PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 57, No. 4
October 2001
pp. 689–700

# Measuring the top quark mass in the $e\mu$ channel: A study

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MS received 2 April 2001; revised 17 May 2001

**Abstract.** We describe a simple method to measure the top quark mass in the $t\bar{t} \to WbW\bar{b} \to e\mu$ channel that may be useful in Run II of DØ detector. The method is validated by applying it to the Run Ib $e\mu$ data.

**Keywords.** Top quark; Tevatron; DØ.

**PACS No. 14.65**

## 1. Introduction

The discovery of the top quark [1] and the measurement of its mass ($174 \pm 5.1$ GeV) [2] were the main scientific achievements of Run I (total integrated luminosity of 76.6 pb$^{-1}$) of DØ detector at Fermilab, USA. (For a thorough review see ref. [3].) Today, the top quark mass is known to a precision that was unanticipated at the start of Run I. Run II promises a much larger data-set (about 40–100 times larger) than that collected in Run I. Such a data-set could, in principle, yield a substantially smaller uncertainty in our knowledge of the top quark mass. However, to achieve that goal it will be necessary to measure the top mass in as many channels as possible, and to explore a variety of methods [6,10]. This paper describes another simple approach to measure top mass that could be useful in Run II. To assess how well it works, we have applied it to the Run Ib $e\mu$ data.

### 1.1 *The method*

We observe that any quantity that depends upon the top quark mass can be used to measure it. In particular, it is not necessary that these quantities be the result of kinematic fits. Moreover, we would expect the precision of the mass estimate to improve as more mass-dependent quantities are included in the analysis, provided that the quantities are sufficiently uncorrelated. This strategy was successfully exploited by CDF in their top quark mass measurement in the di-lepton channel [4]. DØ [5] used a variation of the Kondo [6], Dalitz–Goldstein [7] method.

In this study we consider the decay

$$p\bar{p} \to t\bar{t} \to WbW\bar{b} \to e\mu + X$$

in which one top quark yields a $b$ quark and an electron ($e$) and the other a $b$ quark and a muon ($\mu$). The main disadvantage of the $t\bar{t}$ di-lepton channels is their small branching ratio. For the $e\mu$ channel the branching ratio is 2.4% to be compared with 15% for a single lepton channel. However, the di-lepton channels have smaller backgrounds, which is especially true in the $e\mu$ channel whose principal backgrounds are from $Z \to \tau\tau \to e\mu$ and $WW \to e\mu$.

Our method is based on the following mass-dependent quantities:

$$x_1 = \sqrt{(2p_1 \cdot p_2)}, \tag{1}$$

$$x_2 = \sqrt{(2e \cdot p_1 + m_W^2)}, \tag{2}$$

$$x_3 = \sqrt{(2e \cdot p_2 + m_W^2)}, \tag{3}$$

$$x_4 = \sum_j E_T^j, \tag{4}$$

where $e$, $p_1$ and $p_2$ are, respectively, the four-vectors of the electron and two highest transverse momentum jets and $m_W$ is the mass of the $W$ boson. The quantities $x_2$ and $x_3$ are inspired by the arguments presented in ref. [8]. Ideally, we should keep only one of the two variables: the one in which the lepton and the jet arise from the same top quark. The distribution of that variable is bounded between $m_W$ and $m_t$. We keep both variables, however, because at present we are unable to choose the right pairing with sufficiently high probability [9].

Since there are two leptons in the event, we can form two more variables like $x_2$ and $x_3$ but using the muon four vector. However, we have chosen not to use the muon information because of its relatively modest precision. In Run II, we anticipate a significant improvement in the muon momentum measurement, because of the introduction of a central magnetic field. It should then be possible to use the muon four vector and thus augment the aforementioned variables with two more.

Figures 1–4 show the predicted dependence of the mean values of these variables on the top quark mass. We see that in each case we can approximate the dependence on the top quark mass by a linear function, as indicated in each figure. Plots of the distribution of the estimated top mass (from mean of each of the variables $x_1, \ldots, x_4$) for each MC sample of mass 150, 180, 200, 250 GeV are shown in figure 5. From this figure, it is clear as our simple procedure works in as it provides a good estimate of the top quark mass.

From a sample of $e\mu$ candidates we compute the mean values of $x_1$ to $x_4$ and infer from each quantity a top quark mass $m_i$. We take the weighted sum

$$\hat{m}_t = \sum_{i=1}^{4} w_i m_i, \tag{5}$$

as our final estimate of the top quark mass, where $w_i = 1/\mathrm{Var}[m_i]/\sum_i 1/\mathrm{Var}[m_i]$ and $\mathrm{Var}[m_i]$ is the variance of the distribution of the mass estimate $m_i$. These weights minimize the

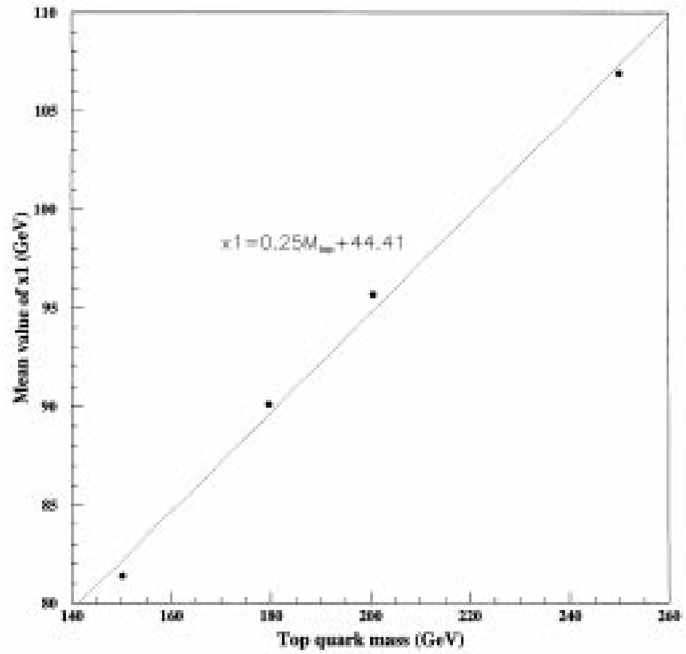


**Figure 1.** Dependence of the mean value of the variable $x_1$ on the top quark mass.

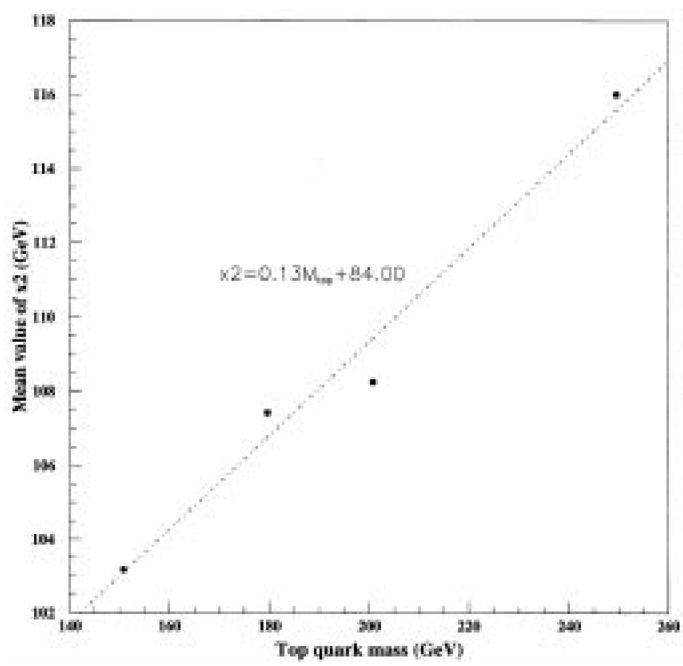


**Figure 2.** Dependence of the mean value of the variable $x_2$ on the top quark mass.

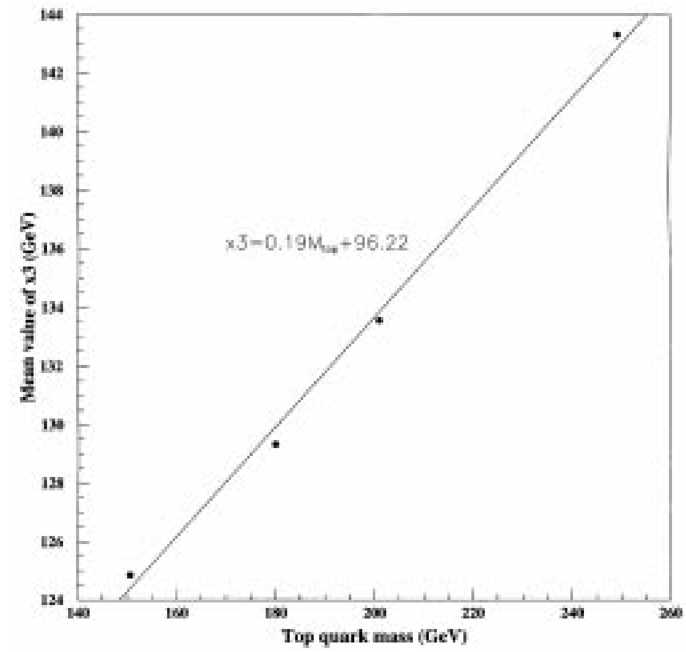


**Figure 3.** Dependence of the mean value of the variable $x_3$ on the top quark mass.

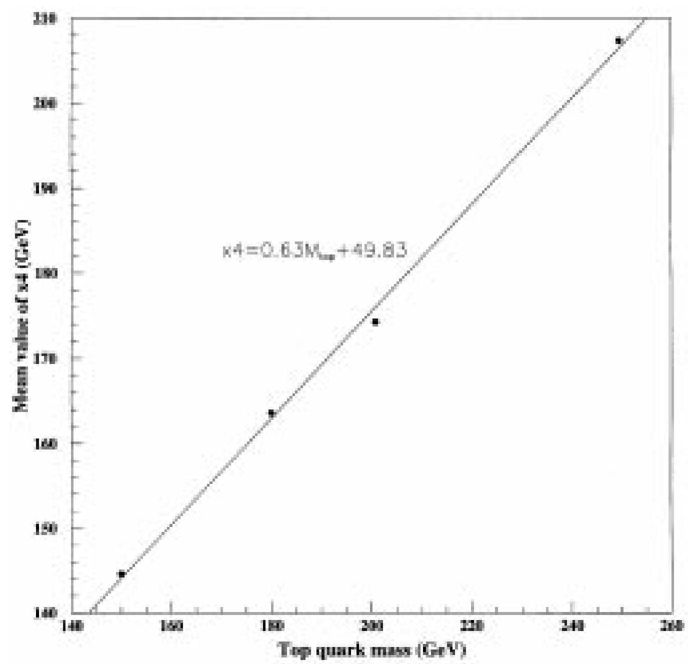


**Figure 4.** Dependence of the mean value of the variable $x_4$ on the top quark mass.

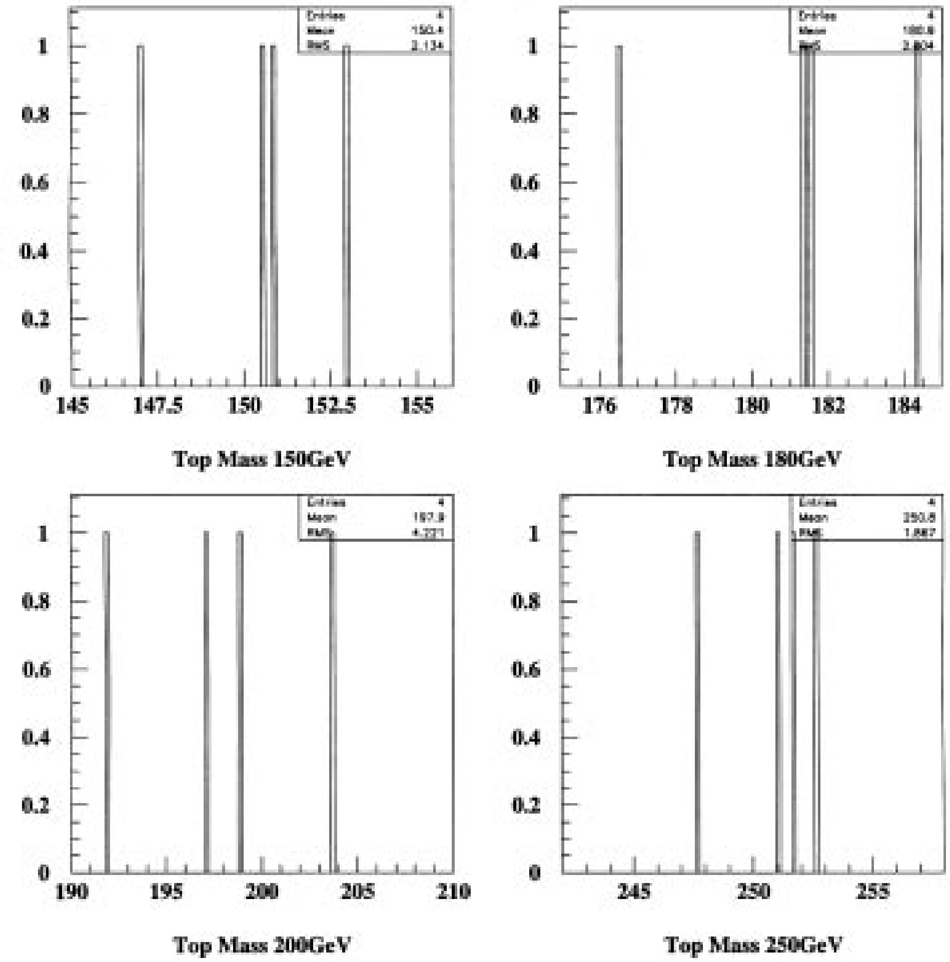


**Figure 5.** Estimated top mass from mean of each of the four vectors for different signal MC.

variance of the distribution of $\hat{m}_t$ when the $m_i$ are uncorrelated. There is, however, some correlation between these variables. For this reason it is better to estimate the error on $\hat{m}_t$ directly from the $\hat{m}_t$ distribution [10].

## 2. Data sets

### 2.1 *Signal and background modeling*

*Signal Monte Carlo*: Event samples ($t\bar{t} \to e\mu$) for four different top quark masses ($m_t = 150, 180, 200, 250$ GeV), generated with ISAJET, have been used in our analysis. These events ($\sim$ 10 K for each top quark mass) were processed with DØGEANT and DØRECO before being subjected to our analysis. The number of events and cross-sections for MC samples of different top quark masses are given in table 1.

*Background Monte Carlo*: The two major backgrounds in the $e\mu$ channel are:

- $Z \to \tau\tau \to e\mu$: This background MC-sample was generated with ISAJET.
- $WW \to e\mu$: This background sample was generated with PYTHIA.

The samples were also processed with DØGEANT and DØRECO. The number of events for the backgrounds $Z\tau\tau$ and $WW$ are **1896** and **2497**, respectively. All the cross-sections used are based on published values rather than those produced by the generators.

### 2.2 *$e\mu$ Data set*

The data used in this analysis were collected taking run with DØ detector. We are using Run Ib data. The Run Ib data were collected using a number of level 2 triggers, which are listed in table 2. An event was required to pass any one of these triggers, that is, the selection procedure follows the logic – OR of the triggers. Table 3 shows the number of events that remain after applying all the object identification criteria (for the electron and the muon), and the level 2 trigger cuts.

**Table 1.** Signal Monte Carlo sample.

| Sample (GeV) | $N_{\text{events}}$ | Cross-section (pb) |
|---|---|---|
| 150 | 8804 | $0.80 \pm 0.14$ |
| 180 | 9052 | $0.29 \pm 0.11$ |
| 200 | 9949 | $0.15 \pm 0.10$ |
| 250 | 10166 | $0.05 \pm 0.08$ |

**Table 2.** Level 2 triggers used in the selection of the $e\mu$ data sample in Run Ib.

| Trigger | Definition |
|---|---|
| MU-ELE | 1 electron, $E_T > 10$ GeV<br>1 muon, $p_T > 8$ GeV |
| ELE-JET-HIGH | 1 electron, $E_T > 15$ GeV, $\lvert\eta\rvert < 2.5$<br>2 jets ($\Delta R = 0.3$), $\lvert\eta\rvert < 2.5$ and $E_T > 10$ GeV<br>$\not{E}_T > 14$ GeV |
| MU-JET-HIGH | 1 muon, $p_T > 10$ GeV<br>1 jet ($\Delta R = 0.7$), $E_T > 15$ GeV, $\lvert\eta\rvert < 2.5$ |
| MU-JET-CENT | 1 muon, $\lvert\eta\rvert < 1.0$ (CF) and $p_T > 10$ GeV<br>1 jet ($\Delta R = 0.7$), $E_T > 15$ GeV |

**Table 3.** Number of data events passing trigger + selection cuts.

| Selection cut | Event yield (76.6 pb$^{-1}$) |
|---|---|
| Initial selection: $E_T^e \geq 10$ GeV, $\lvert \det \cdot \eta^e \rvert < 3.3$ and $p_T^\mu \geq 7$ GeV, $\lvert \eta^\mu \rvert < 2.5$ (no id) | $\approx 500$ K |
| $E_T^e > 15$ GeV, $p_T^\mu > 15$ GeV/c + $\lvert \eta \rvert$ + trigger and full $\mu$ and $e$ ID cuts | 122 |
| Final selection: selection cuts (Section 2.2) | 2 |

To reduce the backgrounds while maximizing the top signal detection efficiency (to have a good signal to background ratio), we tried to optimize the kinematical (selection) cuts. The event selection criteria are:

(i) Require isolated muons: $\Delta R_{\mu,\text{jet}} \geq 0.5$,
(ii) $\not\!\!E_T > 10$ GeV (muon corrected),
(iii) $\not\!\!E_T^{\text{cal}} > 20$ GeV (calorimeter),
(iv) $\Delta R_{e\mu} > 0.25$,
(v) Require 2 jets, $E_T^{\text{jet}} > 15$ GeV,
(vi) $H_T$ $(E_T^e + \sum_{\text{all jets}} E_T^{\text{jet}}) > 120$ GeV.

The same cuts are applied to the MC samples, except the object identification (ID) and trigger cuts. After applying all the cuts (trigger + selection cuts) we are left with 2 candidate events in this channel as shown in table 3. The four vector distributions for the $e\mu$ events finally left are shown in figure 6.

### 2.3 *Data vs Monte Carlo models*

In order to assess how well our Monte Carlo model agrees with the data, we fitted a weighted sum of signal and background distributions to the corresponding distribution for data using a Bayesian method [11]. The results from the fits agree with what we expect from the assumed cross-sections for $Z$ and $WW$. One such fit is shown in figure 7, which shows reasonable agreement between the Monte Carlo background model and the data.

### 2.4 *Final sample*

The Bayesian fit of signal and background to data gives:

$$n_{Z\tau\tau} = 53.7^{+10.9}_{-9.9} \text{ events}$$

and

$$n_{WW} = 43.3^{+11.4}_{-9.6} \text{ events,}$$

which yields estimates of the number of background events in 76.6 pb$^{-1}$, for $WW \to e\mu$ and $Z \to \tau\tau \to e\mu$, of 0.283 and 0.069 events, respectively, to be compared with the 2 data events that survive the cuts.

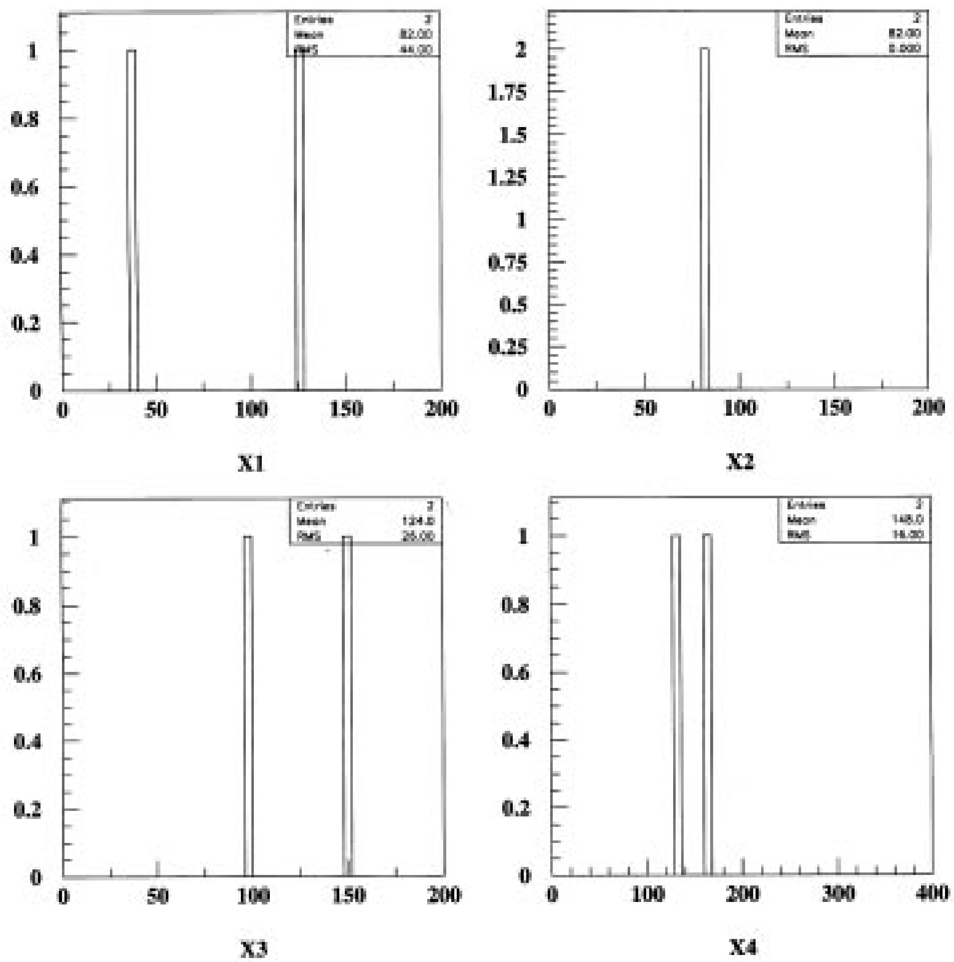


**Figure 6.** Four vector distributions for the $e\mu$ events finally left.

## 3. Analysis

### 3.1 *Data*

The distributions of the variables that have been used in this analysis are shown in figure 8. The variables used are: $E_T$ and $\phi$ for the leading electron and leading muon ($E_T^e$, $p_T^\mu$, $\phi_e$ and $\phi_\mu$), the number of jets ($N_{\text{jet}}$), the invariant mass ($M_{e\mu}$) of the $e\mu$ pair and the azimuthal separation between the leptons, $\Delta\phi_{e\mu}$. The values of $x1, \ldots, x4$ obtained using the procedure described above (section 1.1.1) for the two $e\mu$ events are as shown in figure 6:

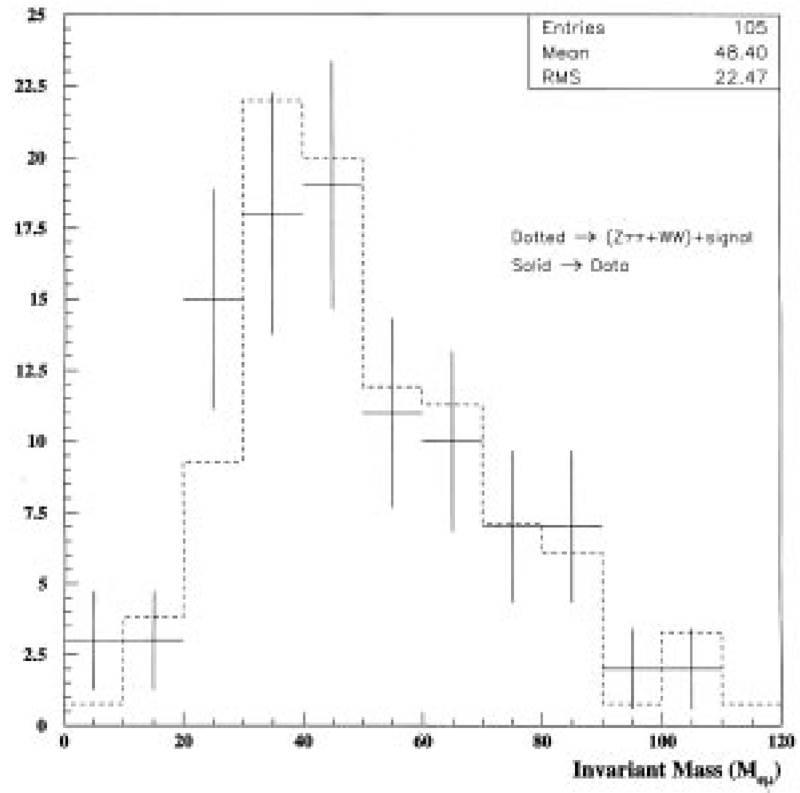


**Figure 7.** Bayesian fit of signal + background to data. The vertical bars represent the statistical errors on data.

| | Event 1 (GeV) | Event 2 (GeV) |
|---|---|---|
| $x1$ | 40 | 124 |
| $x2$ | 82 | 82 |
| $x3$ | 98 | 150 |
| $x4$ | 126 | 160 |

These values should be corrected for the background. However, we neglect this correction since its effect is small compared with that arising from the small sample size. From $x1,\ldots,x4$, we produce four estimates of the top quark mass:

| Mean $xi$ (GeV) | Mass estimate (GeV) |
|---|---|
| 82.00 | 149.42 |
| 82.00 | −15.78 |
| 124.00 | 148.48 |
| 148.00 | 156.22 |

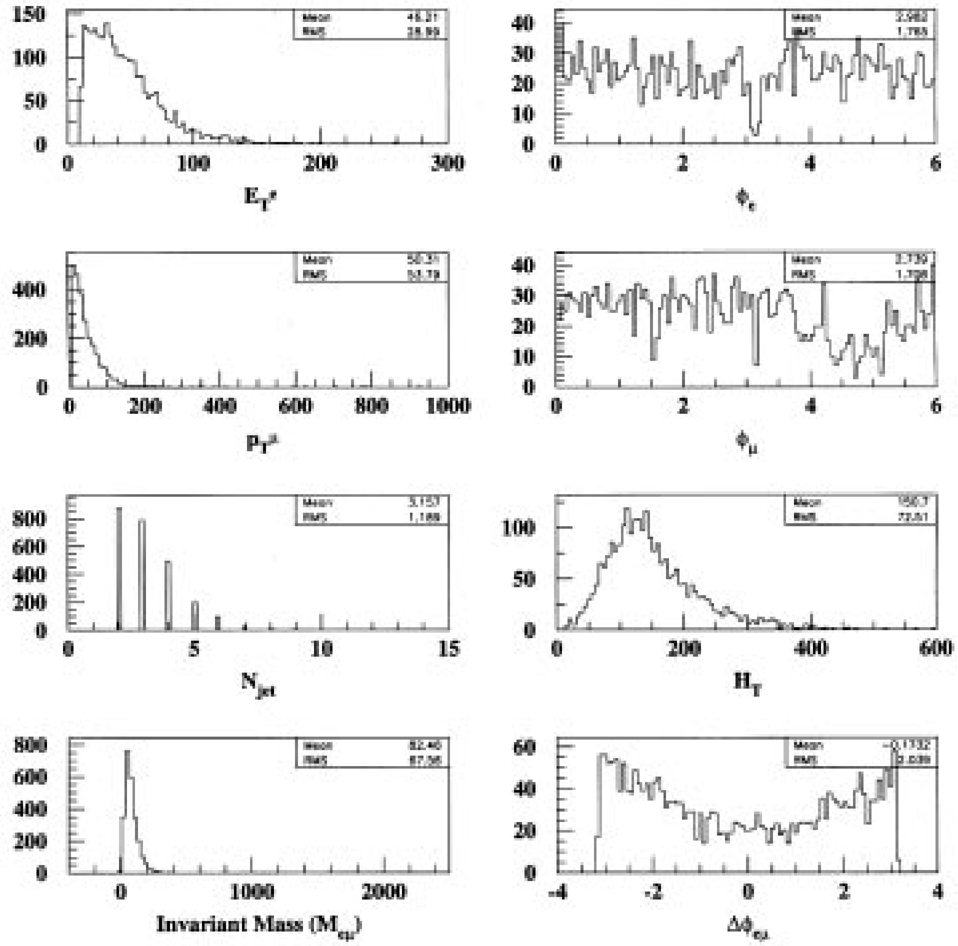

**Figure 8.** Distributions of the variables for MC sample for mass 180 GeV.

We note that $x2$ has led to a negative top mass estimate! We can either keep or throw such estimates away. We chose to exclude unphysical estimates.

In the following we describe simple Monte Carlo experiments that have been used to estimate the uncertainties in these estimates.

### 3.2 *Monte Carlo*

For each top quark mass, we generated many sub-samples of events whose sample size is fixed at the observed sample size, namely 2. Each sub-sample is analysed in the same way as the observed sample. In particular, we calculate the means of $x_1, \ldots, x_4$ and deduce 4 estimates of the top quark mass, excluding unphysical estimates. Finally, we combine

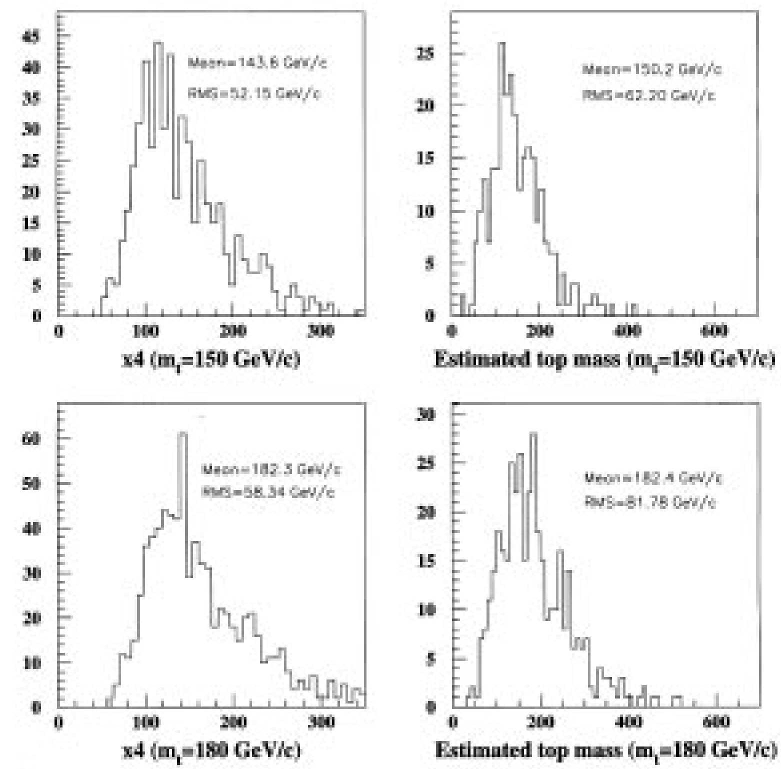


**Figure 9.** Distributions of estimated top masses from one of the four vector variable.

these 4 estimates into an overall estimate using eq. (5). Plots of the distribution of the estimated top mass (from the variable $x_4$) for each mass 150, 180 GeV are shown in figure 9. From this figure, it is clear that our simple procedure works in that it provides a good estimate of the top quark mass. These plots also indicate the typical (frequentist) uncertainty on our mass estimates.

### 3.3 *Results*

Taking the weighted sum according to eq. (5), our overall top quark mass estimate, using the 2 observed events, comes out to be 153 GeV. In figure 9, the plots indicate the distribution of estimated top mass based on samples of size 2 consisting of a single measurement of the observable $x_4$ for each mass 150 GeV and 180 GeV. Estimated uncertainties ($\sim$ 60–80 GeV) as indicated on the distributions are acceptable at this stage as we are devising the method which can be used with larger data-sets from Run II. The uncertainties in the top mass estimates is indeed insensitive to the experimental resolution being essentially the standard deviation divided by $\sqrt{N}$, and this value of $N$ will be quite large in Run II, and hence the small uncertainty.

## 4. Conclusions

We have described a very simple method that identifies several mass-dependent variables each yielding a mass estimate. A weighted average of the mass estimates was shown to yield a useful estimate of the top quark mass. The method is particularly suited to very pure samples of events. In that regard, $e\mu$ events are attractive because the backgrounds can be rendered negligible, especially if one uses more sophisticated event selection methods than the one we have used. With the larger data sets anticipated from Run II it ought to be possible to produce pure samples of $t\bar{t} \to e\mu$ events and to reduce the uncertainty in the top quark mass.

## Acknowledgements

The authors would like to thank all their collaborators and staff of DØ. Special thanks to H B Prosper for useful discussions and encouragement. They thank Vipin Bhatnagar for his help. Financial support by DST is acknowledged.

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[9] It would be better to weigh each variable according to the probability that it correctly pairs the lepton and *b*-jet. However, in this study we have not pursued this option
[10] One can also do things right: minimize eq. (5), subject to the constraint $\sum_i w_i = 1$, taking into account the fact that the $m_i$ are correlated
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