

## A three body approach to study the structural properties of 2-*n* halo nuclei and the search for Efimov states

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**Abstract.** The discovery of neutron rich isotopes of the lightest elements on the neutron drip line exhibiting a halo structure has opened up new vistas in research activities. The novel structural features associated with the halo phenomena have been the subject for extensive theoretical and experimental investigations in recent times. In this talk, I propose to present a broad overview of the recent developments in this field, bringing out the striking features which show that a large number of light nuclei near the neutron drip line are characterized by a clear separation between a ‘normal’ core nucleus and a loosely bound low density veil of neutrons. Specifically, the two neutron halos offer a natural premises, from a theoretical standpoint, to employ three body techniques for studying their detailed structural properties. A considerable part of the talk will be devoted to report and highlight the results on a number of light halo nuclei such as  $^{11}\text{Li}$ ,  $^{14}\text{Be}$ ,  $^{19}\text{B}$  and  $^{22}\text{C}$  on which we have been carrying out investigations employing a simple but realistic three body model. These three body systems which have been termed as ‘Borromean’ (i.e while three body systems are bound, the corresponding binary subsystems on the other hand are unbound) are characterized by large spacial extension and very low separation energy of the neutron. They are, therefore, ideally suited for exploring the possibility of the existence of Efimov states in two neutron halo nuclei. We have recently carried out the three body analyses to predict the possibility of the occurrence of such states on which experimental work at various laboratories is underway.

**Keywords.** Three body calculations; Efimov states; Borromean; halo nuclei.

**PACS Nos** 21.45.+v; 21.10.Dr; 21.60.Gx; 27.20.+n

### 1. Introduction

With the upcoming facilities for producing intense radioactive nuclear beams through high energy fragmentation alongwith the contemporary developments in the state of the art involving detectors and isotope separators, experiments have been carried out which have opened [1–4] a new rich field of nuclear physics: *the structure of nuclei near the driplines*. The chart of nuclides, in which the neutron number  $N$  is plotted against the proton number  $Z$  shows that stable nuclei form a diagonal band near  $Z = N$  line. On either side of this ‘valley of stability’ lies a wider region of unstable nuclei. (These are nuclei that, if formed, will exist for a finite time before  $\beta$ -decaying to more stable forms.) The outer edges of this unstable region are known as the *proton and neutron driplines*.

Using the empirical mass formula for the binding energy:

$$B(N, Z) = b_v A - b_s A^{2/3} - \frac{1}{2} b_{\text{sym}} \frac{(N - Z)^2}{A} - \frac{3}{5} \frac{Z^2 e^2}{R_c} \quad (1)$$

the minimization condition i.e,

$$\frac{\partial B}{\partial N} \Big|_{A=\text{const}} = 0$$

gives

$$(N - Z)_{\beta\text{-stable}} = \frac{\frac{3}{5} \frac{e^2}{R_c} A - (M_n - M_p)}{2 \frac{b_{\text{sym}}}{A} + \frac{3}{5} \frac{e^2}{R_c}} \approx 6 \times 10^{-3} A^{5/3}. \quad (2)$$

The nuclei which satisfy this relationship gives rise to *the  $\beta$ -stability valley*. The neutron (proton) dripline which signify the limits of the nuclear periodic table with separation energies  $S_n = 0$  ( $S_p = 0$ ) are obtained through the conditions

$$\begin{aligned} \frac{\partial B}{\partial N} \Big|_{Z\text{const}} = 0 & \quad \text{for the neutron dripline,} \\ \frac{\partial B}{\partial N} \Big|_{N\text{const}} = 0 & \quad \text{for the proton dripline.} \end{aligned}$$

These lines are located far off from the stability valley and there appears to be a large number of nuclei lying beyond the stability line.

Through several decades of studying the charge and matter distribution of stable nuclei, we have learnt *three characteristics of nuclear density* common to all nuclei.

1. The proportionality of the nuclear radius :  $R \propto A^{1/3}$   
i.e  $R = r_0 A^{1/3}$  :  $r_0 = 1.2 \times 10^{-15} \text{m}$ .
2. Surface thickness of about 1 femtometer (the density over which the nuclear density drops about 70 percent to 30 percent of its maximum value).
3. An almost complete spatial overlap of the proton and neutron density distributions.

The pioneering experiments which were carried out to investigate the properties of these light neutron rich nuclei such as  $^8\text{He}$ ,  $^{11}\text{Li}$ ,  $^{11}\text{Be}$ ,  $^{14}\text{Be}$  etc. seem to suggest that all the three rules mentioned above get violated. Thus, for instance, in the case of  $^{11}\text{Li}$ , its rms radius is 3.16 fm – as large as that of  $^{32}\text{S}$ ; the surface thickness, or diffuseness is extremely large compared with that of the protons alone, which is more typical of a normal nuclear density distribution. Very weak binding of the outermost neutrons in the nucleus leads to neutron density distribution which extends well beyond the matter radius expected from systematics – indicating the formation of ‘cloud’ (quantum mechanical tunneling) commonly known as ‘halo’. One can clearly see in the density distribution [5] a long tail formed by the last two neutrons. Such a description is indeed supported by the experimental observations of abnormally large total reaction and Coulomb dissociation cross-section, very sharply peaked angular distribution of the neutrons observed in coincidence with other fragments in the breakup reactions and sharp narrow structure observed in the momentum distribution of the fragments.

## 1.1 Excitation and structure of the halo nucleus

The discovery of the neutron halo has also brought to light new kinds of *excitation modes* in nuclei. Among the known collective nuclear excitations, the giant E1 excitation which results from the oscillation of the proton and neutron distributions against each other is the most pronounced nuclear excitation. In a nucleus with a neutron halo, one has a similar additional oscillation – that of the core against the halo. This excitation is known as the *soft E1 mode* because of the gradual decrease of the halo density at large distances, the restoring force is weak and the oscillation frequency is extremely low. In fact, several model calculations [6] predict the energy for this excitation to be about 1 MeV, much lower than the typical E1 giant resonance energy of around 20 MeV. Experimentally observed large dissociation cross-section of a halo nucleus like,  $^{11}\text{Li}$ , by a high  $Z$ -target strongly suggests the existence of such a low frequency excitation mode.

Experiments also provide compelling evidence for halo clusterization of matter into a normal core nucleus and a loosely bound low density veil of neutrons around the core. For instance, the experimental measurements [7] on the magnetic dipole and dielectric quadrupole moments of  $^{11}\text{Li}$  give values which are found to be close to those obtained for  $^9\text{Li}$ . In addition the results [8] on charge changing reaction cross-section for  $^{8,9,11}\text{Li}$  on C target strongly indicate that  $^{11}\text{Li}$  is to be regarded as a three body system with  $^9\text{Li}$  as core and the two neutrons orbiting around it.

The simplest halo is a two body system consisting of one neutron outside the nuclear core. It has been shown that if a neutron halo is defined as a divergent rms radius of the halo for the neutron separation energy,  $S_n$ , approaching zero, then a two body halo can occur only for *s*- and *p*-states. The conditions for the occurrence of 2-*n* halo in three body systems such as in  $^{11}\text{Li}$  where  $S_{2n} > 0$  and  $S_n(N-1) < 0$  are more restrictive. Nevertheless, it is quite clear that at such low energies the effective two body interaction is operative in low partial waves such as  $l = 0, 1$  etc. To describe these binary interactions, the most natural choice, from an empirical standpoint, is the separable potentials which can be easily constructed from the two body data in a given partial wave. These potentials can then prove to be not only realistic enough to describe the binary system through analytic expressions but also turn out mathematically simple enough to solve the three body systems representing the 2-*n* halo nuclei. During the past few years we have been employing this approach [9,10] to study the structural properties of halo nuclei such as  $^{11}\text{Li}$ ,  $^{14}\text{Be}$ ,  $^{19}\text{B}$ ,  $^{22}\text{C}$  and  $^{20}\text{C}$ . Before I present the details to describe this approach and the results obtained on these systems, let me briefly mention some of the other theoretical approaches (with modifications in the standard ones) which have been put forth to account for the properties of the halo nuclei.

## 1.2 Other theoretical approaches

1.2.1. *Shell model calculations*: Here several difficulties appear; first shell model orbitals no longer provide a good starting point if the residual interaction is ignored – the valence particles in that case may not even be bound; second is the problem of treating the interaction in the nuclear exterior where many body effects may be small and the interaction is close to the ‘free’ particle case. Arima *et al* [11] employing the shell model, calculate the

radii of neutron-rich  $p$ -shell nuclei and find rather difficult to correctly reproduce matter of  $^{11}\text{Li}$ .

*1.2.2. Self-consistent Hartree–Fock calculations (including relativistic mean field approach [12]):* These attempts also fail to reproduce separation energies, rms radii, interaction cross-section for  $^{11}\text{Li}$ ,  $^6\text{He}$  etc. However, as observed by Bertsch *et al* [13] only by adjusting the separation energy of the last neutron to the measured value is some agreement possible.

*1.2.3. Three body approaches:* A number of attempts using many variants of the three body approach have been made in the recent past and have been summarized in a review article by Zhukov *et al* [14].

The object of the present work is to employ Faddeev’s three body theory [15] using two body separable potentials to study the structural properties of  $2n$ -halo nuclei.

## 2. Faddeev’s three body approach using separable potentials

In Faddeev’s theory, the eigenfunction of the three-particle Hamiltonian with pair interaction is represented as a sum of three terms, viz,

$$\Psi = \psi^{(1)}(\vec{p}_{23}, \vec{p}_1) + \psi^{(2)}(\vec{p}_{31}, \vec{p}_2) + \psi^{(3)}(\vec{p}_{12}, \vec{p}_3) \quad (3)$$

for each of which there exists a coupled set of equations symbolically expressed as

$$\psi^{(i)} = \phi^{(i)} - G_0 t_{jk} (\psi^{(j)} + \psi^{(k)}) \text{ for } i \neq j \neq k = 1, 2, 3. \quad (4)$$

In this equation  $t_{jk}$ ’s are two-body  $t$ -matrices in the three-particle Hilbert space;  $\phi^{(i)}$ ’s refer to the free states incorporating the appropriate boundary conditions for the three body scattering problems. In the case of bound state problem of our interest, the inhomogeneous terms do not appear and we have eventually the coupled integral equation in momentum space. The momenta appearing in the wave function (eq.(3)) are given by

$$\vec{p}_{ij} = \frac{m_j \vec{P}_i - m_i \vec{P}_j}{m_i + m_j}, \vec{p}_k = \frac{(m_i + m_j) \vec{P}_k - m_k (\vec{P}_i + \vec{P}_j)}{m_i + m_j + m_k}, \quad (5)$$

where  $\vec{P}_1, \vec{P}_2$  represent the momenta of the two neutrons and  $\vec{P}_3$  is the momentum of the core. In the three body c.m system  $\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0$ , we have two independent momenta.

For the two body interaction, we assume separable potentials. The potential

$$V_{12} = -\frac{\lambda_n}{2\mu_{12}} g(p_{12}) g(p'_{12}), \quad g(p) = \frac{1}{(p^2 + \beta^2)} \quad (6)$$

is known to reproduce the low energy scattering data for the  $^1S_0$   $n - n$  state reasonably well with the help of adjustable parameters  $\lambda_n$  and  $\beta$ . For the  $n$ -core system, we assume that the core is spinless and the halo neutron exists in a low lying intruder  $s$ -orbit producing a virtual or a bound state with the core. With this restriction, we assume

$$\begin{aligned}
 V_{23} &= -\frac{\lambda_c}{2\mu_{23}} f(p_{23})f(p'_{23}), \\
 V_{31} &= -\frac{\lambda_c}{2\mu_{31}} f(p_{31})f(p'_{31}), \\
 f(p) &= \frac{1}{(p^2 + \beta_1^2)}. \tag{7}
 \end{aligned}$$

Here since we only know the bound state or the excitation energy of the system, this gives a certain amount of flexibility in the determination of the parameter  $\beta_1$ .

With this input, the solution of the three body wave-function can be analytically expressed as

$$\begin{aligned}
 \Psi(\vec{p}_{12}, \vec{p}_3; E) &= D^{-1}(\vec{p}_{12}, \vec{p}_3; E) [g(p_{12})F(p_3) + f(p_{23})G(p_1) \\
 &\quad + f(p_{31})G(p_2)], \tag{8}
 \end{aligned}$$

where the spectator function  $F(p)$  and  $G(p)$  satisfy the coupled integral equations:

$$[\lambda_n^{-1} - h_n(p)]F(p) = 2 \int d\vec{q} K_1(\vec{p}, \vec{q}; E)G(\vec{q}), \tag{9}$$

$$[\lambda_c^{-1} - h_c(p)]G(p) = 2 \int d\vec{q} K_2(\vec{p}, \vec{q}; E)F(\vec{q}) + \int d\vec{q} K_3(\vec{p}, \vec{q}; E)G(\vec{q}), \tag{10}$$

where the kernels  $K_1, K_2, K_3$  alongwith the other symbols are given in ref.[9].

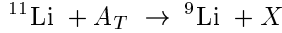
### 3. Results and comparison with experimental data

The above formalism has been employed to study the properties of halo nuclei such as  $^{11}\text{Li}$ ,  $^{14}\text{Be}$ ,  $^{19}\text{B}$ ,  $^{22}\text{C}$  and  $^{20}\text{C}$ . The calculations to extract information on properties such as the binding energy, the momentum distribution of the core and of the halo neutron through the spectator functions as well as the root mean square separations of  $n$ - $n$  and  $n$ -core pairs for  $^{11}\text{Li}$  have been discussed in detail and compared with the experimental data in ref.[9]. Thus, for instance, the equations (9) and (10) which essentially reduce to one variable integral equations after performing the angular integrations, have been numerically solved as an eigenvalue problem in terms of the two body strength parameter  $\lambda_c$  by feeding the value of the three body binding energy  $E$  as input. The numerical values of  $\lambda_c$  so computed are then compared with those determined through the two body analysis. The results are compared in table 1.

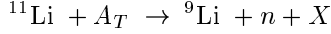
**Table 1.**

| B.E of $^{11}\text{Li}$ (MeV) | $\beta/\alpha$ | $\lambda_n/\alpha^3$ | $\beta_1/\alpha$ | $\lambda_c/\alpha^3$ | $\lambda_c/\alpha^3$ from 3-body |
|-------------------------------|----------------|----------------------|------------------|----------------------|----------------------------------|
| 0.34                          | 5.8            | 18.6                 | 5.0              | 10.32                | 12.92                            |
|                               | 6.3            | 23.4                 | 5.0              | 10.32                | 12.91                            |
|                               | 5.8            | 18.6                 | 5.5              | 14.00                | 17.01                            |
| 0.20                          | 5.8            | 18.6                 | 5.0              | 10.32                | 12.39                            |
|                               | 5.8            | 18.6                 | 5.5              | 14.00                | 16.38                            |

The solutions of the spectator functions  $F(p)$  and  $G(p)$  satisfying the integral eqs (9) and (10) describe the momentum distribution of the spectator particles i.e, of  ${}^9\text{Li}$  and of the halo neutron within the nucleus. Experimentally, momentum distribution of  ${}^9\text{Li}$  fragments following the break-up of  ${}^{11}\text{Li}$ , i.e



has been studied by Orr *et al* [16] to observe the momentum distribution of  ${}^9\text{Li}$ . Similarly, the experiments on the momentum distribution of neutrons through the reaction



using targets such as Be, C and Pb were carried out by Anne *et al* [17] to observe the momentum distribution of halo neutron in the nucleus. Figures (1) and (2) shown in ref. [9] compare reasonably well with the experimental findings.

From our 'master' three body equation (8), we can also make estimate of the correlations between the two halo neutrons and between a halo neutron and  ${}^9\text{Li}$  core within the nucleus. In fact, the function

$$\phi_{nn} = D^{-1}(\vec{p}_{12}, \vec{p}_3; E) g(p_{12}) \quad (11)$$

and

$$\phi_{nc} = D^{-1}(\vec{p}_{12}, \vec{p}_3; E) f(p_{23}) \quad (12)$$

describe the correlation between the two neutrons and between the neutron and  ${}^9\text{Li}$ .

Very recently kinematically complete measurements for Coulomb dissociation of  ${}^{11}\text{Li}$  into  ${}^9\text{Li} + 2n$  have been carried out by Ieki *et al* [18]. The data for the correlation functions deduced from the relative momentum spectrum between two neutrons in coincidence with  ${}^9\text{Li}$  has been compared in figure (3) of ref. [9]. Using these correlation functions we also compute the values of the root mean square radii  $\bar{r}_{nn}$  and  $\bar{r}_{nc}$  as a function of the three body binding energy. These are summarized in table 2.

With the help of these values, we then calculate the rms matter,  $\bar{r}_{\text{matter}}$ , for  ${}^{11}\text{Li}$  using the relation given in ref. [19].

$$\langle r^2 \rangle_{\text{matter}} = \frac{A_c}{A} \langle r^2 \rangle_{\text{core}} + \frac{1}{A} \langle \rho^2 \rangle, \quad (13)$$

where  $\langle r^2 \rangle_{\text{core}}$  is the mean square value of the radius of  ${}^9\text{Li}$  and the radial variable  $\rho^2 = x^2 + y^2$ ,  $\vec{x} = \vec{r}_1 - \vec{r}_2/\sqrt{2}$  and  $\vec{y} = \sqrt{(2A_c/A)} [(\vec{r}_1 + \vec{r}_2/2) - \vec{r}_3]$ . Taking the value of  $\sqrt{\langle r^2 \rangle_{\text{core}}}$  for  ${}^9\text{Li}$  to be 2.3 fm, we obtain the value of the matter radius to be 3.6 fm to be compared with the experimental value of  $3.14 \pm 0.06$  fm.

**Table 2.**

| B.E of ${}^{11}\text{Li}$ (MeV) | $\bar{r}_{nn}$ | $\bar{r}_{nc}$ |
|---------------------------------|----------------|----------------|
| 0.20                            | 10.63          | 10.93          |
| 0.25                            | 9.90           | 9.86           |
| 0.315                           | 8.93           | 8.87           |

3.1 Efimov states in halo nuclei like  $^{14}\text{Be}$ ,  $^{19}\text{B}$ ,  $^{22}\text{C}$  and  $^{20}\text{C}$

The above formalism has also been extended to address an intriguing question as to how far and under what conditions the two neutron halos characterized by large spatial extension and very low separation energy of the halo neutrons actually attain the Efimov limit so as to produce the Efimov states near the three body breakup threshold. Quantitatively, as pointed out by Efimov [20], so long as

$$r_0\sqrt{|E|} \ll 1 \quad \text{and} \quad r_0 \ll a, \quad (14)$$

where  $r_0$  is the range of the two body interaction,  $E$  is the energy of the three body system and  $a$  represents the scattering length of the virtual  $s$ -state of the two particles, the existence of resonating  $s$ -wave potentials results in the appearance of a three body long range interaction which goes as  $1/R^2$  where  $R^2 = r^2 + \rho^2$ . The existence of such a long range force should respond by producing a large number of states particularly near and below the three body break up threshold. With this objective we investigated in ref. [10] the possibility for the occurrence of the Efimov states in  $^{14}\text{Be}$  assuming a possibility for the existence of a low lying  $s$  orbital state for the halo neutrons with  $^{12}\text{Be}$  as core. The basic structure of the integral equations as given in ref. [9] is essentially the same. However, for the purpose of studying the sensitive computational details of the Efimov effect, we transform these equations involving only dimensionless quantities as discussed in ref. [10]. In table 3 are shown the results of three body calculations of energies of  $^{14}\text{Be}$  for the ground states and first two excited states ( $\epsilon_0, \epsilon_1$  and  $\epsilon_2$ ) for different excitation energies of the  $n-^{12}\text{Be}$  system.

From table 3 we note that at 50 keV virtual state energy of  $n-^{12}\text{Be}$  system, the three body binding energy for  $^{14}\text{Be}$ , as obtained from the set of integral equations, is 1350 keV which is in close agreement with the experimentally observed value. (Note that here the parameters of  $n-^{12}\text{Be}$  potential are taken as  $\lambda_1 = 11.71\alpha^3$ ,  $\beta_1 = 5.0\alpha$ .) However, this two body potential does not reproduce any excited state for  $^{14}\text{Be}$  system. As the virtual state energy of the  $n-^{12}\text{Be}$  system is lowered to 2 keV, the resulting three body system starts swelling and enters into the regime of Efimov effect. As a result we get two excited states besides the ground state energy of 1450 keV for the  $^{14}\text{Be}$  system. The same trend appear to follow in the case of Borromean nuclei like  $^{19}\text{B}$  and  $^{22}\text{C}$ . Here we find that by employing the realistic two body potentials for  $n-n$  and  $n$ -core systems, the three body equations can predict the three-body ground state energy in agreement with the experimental values. However, the input two body dynamics, does not allow the three body equation to admit any solution for the occurrence of the excited states near the break-up threshold. It is only

**Table 3.**

| $n-^{12}\text{Be}$ energy<br>(keV) | $\lambda_1$ | $a_s$<br>(fm) | $\epsilon_0$<br>(keV) | $\epsilon_1$<br>(keV) | $\epsilon_{02}$<br>(keV) |
|------------------------------------|-------------|---------------|-----------------------|-----------------------|--------------------------|
| 50                                 | 11.71       | -21           | 1350                  |                       |                          |
| 5.8                                | 12.32       | -62           | 1408                  | 0.1                   |                          |
| 2                                  | 12.46       | -105          | 1450                  | 2.6                   | 0.06                     |
| 1                                  | 12.52       | -149          | 1456                  | 3.8                   | 0.22                     |
| 0.1                                | 12.62       | -483          | 1488                  | 6.1                   | 0.62                     |
| 0.05                               | 12.63       | -658          | 1490                  | 6.4                   | 0.68                     |
| 0.01                               | 12.65       | -1491         | 1490                  | 6.9                   | 0.72                     |

by changing the parameters of the  $n$ -core potential so as to reduce the excitation energy to a few keV, that the Efimov region begins to develop and the excited states start appearing for the Borromean type halo nucleus.

The study of  $^{20}\text{C}$ , which is not Borromean, has been found rather interesting. Here we consider  $n-n-^{18}\text{C}$  system, where  $n-^{18}\text{C}$  has an observed bound state energy of about 160 keV. In this case, with the two body potential for  $n-^{18}\text{C}$ , we find that our equations predict the ground state energy of 3.5 Mev in close agreement with the experimental value. In addition, we get the Efimov states for energies around 150 keV. Thus a realistic potential for a binary  $n-^{18}\text{C}$  system which reproduces the ground state energy in agreement with the experimentally observed value also predicts three excited states around 150 keV. This is a striking result which needs to be confirmed experimentally. This has led us to establish in quantitative terms general criteria for the occurrence of Efimov states in such halo nuclei [21].

#### 4. Conclusion

The physics of the halo phenomena seems to be the *homefield* for *few body cluster models*. It is clear that to understand the structural properties of the  $2n$  halo nuclei, one has to go beyond the meanfield approach to few body theoretical procedures such as the one laid down by *Faddeev three body theory*. By employing the separable potentials for the binary systems we solve the three body systems representing the  $2n$  halo nuclei to compute their structural features. The present approach has the following *distinct advantages*;

1. It provides us direct information about the momentum distribution of single particles within the halo through the knowledge of the spectator function, which can be easily compared with the experimental data.
2. The information about the two body correlation function inside a halo nucleus can be obtained in a rather simple way from the solution of the three body wave-function.
3. It may be emphasized that since the three body observables are computed without resort to any approximation, any discrepancy between the observed and the calculated quantities should directly reflect on the inadequacy or limitation of the input two body potentials used.

To address the *deeper question* as to how these *new clustering features* emerge from the nuclear many body theory is a challenge (to theorists) for the future.

#### Acknowledgment

The financial grant from the UGC is gratefully acknowledged under the research project No.10-4/95(SR-I)

#### References

- [1] P G Hansen, A S Jensen and B Jonson, *Ann. Rev. Nucl. Part. Sci.* **45**, 591 (1995)

- [2] A C Mueller and B M Sherrill, *Ann. Rev. Nucl. Part. Sci.* **43**, 523 (1991)
- [3] I Tanihata *et al*, *Phys. Lett.* **B160**, 380 (1985); **B206**, 592 (1988)
- [4] T Kobayashi *et al*, *Phys. Rev. Lett.* **60**, 2599 (1988); see also *Phys. Lett.* **B232**, 51 (1989)
- [5] Y Suzuki and Y Tosaka, *Nucl. Phys.* **A517**, 599 (1990)  
G F Bertsch and J Foxwell, *Phys. Rev.* **C41**, 1300 (1990)  
G F Bertsch and H Esbensen, *Ann. Phys.* **209**, 327 (1991)
- [6] I Tanihata *et al*, *Phys. Lett.* **B160**, 380 (1985); see also *Nucl. Phys.* **A522**, 275C (1991)
- [7] E Arnold *et al*, *Phys. Lett.* **B197**, 398 (1987); see also *Phys. Lett.* **B281**, 16 (1992)
- [8] B Blank *et al*, *Z. Phys.* **A343**, 375 (1992)
- [9] S Dasgupta, I Mazumdar and V S Bhasin, *Phys. Rev.* **C50**, R550 (1994)
- [10] I Mazumdar and V S Bhasin, *Phys. Rev.* **C56**, R5 (1997)
- [11] A Arima *et al*, *Nucl. Phys.* **A506**, 271 (1990)
- [12] W Koepf *et al*, *Z. Phys.* **A340**, 119 (1991)
- [13] G Bertsch and J Foxwell, *Phys. Rev.* **C42**, 758 (1990)
- [14] M Zhukov *et al*, *Phys. Rep.* **231**, 151 (1993)
- [15] L D Faddeev, *Sov. Phys. JETP* **12**, 1014 (1961)
- [16] N A Orr *et al*, *Phys. Rev. Lett.* **69**, 2050 (1992)
- [17] A Anne *et al*, *Phys. Lett.* **B250**, 19 (1990)
- [18] K Ieki *et al*, *Phys. Rev. Lett.* **70**, 730 (1993)
- [19] D V Fedorov, A S Jensen and K Riisager, *Phys. Lett.* **B312**, 1 (1993)
- [20] V N Efimov, *Comments Nucl. Part. Phys.* **19**, 271 (1990)
- [21] I Mazumdar, Vandana Arora and V S Bhasin, to be published