

Charmonium production at high-energy colliders

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Abstract. I review inclusive charmonium production at the high-energy colliders Tevatron and HERA.

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1. Introduction

The production of charmonium states at high-energy colliders has been the subject of considerable interest during the past few years. New experimental results from $p\bar{p}$, ep and e^+e^- colliders have become available, some of which revealed dramatic shortcomings of earlier model approaches to charmonium production like the colour-singlet model [1]. In theory, a consistent and rigorous framework for inclusive quarkonium production and decays has been developed by Bodwin, Braaten and Lepage [2], superseding the colour-singlet model. The so-called factorization approach is based on the use of non-relativistic QCD (NRQCD) to separate the short-distance physics of heavy quark creation from the long-distance physics of bound state formation. It explicitly takes into account the complete structure of the quarkonium Fock space. The inclusive cross section for the production of a charmonium state H can be expressed as a sum of terms, each of which factors into a short-distance coefficient and a long-distance matrix element:

$$d\sigma(H + X) = \sum_n d\hat{\sigma}(c\bar{c}[n] + X) \langle \mathcal{O}^H[n] \rangle, \quad (1)$$

where n denotes the colour, spin and angular momentum state of an intermediate $c\bar{c}$ pair. The short-distance cross section $d\hat{\sigma}$ can be calculated perturbatively in the strong coupling $\alpha_s(m_c)$. The NRQCD matrix elements $\langle \mathcal{O}^H[n] \rangle \equiv \langle 0 | \mathcal{O}^H[n] | 0 \rangle$ (see [2] for their definition) are related to the non-perturbative transition probabilities from the $c\bar{c}$ state n into the charmonium H . The relative importance of the various terms in (1) can be estimated by using NRQCD velocity scaling rules [3]. For small average velocities v of the heavy quark in the quarkonium rest frame ($v^2 \sim 0.25$ for charmonium) each of the NRQCD matrix elements scales with a definite power of v and the general expression (1) can be organized into an expansion in powers of the heavy quark velocity. The sum (1) is thus a double expansion in $\alpha_s(m_c)$ and v .

The NRQCD formalism implies that so-called colour-octet processes associated with higher Fock state components of the charmonium wave function must contribute to the

cross section. Charm quark pairs that are produced at short distances in a colour-octet state can evolve into a physical charmonium through radiation of soft gluons at late times in the production process, when the charm pair has already expanded to the charmonium size. Such a possibility is ignored in the colour-singlet model where only those charm quark pairs are assumed to form a physical quarkonium that are produced in the dominant Fock state, i.e. in a colour-singlet state and with the spin and angular momentum quantum numbers of the meson. The most profound theoretical evidence that the colour-singlet model is incomplete comes from the presence of infrared divergences in the production cross sections and decay rates for P -wave states. Within the NRQCD approach, this problem finds its natural solution since the infrared singularities are factored into a colour-octet operator matrix element [4]. While colour-octet contributions are needed for a consistent description of P -wave quarkonia, they are phenomenologically even more important for S -wave states like J/ψ or ψ' . According to the velocity scaling rules, colour-octet matrix elements for the production of S -wave charmonia are suppressed by a factor v^4 compared to the leading colour-singlet contributions. However, colour-octet processes can become significant if the short-distance cross section for producing $c\bar{c}$ in a colour-octet state is enhanced. As discussed in detail in the next section, gluon fragmentation into colour-octet charm pairs appears as the most plausible explanation of the large direct J/ψ and ψ' cross sections observed at the Tevatron. The range of applicability of the NRQCD approach to the practical case of charmonium is however still subject to debate, as is the quantitative verification of factorization. Because the charmonium mass is still not very large with respect to the QCD scale, non-factorizable corrections may not be suppressed enough, if the charmonium is not part of an isolated jet, and the expansions in NRQCD may not converge very well. In this situation cross checks between various processes, and predictions of observables such as charmonium polarization and differential cross sections, are crucial in order to assess the importance of different charmonium production mechanisms, as well as the limitations of a particular theoretical approach. In the following, I will focus on the phenomenological implications of the NRQCD approach for J/ψ production at the high-energy colliders Tevatron and HERA.

2. Charmonium production at the Tevatron

The production of S -wave charmonium in $p\bar{p}$ collisions at the Tevatron has attracted considerable attention and has stimulated much of the recent theoretical development in quarkonium physics. The CDF collaboration has measured cross sections for the production of J/ψ and ψ' states not coming from B or radiative χ decays, for a wide range of transverse momenta $5 \text{ GeV} \lesssim p_t(\psi) \lesssim 20 \text{ GeV}$ [5]. Surprisingly, the experimental cross sections were found to be orders of magnitudes larger than the theoretical expectation based on the leading-order colour-singlet model [6]. This result is particularly striking because the data extends out to large transverse momenta where the theoretical analysis is rather clean.

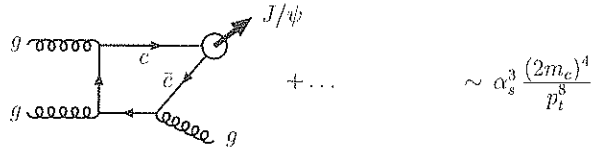
The shortcoming of the colour-singlet model can be understood by examining a typical Feynman diagram contributing to the leading-order parton cross section, figure 1(a). At large transverse momentum, the two internal quark propagators are off-shell by $\sim p_t^2$ so that the parton differential cross section scales like $d\sigma/dp_t^2 \sim 1/p_t^8$, as indicated in the

figure. On the other hand, when $p_t \gg 2m_c$ the quarkonium mass can be considered small and the inclusive charmonium cross section is expected to scale like any other single-particle inclusive cross section $\sim 1/p_t^4$. It was pointed out by Braaten and Yuan [7] that the dominant production mechanism for charmonium at sufficiently large p_t must be via fragmentation, the production of a parton with large p_t which subsequently decays into charmonium and other partons. A typical fragmentation contribution to colour-singlet J/ψ production is shown in figure 1(b). While the fragmentation contributions are of higher order in α_s compared to the fusion process figure 1(a), they are enhanced by a power $p_t^4/(2m_c)^4$ at large p_t and can thus overtake the fusion contribution at $p_t \gg 2m_c$. When colour-singlet fragmentation is included, the p_t dependence of the theoretical prediction is in agreement with the Tevatron data but the normalization is still underestimated by about an order of magnitude [8], indicating that an additional fragmentation contribution is still missing. The crucial step was taken by Braaten and Fleming [9], who argued that gluon fragmentation into colour-octet 3S_1 charm quark pairs, as shown in figure 1(c), should be the dominant source of J/ψ and ψ' at large p_t at the Tevatron. The probability of forming a J/ψ particle from a point like $c\bar{c}$ pair in a colour-octet 3S_1 state is given by the NRQCD matrix element $\langle \mathcal{O}^{J/\psi}[^3S_1^{(8)}] \rangle$ which is suppressed by v^4 relative to the non-perturbative factor of the leading colour-singlet term. However, this suppression is overcompensated for by the gain in two powers of α_s/π in the short-distance cross section for producing colour-octet 3S_1 charm quark pairs as compared to colour-singlet fragmentation. At $\mathcal{O}(v^4)$ in the velocity expansion, two additional colour-octet channels have to be included, figure 1(d), which do not have a fragmentation interpretation at order α_s^3 but which become significant at moderate $p_t \sim 2m_c$ [10]. The importance of the $^1S_0^{(8)}$ and $^3P_J^{(8)}$ contributions cannot be estimated from naive power counting in α_s and v alone, but rather follows from the dominance of t -channel gluon exchange, forbidden in the leading-order colour-singlet cross section.

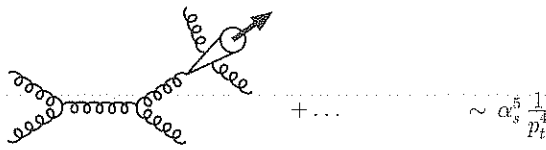
The different contributions to the J/ψ transverse momentum distribution are compared to the CDF data [5] in figure 2. As mentioned above, the colour-singlet model at lowest order in α_s fails dramatically when confronted with the experimental results. When colour-singlet fragmentation is included, the prediction increases by more than an order of magnitude at large p_t , but it still falls below the data by a factor of ~ 30 . The CDF results on charmonium production can be explained by including the leading colour-octet contributions and adjusting the unknown non-perturbative parameters to fit the data. Because the $^1S_0^{(8)}$ and $^3P_J^{(8)}$ short-distance cross sections have a similar p_t dependence, the transverse momentum distribution is sensitive only to a certain linear combination of the corresponding non-perturbative matrix elements. Using heavy-quark spin symmetry to relate the $\langle \mathcal{O}_8^{J/\psi}(^3P_J) \rangle$ matrix elements, the theoretical prediction then depends on two phenomenological parameters only. They are found to be [11] $\langle \mathcal{O}_8^{J/\psi}[^3S_1] \rangle \approx 1.1 \cdot 10^{-2} \text{ GeV}^3$ and $\langle \mathcal{O}_8^{J/\psi}(^1S_0) \rangle + 3.5 \langle \mathcal{O}_8^{J/\psi}(^3P_0) \rangle / m_c^2 \approx 4.4 \cdot 10^{-2} \text{ GeV}^3$. For the colour-octet explanation of the large charmonium cross section at the Tevatron to be viable, the corresponding non-perturbative matrix elements must satisfy an important consistency check. They must be small enough to be compatible with the v^4 suppression relative to the non-perturbative factor of the leading colour-singlet term. Numerically one finds $\langle \mathcal{O}_1^{J/\psi}[^3S_1] \rangle / \langle \mathcal{O}_8^{J/\psi}[^3S_1] \rangle \sim 10^{-2}$, perfectly consistent with the suppression expected from the velocity scaling rules. A rather striking and important conclusion has to be drawn from this analysis: even if the probability for an intermediate colour-octet $c\bar{c}$ pair to evolve into a J/ψ particle is only

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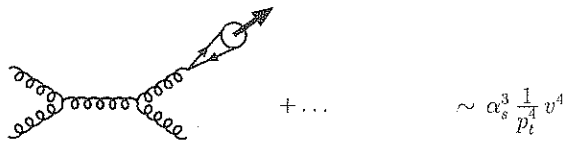
(a) leading-order colour-singlet: $g + g \rightarrow c\bar{c}[{}^3S_1^{(1)}] + g$



(b) colour-singlet fragmentation: $g + g \rightarrow [c\bar{c}[{}^3S_1^{(1)}] + gg] + g$



(c) colour-octet fragmentation: $g + g \rightarrow c\bar{c}[{}^3S_1^{(8)}] + g$



(d) colour-octet t -channel gluon exchange: $g + g \rightarrow c\bar{c}[{}^1S_0^{(8)}, {}^3P_J^{(8)}] + g$

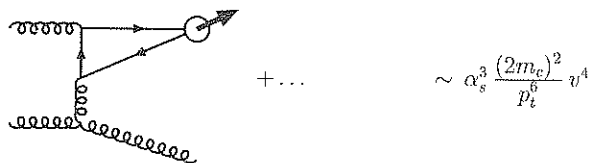


Figure 1. Generic diagrams for J/ψ production in hadron-hadron collisions via colour-singlet and colour-octet channels.

about 1% or less of the corresponding probability for a colour-singlet $c\bar{c}$ pair, the phenomenological consequences of colour-octet contributions may be dramatic. The reason is that the perturbative cross section for producing colour-octet $c\bar{c}$ pairs at short-distances can be orders of magnitude larger than those for producing colour-singlet charm quark pairs. Similar conclusions can be drawn for ψ' production at the Tevatron.

The precise numerical values for the non-perturbative matrix elements obtained from the fit to the Tevatron data are however subject to large theoretical uncertainties. In particular the determination of the $\langle O_8^{J/\psi}({}^1S_0) \rangle$ and $\langle O_8^{J/\psi}({}^3P_0) \rangle$ matrix elements is very

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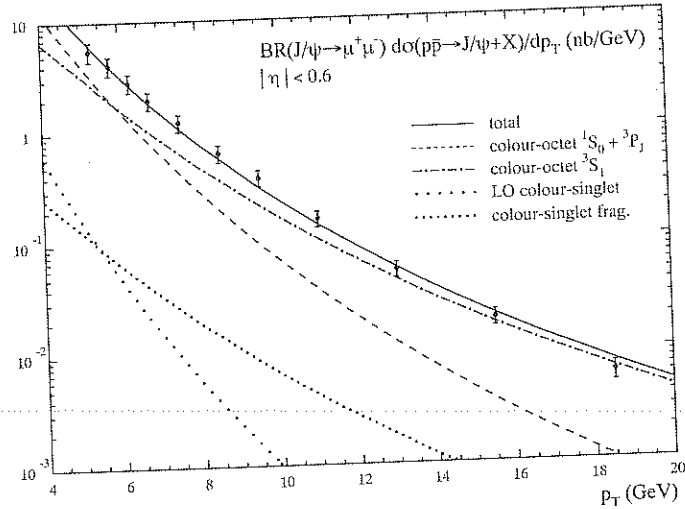


Figure 2. Colour-singlet and colour-octet contributions to direct J/ψ production in $p\bar{p} \rightarrow J/\psi + X$ at the Tevatron ($\sqrt{s} = 1.8$ TeV) [11] together with experimental data from CDF [5].

sensitive to all effects that modify the shape of the J/ψ transverse momentum distribution at relatively small transverse momentum $p_t \lesssim 8$ GeV. The theoretical uncertainties include the small- x behaviour of the gluon distribution [11], the evolution of the strong coupling [11], next-to-leading order contributions, intrinsic transverse momentum of the partons in the proton [12], and also systematic effects inherent in NRQCD, such as the inaccurate treatment of the energy conservation in the hadronization of the colour-octet $c\bar{c}$ pairs [13]. First studies of the above mentioned effects indicate that the determination of $\langle O_8^{J/\psi} [^3S_1] \rangle$ is accurate within a factor of two while $\langle O_8^{J/\psi} (^1S_0) \rangle$ and $\langle O_8^{J/\psi} (^3P_0) \rangle$ may be overestimated considerably.

The discussion so far, although very encouraging, does not strictly prove the phenomenological relevance of colour-octet contributions because free parameters had to be introduced to fit the data. However, if factorization holds quantitatively, the non-perturbative matrix elements are universal and can be used to make predictions for various observables and reactions. A global analysis of different production processes, like J/ψ production at HERA, is one way to test the universality of the colour-octet contributions, and will be discussed in the next section.

The single most important test of the NRQCD approach to be done in the near future however is the analysis of J/ψ polarization at large transverse momentum at the Tevatron. Recall that at large p_t , J/ψ production should be dominated by gluon fragmentation into a colour-octet 3S_1 charm quark pair, figure 1(c). When $p_t \gg 2m_c$ the fragmenting gluon is effectively on-shell and transverse. The intermediate $c\bar{c}$ pair in the colour octet 3S_1 state inherits the gluon's transverse polarization and so does the J/ψ , because the emission of soft gluons during hadronization does not flip the heavy quark spin at leading order in the velocity expansion. Consequently, at large transverse momentum one should observe transversely polarized J/ψ [14]. The polarization can be measured through the angular

distribution in the decay $J/\psi \rightarrow l^+l^-$, given by $d\Gamma/d\cos\theta \propto 1 + \lambda \cos^2\theta$, where θ denotes the angle between the lepton three-momentum in the J/ψ rest frame and the J/ψ three-momentum in the lab frame. Pure transverse polarization implies $\lambda = 1$. Corrections to this asymptotic limit due to spin-symmetry breaking and higher order fragmentation contributions have been estimated to be small [15]. The dominant source of depolarization comes from the colour-octet fusion diagrams, figure 1(d), which are important at moderate p_t . Still, at $\mathcal{O}(v^4)$ in the velocity expansion, the polar angle asymmetry λ can be unambiguously calculated within NRQCD in terms of the three non-perturbative matrix elements that have been determined from the unpolarized cross section [11, 16]. Figure 3 displays the theoretical prediction for λ as function of the J/ψ transverse momentum. No transverse polarization is expected at $p_t \sim 5$ GeV, but the angular distribution is predicted to change drastically as p_t increases. Observation of such a pattern would provide strong support for the gluon fragmentation mechanism and heavy-quark spin symmetry as predicted by the NRQCD approach. On the other hand, if no substantial fraction of transversely polarized J/ψ is observed at large p_t , NRQCD is not applicable at the charm mass scale. The polarization analysis of Tevatron data is under way and experimental results are expected to become available soon.

Furthermore, polarization measurements are the most powerful way to discriminate the NRQCD approach from the so-called ‘colour evaporation model’ [17], an early approach to charmonium production which has been revived recently [18]. In the colour evaporation model the cross section for a specific charmonium state is given as a universal fraction of the inclusive $c\bar{c}$ production cross section integrated up to the open charm threshold. In general, the assumption of a single universal long-distance factor

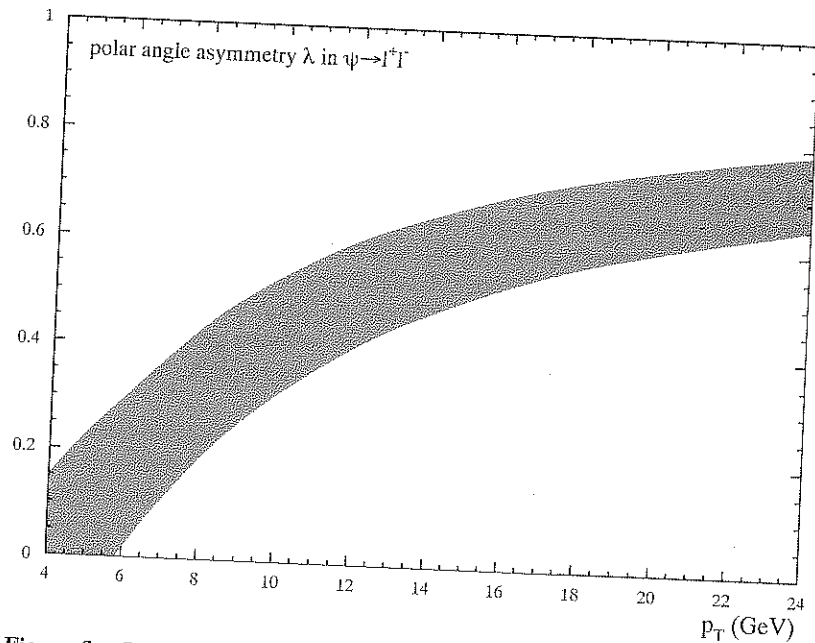


Figure 3. Polar angle asymmetry λ as a function of p_t in direct J/ψ production $p\bar{p} \rightarrow J/\psi(\rightarrow l^+l^-) + X$ at the Tevatron ($\sqrt{s} = 1.8$ TeV) [11].

is too restrictive. It implies a universal $\sigma(\chi_c)/\sigma(J/\psi)$ ratio, which is not supported by the comparison of charmonium production in fixed-target hadron collisions and photoproduction. Still, since the colour evaporation model allows colour-octet charm quark pairs from gluon fragmentation to hadronize into charmonium, it can describe the p_t distribution of the Tevatron data. In contrast to the NRQCD approach, however, the colour evaporation model predicts charmonium to be always produced unpolarized. The model assumes unsuppressed gluon emission from the $c\bar{c}$ pair during hadronization which randomizes spin and colour. This assumption is clearly wrong in the heavy quark limit where spin symmetry is at work and soft gluon emission does not flip the heavy quark spin. Nonetheless, since the charm quark mass is not very large with respect to the QCD scale, the applicability of heavy quark spin symmetry to charmonium physics has to be tested by confronting the NRQCD polarization signature with experimental data.

3. Charmonium production at HERA

The analysis of J/ψ production in high energy ep collisions at HERA appears to be a powerful tool to constrain the colour-octet matrix elements and to test the picture of quarkonium production as developed in the context of the NRQCD factorization approach.

The production of charmonium at HERA is dominated by photoproduction events where the electron is scattered by a small angle producing photons of almost zero virtuality. The cross section is dominated by photon-gluon fusion, where the photon interacts directly with a gluon from the proton. Besides the direct photoproduction channel, J/ψ production at HERA can also take place via resolved photon reactions where the photon behaves as a source of partons which interact with the partons in the proton. Resolved processes are expected to contribute significantly to the lower endpoint of the J/ψ energy spectrum and might be probed at HERA for the first time in the near future. Higher-twist phenomena like diffractive charmonium production [19] are not included in the leading-twist calculation of NRQCD and have to be eliminated by either a cut in the J/ψ transverse momentum $p_t \gtrsim 1$ GeV, or by a cut in the J/ψ energy variable $z \equiv p_p \cdot k_\psi / p_p \cdot k_\gamma \lesssim 0.9$ (in the proton rest frame, z is the ratio of the J/ψ to γ energy, $z = E_\psi/E_\gamma$).

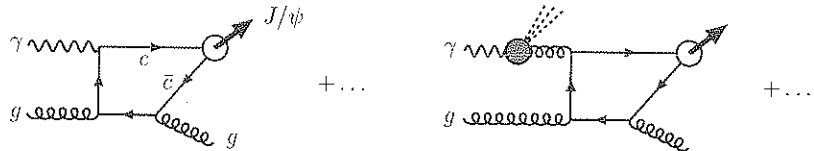
In the following, I assume that the J/ψ transverse momentum $p_t > 1$ GeV, in order to suppress the diffractive contribution and higher-twist corrections in general. The leading-twist parton reactions contributing to inelastic J/ψ production away from $p_t = 0$ (or $z = 1$) are shown in figure 4. At leading order in the velocity expansion J/ψ is produced through the colour singlet channel [1], figure 1(a). Relativistic corrections at $\mathcal{O}(v^2)$ affect the small- p_t region [20] but can be neglected for $p_t \gtrsim 1$ GeV. At $\mathcal{O}(v^4)$ relative to the colour-singlet contribution, the J/ψ can also be produced through intermediate colour-octet 3S_1 , 1S_0 and 3P_J configurations, figure 4(b) [21, 22]. Fragmentation contributions in photon-gluon fusion exist only at the next order in α_s and are suppressed for $p_t \lesssim 10$ GeV [23]. The ${}^3S_1^{(8)}$ -channel is insignificant for the direct photon contribution but dominates in resolved photon interactions at $p_t \gtrsim 5$ GeV because it includes a gluon fragmentation component. The resolved photon amplitudes are identical to those relevant to J/ψ production in hadron-hadron collisions and at HERA energies the relative importance of the various contributions as functions of p_t is nearly the same as at Tevatron energies.

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(a) leading-order colour-singlet:

direct γ : $\gamma + g \rightarrow c\bar{c}[{}^3S_1^{(1)}] + g$

resolved γ : $g_\gamma + g \rightarrow c\bar{c}[{}^3S_1^{(1)}] + g$



(b) inelastic colour-octet:

direct γ : $\gamma + g \rightarrow c\bar{c}[{}^1S_0^{(8)}, {}^3P_J^{(8)}] + g$

resolved γ : $g_\gamma + g \rightarrow c\bar{c}[{}^3S_1^{(8)}] + g$

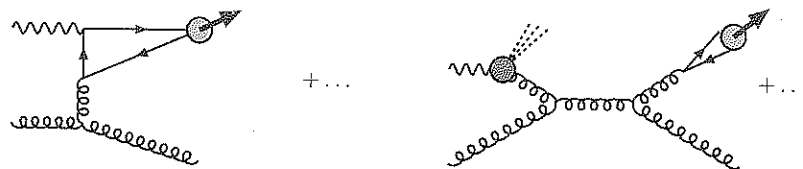


Figure 4. Generic diagrams for J/ψ production in photon-hadron collisions via colour-singlet and colour-octet channels.

The J/ψ energy distribution appears to be particularly sensitive to the different production mechanisms. The colour-singlet and colour-octet contributions to the differential cross section via direct and resolved photon processes are shown in figure 5 [24]. The results are shown as a function of the scaling variable $z = p_\psi \cdot p_p / p_\gamma \cdot p_p$ at a typical HERA energy of $\sqrt{s_{\gamma p}} = 100$ GeV in the restricted range $p_\perp > 1$ GeV. Adopting the NRQCD matrix elements as determined from the Tevatron J/ψ data (see previous section), colour-octet processes are predicted to exceed the colour-singlet contribution by more than an order of magnitude for $z \gtrsim 0.6$ and $z \lesssim 0.2$. Resolved photon interactions are negligible for $z \gtrsim 0.3$, but colour-octet terms are expected to become significant at the lower end of the energy spectrum [25]. The strong colour-octet enhancement at small and large z is a consequence of t -channel gluon exchange in the ${}^1S_0^{(8)}$ and ${}^3P_J^{(8)}$ processes, analogous to ψ production at moderate p_t at the Tevatron. The theoretical predictions are compared to experimental results obtained by the H1 [26] and ZEUS [27] collaborations at HERA. From figure 5 one can conclude that the large colour-octet enhancement in the region $z \gtrsim 0.6$ is not supported by the experimental data. The current experimental results can be described by the colour-singlet channel alone, once the large normalization uncertainty of the colour-singlet cross section due to the choice for $\langle \mathcal{O}_1^{J/\psi}[{}^3S_1] \rangle^1$, the

¹ The colour-singlet matrix element is obtained from the J/ψ wave function at the origin as calculated in QCD-motivated potential models. The reliability of this procedure for the case of charmonium has been called into question recently [28].

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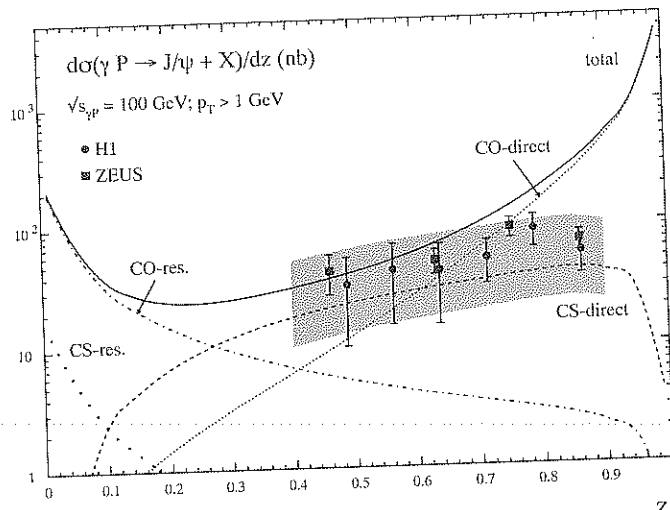


Figure 5. Colour-singlet (CS) and colour-octet (CO) contributions to the J/ψ energy distribution in $\gamma p \rightarrow J/\psi + X$, compared to experimental data [26, 27]. Both direct and resolved photon processes are included. For details see [24].

charm quark mass and the QCD coupling are taken into account, as indicated by the shaded band (see [24, 29] for details).

It is however premature to interpret the discrepancy between the theoretical predictions for colour-octet contributions and the HERA data at large z as a failure of the NRQCD approach itself, as long as not all theoretical uncertainties that reside in the calculation of the short distance cross sections have been assessed carefully. Potentially large QCD effects include higher-twist contributions [30], next-to-leading order corrections² and intrinsic transverse momentum of the partons in the proton [33]. Moreover, close to the boundary of phase space, for $z \gtrsim 0.75$, the shape of the z -distribution cannot be predicted without resumming singular higher-order terms in the velocity expansion [13]. This difficulty is exactly analogous to the well-known problem of extracting the CKM matrix element $|V_{ub}|$ from the endpoint region of the lepton energy distribution in semileptonic B decay. To constrain the colour-octet contributions from the J/ψ energy spectrum, the distribution would have to be averaged over a sufficiently large interval $\Delta z \gg v^2 \sim 0.25$ containing the endpoint $z = 1$.

The low- z region is not expected to be sensitive to higher-order terms in the velocity expansion. Therefore, if the data could be extended to $z \lesssim 0.3$, an important resolved photon contribution should be visible, if the colour-octet matrix elements are not significantly smaller than extracted from the Tevatron data and expected from velocity scaling rules.

With higher statistics data at HERA it will be possible to extract more detailed information on the colour-octet matrix elements and the phenomenological significance of colour-octet contributions. Particularly interesting is the analysis of J/ψ polarization

² Next-to-leading order calculations for J/ψ photoproduction exist only for the colour-singlet channel [31] and for the colour-octet contributions to the total cross section at $z \approx 1$ and $p_t \approx 0$ GeV [32].

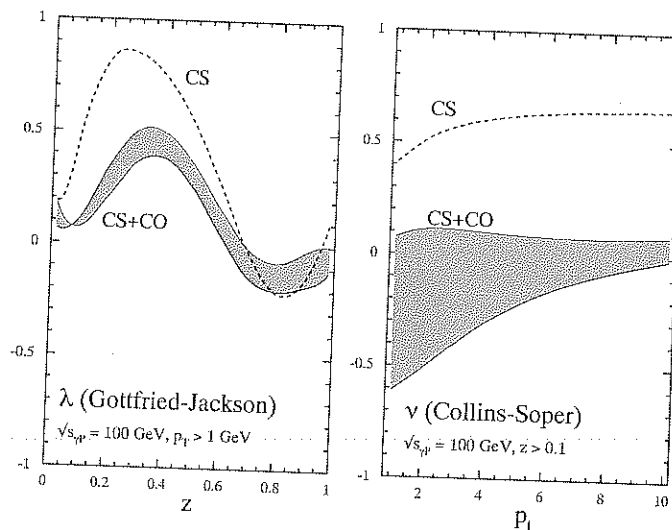


Figure 6. Colour-singlet (CS) and colour-octet (CO) contributions to the polar and azimuthal angular parameters in the leptonic J/ψ decay. For details and the definition of the Gottfried-Jackson and Collins-Soper reference frames see [29].

[29]. Two instructive examples are shown in figure 6, the polar angle parameter λ and the azimuthal angle parameter ν in the leptonic J/ψ decay as a function of the energy fraction z and the transverse momentum p_t , respectively. The polarization signatures predicted from colour-octet processes are distinctive and can be used to extract information on the relative weight of colour-singlet and colour-octet contributions. Polarization analyses can also discriminate between NRQCD and the colour-evaporation model which predicts unpolarized quarkonium.

Various other observables and final states may be accessible in the future at HERA, like photoproduction of ψ' , Υ , χ_c and η_c particles, associated $J/\psi + \gamma$ production, fragmentation contributions at large p_t as well as deep inelastic J/ψ production (for references see [24]).

4. Summary and conclusion

The NRQCD factorization approach is a rigorous framework for inclusive quarkonium production and decays and provides a striking and elegant explanation for the large direct J/ψ and ψ' cross section measured in $p\bar{p}$ collisions at the Fermilab Tevatron. Still, more theoretical work and experimental information is needed to establish the applicability of NRQCD at the charm mass scale and the phenomenological significance of colour-octet processes. The non-observation of large colour-octet contributions in J/ψ photoproduction represents a serious challenge to the NRQCD approach, but no firm conclusions can be drawn without a careful assessment of potentially large theoretical uncertainties. The theoretical issues to be addressed in the future include the calculation of higher-order QCD corrections, the resummation of higher-order terms in the velocity expansion and quantitative insights in the effect of higher-twist contributions. Phenomenological tests of

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NRQCD will be done by a global analysis of different production processes and observables. Charmonium production at HERA should play an important role in the future in this regard. The single most crucial test of the NRQCD approach is the analysis of J/ψ and ψ' polarization at large p_T at the Tevatron. The polarization analysis of the Tevatron data is under way and experimental results are eagerly awaited.

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