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# Relevance of thermally populated first excited state of $^{44}$Ti to the abundance problem of Cassiopiea A: A model study

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**Abstract.** We examine the possible role of electron-capture on the thermally populated first $2^+$ excited state of $^{44}$Ti in hot astrophysical environments pertaining to post explosive nucleosynthesis supernova debris. We find in a simple schematic model that the astrophysical weak interaction rate for electron-capture decay of $^{44}$Ti can depend considerably on temperature and hence on time. We propose a time varying decay rate for the evolving supernova debris and demonstrate its consequence for the $^{44}$Ti mass yield of the supernova Cas A, observed through the measured 1.157 MeV $\gamma$-ray flux from the electron-capture decay of $^{44}$Ti.

**Keywords.** Nuclear structure; electron capture; nucleosynthesis; supernova.

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## 1. Introduction

Recent measurement [1] of 1.157 MeV $\gamma$-ray flux following the decay of $^{44}$Ti by the space-borne instrument aboard Compton Gamma Ray Observatory (CGRO) leads to an estimate of the amount of $^{44}$Ti produced in explosive nucleosynthesis in supernova Cassiopiea A (Cas A). It is believed that $^{44}$Ti yield can be used to test and calibrate the supernova models. The measurement shows a large absolute amount of $^{44}$Ti mass ($M_{^{44}\mathrm{Ti}} > 1\times 10^{-4} M_{\odot}$) and a high $M_{^{44}\mathrm{Ti}}/M_{^{56}\mathrm{Ni}}$ abundance ratio [2,3], when compared with theoretical predictions of spherically symmetric collapse-driven supernova nucleosynthesis models. Recently Nagataki *et al* [3] proposed axisymmetric models for the Cas A and obtained larger $^{44}$Ti yield. Thus the observed $^{44}$Ti yield should be known as accurately as possible before it can be used as a probe for supernova models.

In calculating the total $^{44}$Ti yield of the supernova using the measured line flux of CGRO one needs also the distance, explosion date of the supernova and the half-life of $^{44}$Ti. Uncertainty in the measured terrestrial half-life of $^{44}$Ti was very large, ranging from 39 to 66.6 years and the results of all the three recent measurements with higher precision suggest a half-life of 59.2±0.6 y for $^{44}$Ti [2,4].

An accurate determination of terrestrial half-life of $^{44}$Ti will reduce the uncertainty of the observed yield but actual half-life of $^{44}$Ti may be different in the astrophysical environments. When nuclear statistical equilibrium prevail, electron-capture (EC) or $\beta$-decay may occur on thermally populated states as well [5].

Contribution of these excited states to the total decay rate may sometimes dominate over that from the ground state depending upon the value of lepton-fraction $Y_e$ (lepton/baryon), density ($\rho$) and temperature ($T$) [5]. Explosive nucleosynthesis occurs in a time scale of $\simeq$ 1 s and the synthesized nuclei are left in sufficiently hot environment for the next 10–16 sec in expanding debris. However, the debris cools over a very long time and thins out when $\gamma$-rays can escape. $^{44}$Ti is believed to be synthesized during $\alpha$-rich freeze-out and the peak temperature may be around $5.5 \times 10^9$ K (0.474 MeV) and density in the range $10^6$–$10^8$ gm/cc [6,3]. Around the temperature 0.474 MeV the thermally populated first $2^+$ excited state at 1.08299 MeV of $^{44}$Ti in nuclear statistical equilibrium may decay by electron-capture to $2^+$ ground state of $^{44}$Sc. This decay may even occur through an allowed Gamow-Teller electron-capture transition with log *ft* value less than the two non-unique first forbidden transitions from the ground state of $^{44}$Ti to the $1^-(E_x = 0.0679$ MeV, $I^\epsilon =$ $0.7$%) and $0^-(E_x = 0.1463$ MeV, $I^\epsilon = 99.3$%) excited states of $^{44}$Sc. These two non-unique first forbidden transitions occur via orbital electron-capture and the corresponding log *ft* values too can be different from the terrestrial ones in astrophysical environments [5]. In this work we examine in a schematic model, the role of continuum electron-capture on the first excited state and find its consequence for the $^{44}$Ti yield of the explosion.

## 2. Formulation

In deriving a relation between the measured $\gamma$-flux and the $^{44}$Ti yield one assumes the decay rate of the $^{44}$Ti nucleus independent of time (always equal to its terrestrial value, $\log\lambda_L = -9.431, \lambda_L$ in sec$^{-1}$) during the evolution of the debris for a period of $t_a$ y, the age of the supernova. This appears quite surprising and we expect that in most general situation the decay rate at time $t$ of $^{44}$Ti synthesized in the explosion and decaying via weak interaction should be given by

$$-\frac{\mathrm{d}N}{\mathrm{d}t} = 4\pi D^2 F_\gamma(t) = \int_V n(\vec{r},t)\lambda'[Y_e(\vec{r},t),\rho(\vec{r},t),T(\vec{r},t)]\mathrm{d}V, \tag{1}$$

where $N$ is the total number of $^{44}$Ti nuclei at time $t$ in the supernova debris, $D$ is the distance of the supernova from the detector in Kpc, $F_\gamma(t)$ is the flux of characteristic $\gamma$-line following decay of $^{44}$Ti detected in cm$^{-2}$ sec$^{-1}$, $n$ is the number density of the $^{44}$Ti nucleus at a distance $\vec{r}$ from the centre of the supernova at time $t$, $\lambda'$ is the decay rate in d$V$. The decay rate $\lambda'$ is a function of $(Y_e, \rho, T)$ at $(\vec{r}, t)$. The volume integral may be taken over the whole volume $V$ of the debris. Evaluation of the integral on the right hand side of eq. (1) requires knowledge of stellar evolution trajectory of the debris at time $t$. Here we replace $\lambda'[Y_e(\vec{r},t),\rho(\vec{r},t),T(\vec{r},t)]$ in eq. (1) by the value $\lambda(t)$, same throughout the volume of the layer in the debris, where $^{44}$Ti was synthesized during explosion (Si-shell) and obtain,

$$-\frac{\mathrm{d}N}{\mathrm{d}t} = 4\pi D^2 F_\gamma(t) = N(t)\lambda(t). \tag{2}$$

From eq. (2) we obtain for $^{44}$Ti the final expression for the mass yield as

$$M_{^{44}\mathrm{Ti}} = 1.39 \times 10^{-4} F_\gamma(t_a) D^2 \lambda^{-1}(t_a) \mathrm{e}^{\int_0^{t_a} \lambda(t)\mathrm{d}t}, \tag{3}$$

where $M_{^{44}\mathrm{Ti}}$ is in solar masses $M_\odot$, and the decay rate $\lambda(t)$ of $^{44}$Ti at time $t$ is in $\mathrm{y}^{-1}$.

In the post explosive nucleosynthesis expansion of the supernova, temperature in any volume element changes with time. So the time dependence of the astrophysical decay rate may come from its temperature dependence ($T = T(\vec{r}, t)$). Assuming nuclear statistical equilibrium the expression for the decay rate of $^{44}$Ti at temperature $T$ (in MeV) is

$$\lambda(t) \simeq \lambda(T(\vec{r},t)) = \frac{\ln(2)}{G(T(\vec{r},t))}\left[\frac{1}{t_{1/2}(\mathrm{gs})} + \frac{5f(Q_{\mathrm{gs}} + 1.08299, \epsilon(\vec{r},t), T(\vec{r},t))}{\langle ft_{1/2}(2^+ \to 2^+)\rangle}\mathrm{e}^{-1.08299/T(\vec{r},t)}\right]. \tag{4}$$

In eq. (4) the nuclear partition function $G(T(\vec{r},t)) = 1 + 5\mathrm{e}^{-1.08299/T(\vec{r},t)}$, $t_{1/2}(\mathrm{gs})$ is the terrestrial half-life, $f(Q_{\mathrm{gs}} + 1.08299, \cdots)$ is the $f$-factor for the transition from the first $2^+$ excited state of $^{44}$Ti to the $2^+$ ground state of $^{44}$Sc, with $Q_{\mathrm{gs}}$ ($= 0.2645$ MeV), the $Q$-value for the decay from the ground state and $\langle ft_{1/2}(2^+ \to 2^+)\rangle$ is the $ft$-value for this transition. An estimate of the continuum electron capture (cec) rate on the first $2^+$ state of $^{44}$Ti, using the allowed Gamow-Teller nuclear matrix element corresponding to log $ft = 4$ and 5, suggest that $\lambda_{\mathrm{cec}}$ can take drastically very low value ($10^{-2}$–$10^{-3}$ $\mathrm{sec}^{-1}$) around the peak temperature 0.474 MeV and then quickly falls off with temperature.

If the actual log *ft* value fall within the allowed GT range, the $t_{1/2}(T)$ may be reduced considerably for $T > 0.10$ MeV. Consequently a non-negligible fraction of the $^{44}$Ti produced in the explosion will decay, even if the duration of this finite temperature situation is small, and will contribute to the early phase of the supernova light-curve. This implies more $^{44}$Ti yield than calculated from eq. (3) with $\lambda(t) = \lambda_L$, the terrestrial decay rate.

We now use a simple model for the proposed time dependence of the astrophysical decay rate $\lambda(t)$ to calculate analytically the $^{44}$Ti mass using eq. (3). This is based on the behaviour of $t_{1/2}(T)$ we study using eq. (4) and is given by

$$\lambda(t) = 1 + (\lambda_0 - 1)\mathrm{e}^{-t/\Delta}, \tag{5}$$

where decay rates are expressed in units of the terrestrial decay rate $\lambda_L$. $\lambda_0$ is the value of the astrophysical rate $\lambda(\vec{r}, t = 0)$ just after the explosive nucleosynthesis is over, chosen as the origin of time ($t = 0$) for the evolution of the debris. The parameters $\Delta$, can be viewed as a time constant with which the astrophysical decay rate drops from the peak value $\lambda_0$ to the terrestrial value during the expansion phase of the debris.

## 3. Results and discussion

We choose the peak values of $\lambda_0 = 10^{6.431}, 10^{7.431}$ in unit of terrestrial rate corresponding to an effective astrophysical rate $10^{-3}$ and $10^{-2}$ $\mathrm{sec}^{-1}$.

With eq. (5) for the $\lambda(t)$ we calculate $(\mathrm{e}^{\lambda_L \int_0^{t_a} \lambda(t)\mathrm{d}t} - \mathrm{e}^{\lambda_L t_a})/e^{\lambda_L t_a}$ and obtain from eq. (3) an expression for the change of the $^{44}$Ti yield only due to the proposed time dependence,

$$\frac{\Delta M_{^{44}\mathrm{Ti}}}{M_{^{44}\mathrm{Ti}}} = \left[\mathrm{e}^{(\lambda_0 - 1)(\Delta/t_m)(1 - \mathrm{e}^{-t_a/\Delta})} - 1\right]. \tag{6}$$

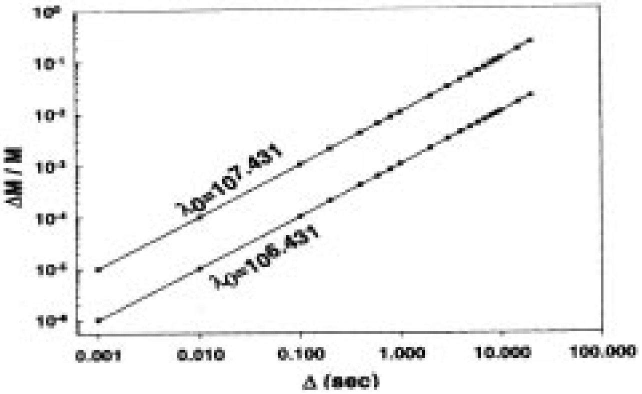

**Figure 1.** Variation of $\Delta M_{^{44}\mathrm{Ti}}/M_{^{44}\mathrm{Ti}}$ with $\Delta$. $\lambda_0$ is in unit of terrestrial rate $\lambda_L$ and is obtained from eq. (4).

Here $t_m = 1/\lambda_L$ is the terrestrial mean life of $^{44}$Ti in y. We have also tried with $\lambda(t) = \lambda_0 - (\lambda_0 - 1)\tanh(t/\Delta)$, with results qualitatively similar to that of the above model.

In figure 1 we show the variation of $\Delta\ M_{^{44}\mathrm{Ti}}/M_{^{44}\mathrm{Ti}}$ as a function of $\Delta$ for the supernova Cas A for different values of $\lambda_0$ corresponding to log *ft* $=$ 4, 5. We take $t_a = 312$ y for the age of the supernova Cas A. The figure demonstrates that if there is EC at all on the first $2^+$ state of $^{44}$Ti, and unless the decay rate from the ground state is reduced due to stellar environmental effects, the value of $M_{^{44}\mathrm{Ti}}$, will be larger than the value estimated using constant terrestrial decay rate and 1.157 MeV $\gamma$-flux. It may be necessary therefore to pay attention to this aspect of the Cas A abundance problem before using the observed $^{44}$Ti mass as a probe for supernova models.

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