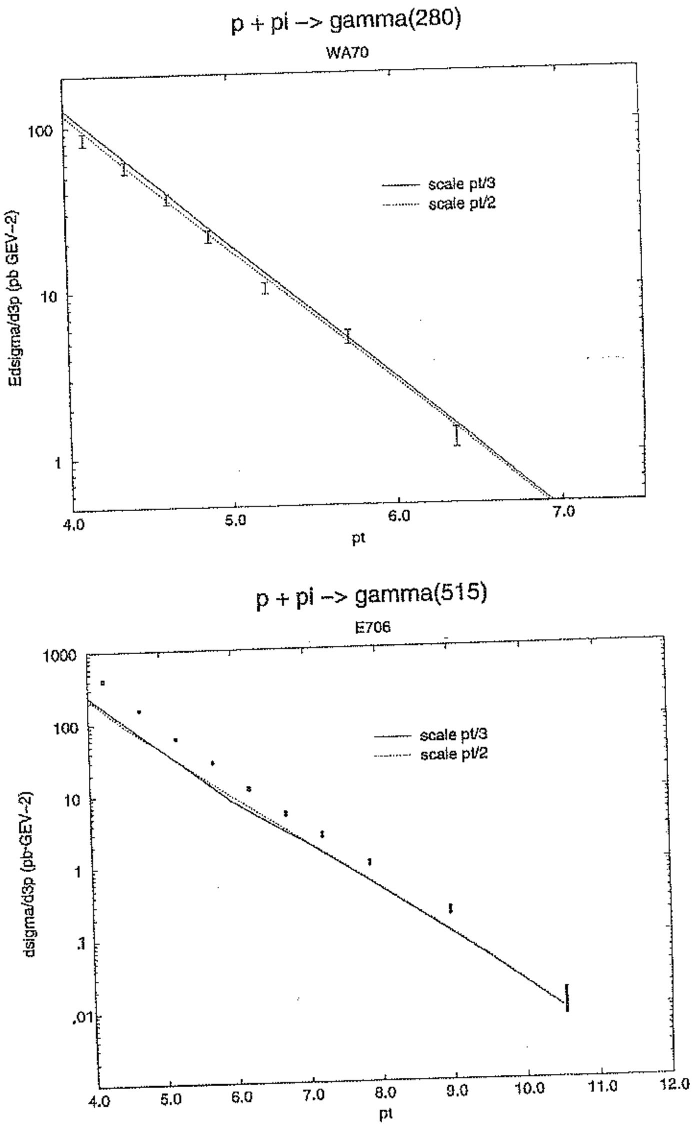

Figure 8. Direct photon cross-sections compared with WA70 and E706 data.

This determination is compatible with jet cross-sections at Tevatron and adjusting the gluon distribution to fit E706 data appears to be a difficult exercise. In any case the theoretical prediction allows us to compare WA70 and E706 data (assuming that no new phenomena will change the prediction when we pass from $\sqrt{s} = 23$ GeV to $\sqrt{s} = 31$ GeV) and to underline an incompatibility between these two experiments. The solution proposed by the E706 collaboration consists in introducing the effect of the primordial transverse momentum $k_\perp$ of the ingoing partons which must be large enough to enhance the cross sections and bring them in agreement with experiment. But it is clear that this effect will also bring the theoretical predictions above the WA70 data, unless $\langle k_\perp \rangle$ strongly decreases when going from the E706 center of mass energy to the WA70 CM energy.

5. Conclusion

There is no strong evidence in favor of a large primordial $k_\perp$ from Tevatron [45] or fixed target experiments. However the disagreement between the WA70 and E706 data is puzzling. Were the authors of ref. [43, 49] right, the need for a large $k_\perp$ effect would open a new field of research at the border between perturbative and nonperturbative QCD; the theoretical description of the $k_\perp$-kick would become an important goal. Future UA6 results [50] on $pp \to \gamma X$ and $p\bar{p} \to \gamma X$ reactions should allow us to clarify the situation.

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