

Astrophysical S -factors from asymptotic normalization coefficients

R E TRIBBLE, A AZHARI, H L CLARK, C A GAGLIARDI, Y-W LUI,
A M MUKHAMEDZHANOV, A SATTAROY, X TANG, L TRACHE, V BURJAN,
J CEJPEK*, V KROHA*, Š PISKOŘ*, J VINCOUR* and F CĂRSTOIU†

Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA

*Institute for Nuclear Physics, Czech Academy of Sciences, Prague-Řež, Czech Republic

†Institute for Atomic Physics, Bucharest, Romania

Abstract. S -factors for direct capture reactions can be found at astrophysical energies from asymptotic normalization coefficients which provide the normalization of the tail of the overlap function. For example the overlap for ${}^8\text{B} \rightarrow {}^7\text{Be} + p$ defines the S -factor for ${}^7\text{Be}(p, \gamma){}^8\text{B}$. Peripheral transfer reactions offer a technique to determine these asymptotic normalization coefficients. As a test of the technique, the ${}^{16}\text{O}({}^3\text{He}, d){}^{17}\text{F}$ reaction has been used to determine asymptotic normalization coefficients for transitions to the ground and first excited states of ${}^{17}\text{F}$. The S -factors for ${}^{16}\text{O}(p, \gamma){}^{17}\text{F}$ calculated from these ${}^{17}\text{F} \rightarrow {}^{16}\text{O} + p$ asymptotic normalization coefficients are found to be in very good agreement with recent measurements. Following the same technique, the ${}^{10}\text{B}({}^7\text{Be}, {}^8\text{B}){}^9\text{Be}$ and ${}^{14}\text{N}({}^7\text{Be}, {}^8\text{B}){}^{13}\text{C}$ reactions have been used to measure the asymptotic normalization coefficient for ${}^7\text{Be}(p, \gamma){}^8\text{B}$. This result provides an indirect determination of $S_{17}(0)$.

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1. Introduction

Nuclear capture reactions such as (p, γ) and (α, γ) , play a major role in defining our universe. Until recently, the only reliable method to determine a reaction rate that is dominated by direct capture has been to measure it at laboratory energies with a low energy particle beam and extrapolate the result to astrophysical energies. Often the reaction of interest involves a radioactive target which makes measurements quite difficult. Hence it is important to develop alternative techniques to determine rates. Coulomb dissociation may provide an indirect method for obtaining this information, but it has not yet been subjected to a suitable reliability test.

Direct capture reactions of astrophysical interest usually involve systems where the binding energy of the captured proton is low. Hence at stellar energies, the capture proceeds through the tail of the nuclear overlap function. The shape of the overlap function in this tail region is completely determined by the Coulomb interaction, so the amplitude of the overlap function alone dictates the rate of the capture reaction. The ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction

is an excellent example of such a direct capture process. Indeed recent calculations of the normalization constant have been used to predict the capture rate [1,2]. But new measurements, both direct and indirect, are still needed as was underscored in a recent review of stellar reaction rates [3] which includes a detailed discussion of the uncertainties in our present knowledge of $S_{17}(0)$ and its importance to the solar neutrino problem.

The asymptotic normalization coefficient C for $A + p \leftrightarrow B$ specifies the amplitude of the single-proton tail of the wave function for nucleus B when the core A and the proton are separated by a distance large compared to the nuclear radius. In previous communications [1,4], we have pointed out that astrophysical S -factors for peripheral direct radiative capture reactions can be determined through measurements of asymptotic normalization coefficients (ANC) using traditional nuclear reactions such as peripheral nucleon transfer. Direct capture S -factors derived with this technique are most reliable at the lowest incident energies in the capture reaction, precisely where capture cross sections are smallest and most difficult to measure directly. Of course it is extremely important to test the reliability of the technique in order to know the precision with which it can be applied. Determining the S -factors for $^{16}\text{O}(p, \gamma)^{17}\text{F}$ from its ANC's has been recognized as a suitable test for this method [3] because the results can be compared to existing direct measurements of the cross sections [5,6]. Furthermore, the $^{16}\text{O}(p, \gamma)^{17}\text{F}$ reaction has substantial similarities to the $^7\text{Be}(p, \gamma)^8\text{B}$ reaction. Below we briefly discuss the results of a measurement of the $^{16}\text{O}(^3\text{He}, d)^{17}\text{F}$ reaction to determine the ANC's for the ground and first excited states in ^{17}F . From these ANC's we calculate S -factors for $^{16}\text{O}(p, \gamma)^{17}\text{F}$ and compare to experimental results. We then discuss our measurement of the $^{10}\text{B}(^7\text{Be}, ^8\text{B})^9\text{Be}$ and $^{14}\text{N}(^7\text{Be}, ^8\text{B})^{13}\text{C}$ reactions, the extraction of the ANC's for $^8\text{B} \rightarrow ^7\text{Be} + p$ and our determination of $S_{17}(0)$.

2. A test case

The $^{16}\text{O}(^3\text{He}, d)^{17}\text{F}$ reaction was measured previously at a beam energy $E_{^3\text{He}} = 25$ MeV [7]. We repeated the measurement at 29.75 MeV in order to obtain better angular coverage and to have a measurement at a second energy, both of which were necessary for extracting reliable ANC's. Data at laboratory scattering angles between 6.5° and 25° were obtained using Si solid state detectors and a ^3He beam, incident on a $134 \mu\text{g}/\text{cm}^2$ Mylar target, from the U-120M isochronous cyclotron of the Nuclear Physics Institute (NPI) of the Czech Academy of Sciences. Additional data at laboratory angles between 1° and 11° were obtained using the MDM magnetic spectrometer and a molecular $(^3\text{He} - d)^+$ beam, incident on a $540 \mu\text{g}/\text{cm}^2$ Mylar target, from the Texas A&M University K500 superconducting cyclotron. Absolute cross sections were determined at the NPI using their detection system which has been well calibrated for $(^3\text{He}, d)$ reaction studies. The data obtained at Texas A&M University were normalized to the data from the NPI measurement in the region where the two data sets overlapped. The angular distributions for the combined data sets are shown in figure 1. More details of the experiments can be found in [8].

For a peripheral transfer reaction, ANC's are extracted from the angular distributions by comparison to a DWBA calculation. Consider the proton transfer reaction $a + A \rightarrow c + B$, where $a = c + p$ and $B = A + p$. The experimental cross section is related to the DWBA cross section according to

$$\frac{d\sigma}{d\Omega} = \sum (C_{Ap}^B)^2 (C_{cp}^a)^2 R, \quad (1)$$

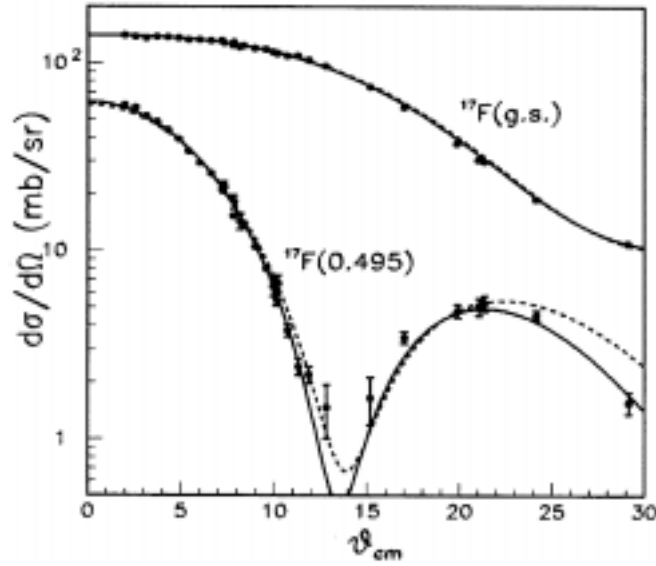


Figure 1. Angular distributions for the ground and first excited states of ^{17}F from the $^{16}\text{O}(^3\text{He}, d)^{17}\text{F}$ reaction. The curves are fits from DWBA calculations using two different optical potential sets.

where

$$R = \frac{\tilde{\sigma}^{\text{DW}}}{b_{Ap}^2 b_{cp}^2}. \quad (2)$$

$\tilde{\sigma}^{\text{DW}}$ is the calculated DWBA cross section and the b 's are the asymptotic normalization constants for the single particle orbitals used in the DWBA. The sum in eq. (1) is taken over the allowed angular momentum couplings, and the C 's are the ANC's for $B \rightarrow A + p$ and $a \rightarrow c + p$. The normalization of the DWBA cross section by the ANC's for the single particle orbitals makes the extraction of the ANC for $B \rightarrow A + p$ essentially independent of the parameters used in the single particle potential wells (see [9,10] for additional details).

The DWBA calculations were carried out with the finite range code PTOLEMY [11], using the full transition operator. The single particle orbitals were calculated in Woods–Saxon potentials with r_0 in the range 1.15–1.35 fm and a_0 in the range 0.55–0.75 fm, and indicated that the calculated DWBA cross sections are insensitive to assumptions about the ^{17}F wave functions in the nuclear interior. A range of optical model parameter sets were studied for both the entrance and exit channels, as detailed in [8], and the resulting fits to the ground and excited state angular distributions using two different combinations of parameter sets are shown in figure 1. Normalizing the DWBA calculations to the data and dividing by the ANC's for the single particle orbital yields the product of the ANC's for the $^{17}\text{F} \rightarrow ^{16}\text{O} + p$ and $^3\text{He} \rightarrow d + p$ systems. Dividing this product by the known ANC for $^3\text{He} \rightarrow d+p$ [12,13] provides C^2 for $^{17}\text{F} \rightarrow ^{16}\text{O} + p$. The dominant contribution to the uncertainties is due to the variation in the extracted ANC's with different optical model parameter sets. The fits shown in figure 1 are for the two combinations of parameter sets

which give the largest variation in C^2 . This difference was used to obtain the uncertainty in C^2 due to the choice of optical model parameters. Our final adopted ANC for the ground state is $C^2 = 1.08 \pm 0.10 \text{ fm}^{-1}$. The uncertainty includes $\pm 4.8\%$ from the absolute normalization and angle accuracies, plus the statistics of the fits, and $\pm 7.6\%$ associated with the choice of optical model parameters and single particle orbital. Our final adopted ANC is $C^2 = 6490 \pm 680 \text{ fm}^{-1}$ for the excited state. The corresponding contributions to its uncertainty are $\pm 5.4\%$ and $\pm 9.0\%$.

The relation of the ANC's to the direct capture rate at low energies is straightforward. The cross section for the direct capture reaction $A + p \rightarrow B + \gamma$ can be written as

$$\sigma = \lambda |\langle I_{Ap}^B(\mathbf{r}) | \hat{O}(\mathbf{r}) | \psi_i^{(+)}(\mathbf{r}) \rangle|^2, \quad (3)$$

where λ contains kinematical factors, I_{Ap}^B is the overlap function for $B \rightarrow A + p$, \hat{O} is the electromagnetic transition operator, and $\psi_i^{(+)}$ is the scattering wave in the incident channel. If the dominant contribution to the matrix element comes from outside the nuclear radius, the overlap function may be replaced by

$$I_{Ap}^B(r) \approx C \frac{W_{-\eta, l+1/2}(2\kappa r)}{r}, \quad (4)$$

where C defines the amplitude of the tail of the radial overlap function I_{Ap}^B , W is the Whittaker function, η is the Coulomb parameter for the bound state $B = A + p$, and κ

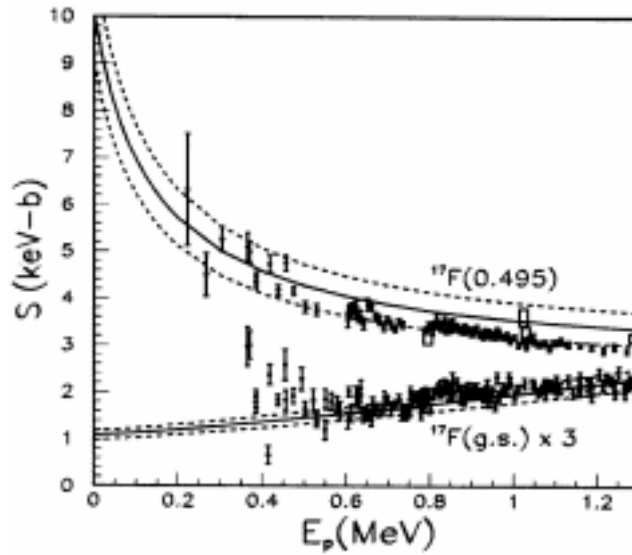


Figure 2. A comparison of the experimental S -factors to those determined from the ANC's found in $^{16}\text{O}(^3\text{He}, d)^{17}\text{F}$. The solid data points are from [5], and the open boxes are from [6]. The solid lines indicate our calculated S -factors, and the dashed lines indicate the $\pm 1\sigma$ error bands. Note that the experimental ground state S -factor may be contaminated by background [15] at energies below 500 keV.

is the bound state wave number. For $^{16}\text{O}(p, \gamma)^{17}\text{F}$, the required C 's are just the ANC's found above from the transfer reaction. Thus, the direct capture cross sections are directly proportional to the squares of these ANC's.

Following the prescription outlined above, the S -factors describing the capture to both the ground and first excited states for $^{16}\text{O}(p, \gamma)^{17}\text{F}$ were calculated, with no additional normalization constants, using the standard definition of S [14]. The results are shown in figure 2 compared to the two previous measurements of $^{16}\text{O}(p, \gamma)^{17}\text{F}$ [5,6]. Both E1 and E2 contributions have been included in the calculations, but the E1 components dominate the results. The theoretical uncertainty in the S -factors is less than 2% for energies below 1 MeV. Above 1 MeV the nuclear interaction begins to be important in the evaluation of the scattering wave function. The agreement between the measured S -factors and those calculated from our $^{17}\text{F} \rightarrow ^{16}\text{O} + p$ ANC's is quite good, especially for energies below 1 MeV where the approximation of ignoring contributions from the nuclear interior should be very reliable and the optical potential uncertainties are negligible. Overall, the results verifying the technique is valid for determining S -factors to accuracies of at least 9%.

3. ANC for $^8\text{B} \rightarrow ^7\text{Be} + p$

We have measured the ($^7\text{Be}, ^8\text{B}$) reaction on a 1.7 mg/cm² ^{10}B target and a 1.5 mg/cm² Melamine target in order to extract the ANC for $^8\text{B} \rightarrow ^7\text{Be} + p$. The radioactive ^7Be beam was produced at 12 MeV/u by filtering reaction products from the $^1\text{H}(^7\text{Li}, ^7\text{Be})n$ reaction in the recoil spectrometer MARS, starting with a primary ^7Li beam at 18.6 MeV/A from the TAMU K500 cyclotron. The beam was incident on an H_2 cryogenic gas target, cooled by LN_2 , which was kept at 1 atmosphere (absolute) pressure. Reaction products were measured by 5 cm \times 5 cm Si detector telescopes consisting of a 100 μm ΔE strip detector, with 16 position sensitive strips, followed by a 1000 μm E counter. Two different detector configurations were used to take data. In one mode, two detector telescopes were mounted symmetrically about the beam axis as shown in figure 3. The angular coverage in this mode was from $\approx 4^\circ$ to $\approx 18.5^\circ$ in the laboratory frame for each telescope. In the second mode, a single detector telescope stack was placed at 0° and the beam was stopped just in front of the ΔE detector. The effective angular coverage in the lab for this mode was from $\approx 3.8^\circ$ to $\approx 14^\circ$.

A single 1000 μm Si strip detector was used for initial beam tuning. This detector, which was inserted at the target location, allowed us to optimize the beam shape and to normalize the ^7Be flux relative to a Faraday cup that measured the intensity of the primary ^7Li beam. Following optimization, the approximate ^7Be beam size was 6 mm \times 3 mm (FWHM), the energy spread was ≈ 1.5 MeV, the full angular spread was $\Delta\theta \approx 28$ mrad and $\Delta\phi \approx 62$ mrad, and the purity was $\geq 99.5\%$ ^7Be for the experiment with the ^{10}B target. The beam size and angular spread were improved for the experiment with the ^{14}N target to 4 mm \times 3 mm (FWHM), $\Delta\theta \approx 28$ mrad and $\Delta\phi \approx 49$ mrad. Periodically during the data acquisition, the beam detector was inserted to check the stability of the secondary beam tune. The system was found to be quite stable over the course of the experiment with maximum changes in intensity observed to be less than 5%. The typical rate for ^7Be was ≈ 1.5 kHz/pnA of primary beam on the production target. Primary beam intensities of up to 80 pnA were obtained on the gas cell target during the experiments.

Preliminary results for the elastic scattering angular distributions from the two targets

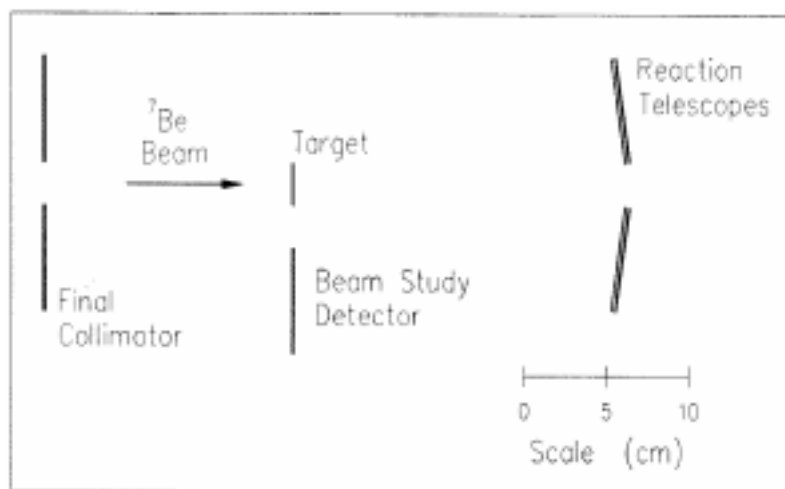


Figure 3. Detector geometry used for the (⁷Be, ⁸B) reaction studies.

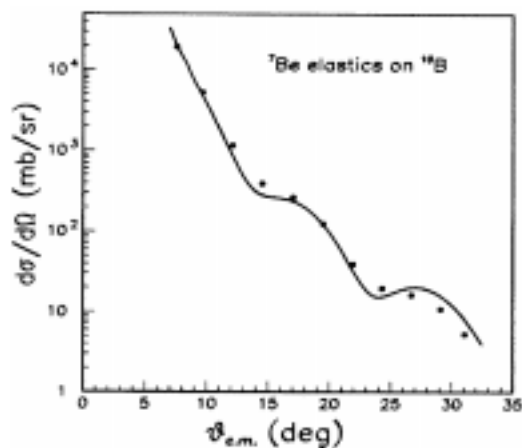


Figure 4. Angular distribution for elastic scattering from the ¹⁰B target. The curve is from optical model calculations smoothed over the angular acceptance of each bin.

are shown in figures 4 and 5. For the ¹⁰B target, the elastic scattering yield includes contributions from three target components, B (86%), C (10%) and O (4%), while the Melamine target includes N (67%) and C (28%). A Monte Carlo simulation described below was used to generate the solid angle factor for each angular bin and the smoothing needed for the calculation to account for the finite angular resolution of the beam. The absolute cross section is then fixed by the target thickness, number of incident ⁷Be, the yield in each bin, and the solid angle. The curves shown with the elastic scattering were found from optical model calculations described below by adding together the cross section

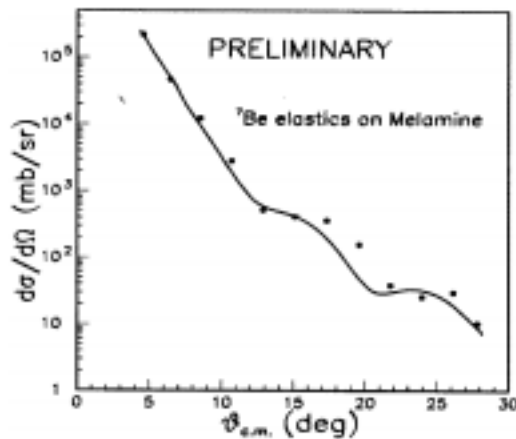


Figure 5. Angular distribution for elastic scattering from the Melamine target. The curve is from optical model calculations smoothed over the angular acceptance of each bin.

predictions for the target components in the laboratory frame and then transforming the result to the center of mass assuming kinematics appropriate for either the ^{10}B or ^{14}N targets.

The ^8B Q -value spectra, shown in figure 6, were obtained by assuming either a $^{10}\text{B}(^7\text{Be}, ^8\text{B})^9\text{Be}$ or $^{14}\text{N}(^7\text{Be}, ^8\text{B})^{13}\text{C}$ reaction and correcting the ^8B reaction products for kinematic energy shifts as a function of scattering angle. In the case of the ^{10}B target, the major contributions to the energy resolution seen in figure 6 are the beam energy spread, the target thickness and the nonuniformity of the target. The beam energy spread and differential energy loss in the target dominated the energy resolution for the Melamine target. Since the ground state of ^9Be is not cleanly separated from excited states, a Monte Carlo simulation of the experiment has been used to fix the line shape and determine cross sections. The simulation, which is fine tuned to reproduce the measured beam properties and the resolution observed in elastic scattering, includes the geometry of the experimental setup, reaction kinematics, nonuniform energy loss in the target and the size, angular spread and energy spread of the beam. The beam location and angle at the target are determined by symmetry requirements on ^7Be elastic scattering data. The three peaks shown in figure 6 for the ^{10}B target correspond to the excitation of the ground and second excited states of ^9Be and the ground state of ^{15}N from the ^{16}O contamination in the target. They were obtained by including the predicted angular distributions for the states in the Monte Carlo simulation and then extracting the associated Q -value spectrum. The normalization of the three peaks was done by a χ^2 minimization to the data. In the Melamine case, the ground state of ^{13}C is cleanly resolved from excited states making the normalization of the Q -value spectrum via the Monte Carlo simulation straight forward.

The ANC for $^8\text{B} \rightarrow ^7\text{Be} + p$ was extracted based on the fit to the Q -value spectra and the ANCs for $^{10}\text{B} \rightarrow ^9\text{Be} + p$ [9] and $^{14}\text{N} \rightarrow ^{13}\text{C} + p$ [10] following the procedure outlined above in our test case. Two orbitals, $1p_{1/2}$ and $1p_{3/2}$, contribute to the transfer reaction but the $1p_{3/2}$ dominates in both cases. In calculating the angular distribution, we used

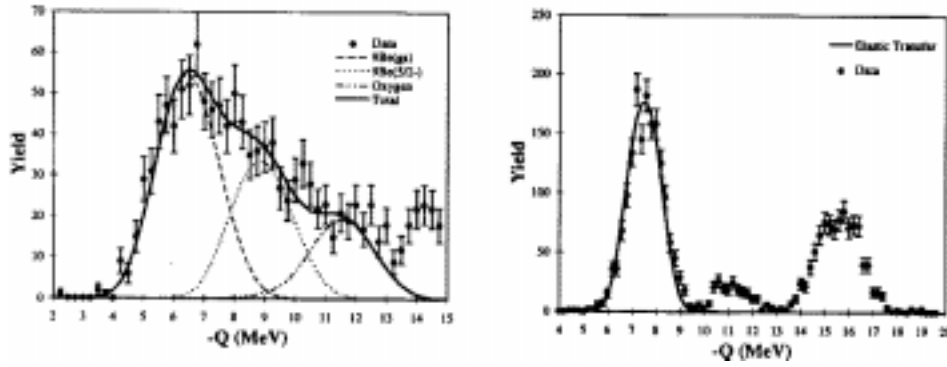


Figure 6. Q -value spectra for ^8B reaction products. The three peaks for the ^{10}B target correspond to the excitation of the ground state and second excited states of ^9Be and the ground state in ^{15}N from the ^{16}O contamination in the target. Data for the Melamine target show a clear isolation of the ground state for ^{13}C .

the ratio for the two orbitals as given by a microscopic description of the ^8B ground state [4]. No optical model parameters are available for either the entrance or exit channel in this reaction. As part of this program, we have obtained elastic scattering data on several beam–target combinations in this mass and energy range and have been developing appropriate optical model parameters that can be extended to ^7Be and ^8B projectiles [16]. Consequently we have chosen parameters obtained from renormalized microscopic folding potentials using the JLM effective NN interaction [17]. The surface region of these folding potentials was fit with volume Woods–Saxon forms and the parameters found for the entrance (exit) channel for the ^{10}B target were $V = 63.8(67.0)$ MeV, $W = 29.4(31.8)$ MeV, $R_V = 3.18(3.18)$ fm, $a_V = 0.85(0.876)$ fm, $R_I = 3.49(3.54)$ fm, $a_I = 0.95(0.99)$ fm. The corresponding values for ^{14}N were $V = 79.1(85.2)$ MeV, $W = 36.0(39.3)$ MeV, $R_V = 3.30(3.30)$ fm, $a_V = 0.88(0.91)$ fm, $R_I = 3.62(3.67)$ fm, $a_I = 0.98(1.02)$ fm. The entrance channel parameters were the same as those used in calculating the elastic scattering angular distributions for ^7Be on ^{10}B in figure 4 and on ^{14}N in figure 5. We have checked the sensitivity of the calculations by varying the normalization parameter. As in previous studies, the results are insensitive to bound state single particle well parameters in the DWBA calculations.

Angular distributions for the (^7Be , ^8B) reactions populating the ground states of ^9Be and ^{13}C were extracted using the same procedure as for the elastic scattering. The results are compared to DWBA calculations in figures 7 and 8. The normalization factors between the data and calculations were obtained from the fits to the respective Q -value spectra.

The astrophysical S -factor for $^7\text{Be}(p,\gamma)^8\text{B}$ has been determined from the ANC which includes a 10% uncertainty for optical model parameters, a 11% uncertainty for experimental fits and normalization of the absolute cross section and the uncertainty in the ANC's for $^{10}\text{B} \rightarrow ^9\text{Be} + p$ and $^{14}\text{N} \rightarrow ^{13}\text{C} + p$. The relative contribution of the two angular momentum couplings to the S -factor is straightforward to calculate and introduces a negligible additional uncertainty in our result [1,4]. The preliminary value that we find for the ^{10}B target is $S_{17}(0) = 17.2 \pm 2.6$ eV b and 18 ± 3 eV b for the ^{14}N target which are in good

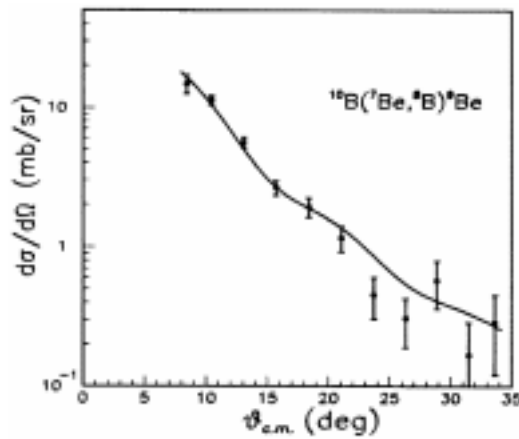


Figure 7. Angular distribution for ^8B populating the ground state of ^9Be from the ^{10}B target. The curve is from a DWBA calculation smoothed over the angular acceptance of each bin.

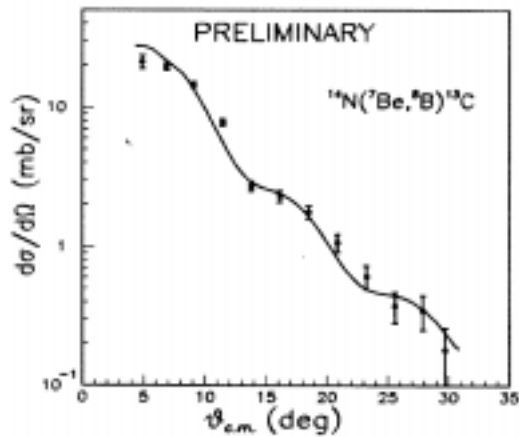


Figure 8. Angular distribution for ^8B populating the ground state of ^{13}C from the Melamine target. The curve is from a DWBA calculation smoothed over the angular acceptance of each bin.

agreement with the recommended value of 19^{+4}_{-2} eV b [3]. The simulation for the ^{14}N target has not been optimized so the result quoted for it and the uncertainty in the result are very preliminary.

One of the primary sources of uncertainty in the values quoted above for $S_{17}(0)$ is the optical model parameters that are used to predict the angular distribution. As indicated, we are working toward a set of global optical model parameters to be used for radioactive beams in this mass and energy region. Once this is complete, the calculations will be

redone and a new ANC and hence $S_{17}(0)$ will be determined. The ANC's for $^{10}\text{B} \rightarrow ^9\text{Be} + p$ and $^{14}\text{N} \rightarrow ^{13}\text{C} + p$ represent the other major source of uncertainty. Measurements of these ANC's have been carried out using ($^3\text{He}, d$) reactions and the results should be forthcoming.

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