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Analysis of a shielded $\mathbf{TE_{011}}$ mode composite dielectric resonator for stable frequency reference

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Abstract. Analysis of a TE_{011} mode composite sapphire–rutile dielectric resonator has been carried out to study the temperature variation of resonance frequency, close to the Cs atomic clock hyperfine frequency of 9.192 GHz. The complementary behavior of dielectric permittivity with temperature of the composite has been exploited to obtain the desired turning point in the resonant frequency. The frequency of the composite structure is found to be independent of the shield diameter beyond four times the puck diameter.

Keywords. Shielded dielectric resonator; high temperature superconductor.

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1. Introduction

The paper presents the numerical analysis of a cryogenically cooled, shielded composite dielectric resonator towards the design of a stable frequency reference at 9.192 GHz, close to Cs atomic clock hyperfine frequency. We address the requirements of a stable frequency reference using HTS–composite–HTS dielectric resonators (DR). This device shows great promise in complementing the extremely high stability achieved with atomic frequency standards for long averaging time. The microwave resonator based on composite dielectric materials, with opposite temperature coefficient of permittivity (TCP), can provide a turning point in the resonant frequency with temperature [1] that relaxes the temperature control requirement to permit high frequency stability. Such technique incorporating dielectric compensation to a whispering gallery mode has been reported [2,3].

2. Results and discussion

A systematic analysis of TE_{011} mode composite DR, shown in figure 1, has been carried out to study the temperature dependence of the resonant frequency. The TCP of sapphire and rutile has complimentary behavior given by the following expressions:

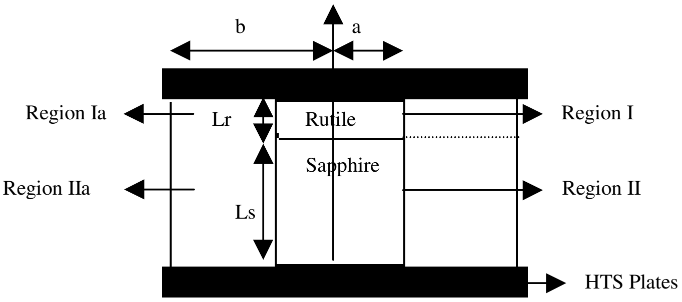

Figure 1. Schematic of a composite dielectric resonator.

$$\varepsilon_\mathrm{r}(T) = 113.446 + 0.043T - 0.002T^2 + 7.724 \times 10^{-6}T^3 - 1.072 \times 10^{-8}T^4$$

$$\varepsilon_\mathrm{s}(T) = 9.6 + 2.54 \times 10^{-11}T^4 .$$

On applying the continuity conditions of the electromagnetic fields at the interfaces of various regions in the DR a set of following eigenvalue equations are obtained, the roots of which yield, using numerical technique in polar coordinates, the eigen frequencies

$$P1(f) = \xi_\mathrm{s} J0(\xi_\mathrm{s} \cdot a) G1(a) + \xi_\mathrm{IIa} G0(a) J1(\xi_\mathrm{s} \cdot a)$$

$$P2(f) = \xi_\mathrm{s} J1(\xi_\mathrm{r} \cdot a) F0(a) + \xi_\mathrm{Ia} F1(a) J0(\xi_\mathrm{r} \cdot a)$$

and $R1(f) = P1(f) + P2(f)$.

$$\xi_\mathrm{s}^2 = \frac{\omega^2 \varepsilon_\mathrm{s}^2(T)}{c^2} - \beta_s^2 \qquad \text{and} \qquad \xi_\mathrm{r}^2 = \frac{\omega^2 \varepsilon_\mathrm{r}^2(T)}{c^2} - \beta_\mathrm{r}^2 ,$$

$$\beta_\mathrm{s} = \frac{n\pi}{L_\mathrm{s}} \qquad \text{and} \qquad \beta_\mathrm{r} = \frac{n\pi}{L_\mathrm{r}} .$$

Here ξ_s, ξ_r and ξ_a are the radial wave numbers in sapphire, rutile and air, respectively and β's are the corresponding propagation constants in the z-direction, ω is the angular resonance frequency and c is the velocity of light. Further

$$F0(\rho) = I0(\xi_\mathrm{Ia} \cdot \rho) + K0(\xi_\mathrm{Ia} \cdot \rho) \frac{I1(\xi_\mathrm{Ia} \cdot a)}{K1(\xi_\mathrm{Ia} \cdot a)} ,$$

$$F1(\rho) = -I1(\xi_\mathrm{Ia} \cdot \rho) + K1(\xi_\mathrm{Ia} \cdot \rho) \frac{I1(\xi_\mathrm{Ia} \cdot a)}{K1(\xi_\mathrm{Ia} \cdot a)} ,$$

$$G0(\rho) = I0(\xi_\mathrm{IIa} \cdot \rho) + K0(\xi_\mathrm{IIa} \cdot \rho) \frac{I1(\xi_\mathrm{IIa} \cdot a)}{K1(\xi_\mathrm{IIa} \cdot a)} ,$$

$$G1(\rho) = -I1(\xi_\mathrm{IIa} \cdot \rho) + K1(\xi_\mathrm{IIa} \cdot \rho) \frac{I1(\xi_\mathrm{IIa} \cdot a)}{K1(\xi_\mathrm{IIa} \cdot a)} .$$

Here $J0$, $J1$, $I0$, $I1$, $K0$ and $K1$ are the Bessel and Hankel functions of the first and second kind, and a represents the radius of the composite dielectric puck. Figure 2 gives the

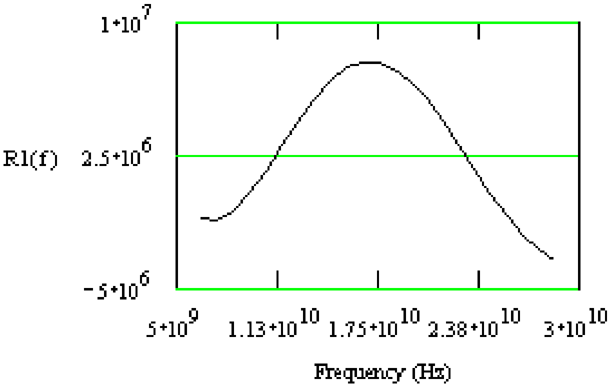

Figure 2. Variation of eigenvalue equation with frequency for $a = 1$ mm, $b = 8.45$ mm, $L_r = 2.39$ mm and $L_s = 5.1$ mm.

variation of eigenvalue equation with frequency for optimized dimensions, viz., a, b, L_r and L_s to get a desired resonance frequency. The frequency is found very sensitive to change in the aspect ratio $2a/L$ and shield diameter b. The first root of $R1(f)$ gives the resonance frequency of the fundamental mode. The next harmonic is found at around 23 GHz. This diagram gives important clue about the existence of higher harmonics in the resonator.

Figure 3 shows the variation in the fundamental mode resonant frequency with temperature. The point of inflection of frequency is obtained at around 66.25 K. The fractional frequency stability of $\sim 10^{-14}$ can be obtained provided temperature stability of 1 K is maintained. The point of inflection is due to the competing temperature dependences of dielectric permittivity $d\varepsilon/dT$ of sapphire and rutile and that of the penetration depth of HTS thin films. The variation of penetration depth with temperature, following two-fluid model, changes the effective length of the resonator. The point of inflection

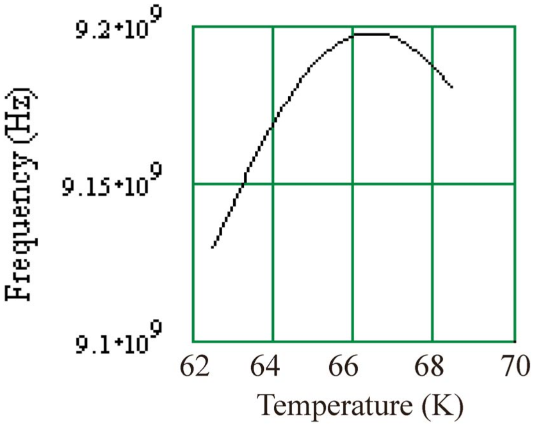

Figure 3. Variation of fundamental mode resonant frequency with temperature.

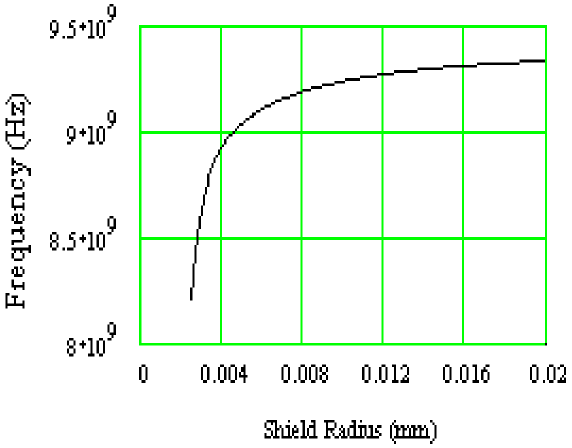

Figure 4. The dependence of resonant frequency on shield radius.

changes towards higher temperature values on decreasing the aspect ratio. Thus a desired turning point in temperature can be designed by tailoring the aspect ratio and the shield diameter.

Figure 4 shows the effect of the shield diameter on the resonance frequency. As can be seen the frequency tends to a stable value for $b > 8a$ for a fixed aspect ratio. The stability and the performance of the composite DR can be further improved by inserting a low permittivity region, say air, above the rutile puck. This low permittivity region would act as dielectric mirrors confining the EM energy into the rutile puck thus reducing the effect of the superconducting reactance. Gallop *et al* [1] has experimentally reported the fractional frequency stability for such a composite structure. The composite DR is highly compact being $\sim$ 40 times smaller in volume than a sapphire DR and $\sim$ 12 times smaller than a rutile DR at the same frequency and resonant mode.

3. Conclusions

The paper analyses the composite dielectric resonator in the fundamental mode. The studies show that the composite structure, along with rendering a point of inflection of frequency with temperature makes the structure compact in order to meet the cutting edge technology of miniaturization. The point of inflection of frequency with temperature has been obtained at the expense of Q factor. In order to make the DR less lossy, a low permittivity region can be introduced above the high permittivity region.

References

[1] J C Gallop, C D Langham, L Hao and F Abbas, *IEEE Trans. Instrum. Meas.* **46**, 122 (1997)
[2] M E Tobar, J G Harnett, E N Ivanov, D Cross, P Blondy and P Gullion, *IEEE Trans. Microwave Theory Tech.* **48**, 1265 (2000)
[3] C D Langham and J C Gallop, *IEEE Trans. Instrum. Meas.* **48**, 130 (1997)