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Ultrasonic attenuation in cuprate superconductors

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Abstract. We calculate the longitudinal ultrasonic attenuation rate (UAR) in clean d-wave superconductors in the Meissner and the mixed phases. In the Meissner phase we calculate the contribution of previously ignored processes involving the excitation of a pair of quasi-holes or quasi-particles. There is a contribution $\propto T$ in the regime $k_{\mathrm{B}}T \ll Qv_{\mathrm{F}} \ll \Delta_0$ and a contribution $\propto 1/T$ in the regime $Qv_{\mathrm{F}} \ll k_{\mathrm{B}}T \ll \Delta_0$. We find that these contributions to the UAR are large and cannot be ignored. In the mixed phase, using a semi-classical description, we calculate the electronic quasi-particle contribution to the UAR which at very low T, has a T independent term proportional to $\sqrt{H}$.

Keywords. Ultrasonic attenuation; cuprates; superconductivity.

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1. Introduction

The calculation of the UAR in d-wave superconductors has been considered earlier, both within the clean limit [1,2] and in the opposite hydrodynamic limit [3]. There exists evidence [4] that the superconducting state has well-defined quasiparticles corresponding to a d-wave BCS state. In the Meissner phase, the quasiparticles have a gap function that has nodes on the Fermi surface at the points $k_x = \pm k_y$. The low energy electronic excitations near the nodes are Dirac quasiparticles whose energies are $E^0_{\mathbf{k}} = \sqrt{v_{\mathrm{F}}^2 k_1^2 + v_\Delta^2 k_2^2}$. The UAR is determined primarily by the phonon lifetime. We consider here the phonon damping through the creation of electronic excitations.

2. Longitudinal ultrasonic attenuation in the Meissner phase

This calculation has been performed earlier by Vekhter *et al* [1]. They obtain a UAR which is linearly proportional to T at $T \ll \Delta_0$ where Δ_0 is the maximum amplitude of the superconducting gap. They normalize their result by dividing the low temperature UAR ($\alpha_S(T)$) by the UAR at T_{c}. Their calculation of $\alpha_S(T_{\mathrm{c}})$ assumes a parabolic band structure which is believed [5] to poorly describe the cuprates. Thus the expression in [1] used to extract the v_{F}/v_Δ ratio is erroneous as there is an unknown, Fermi surface sensitive, wave-vector dependence in their $\alpha_S(T_c)$ which has been missed. We indicate below how these shortcomings can be redressed.

The calculation of Vekhter *et al* ignores the contribution of terms corresponding to the creation of a pair of quasiparticles or quasiholes to the UAR. These terms are proportional to $\chi''_{\mathrm{rem}}(\mathbf{Q},\Omega) = X1 - X2$ where

$$X1 = \Sigma_{\mathbf{k}}[1 - n(E^0_{\mathbf{k}}) - n(E^0_{\mathbf{k+Q}})] \times \left(u^2_{\mathbf{k+q}} v^2_{\mathbf{k}} + \frac{\Delta_{\mathbf{k}}\Delta_{\mathbf{k+Q}}}{4E^0_{\mathbf{k}}E^0_{\mathbf{k+Q}}}\right)\delta(\Omega - E^0_{\mathbf{k}} - E^0_{\mathbf{k+Q}}) \tag{1}$$

and

$$X2 = \Sigma_{\mathbf{k}}[1 - n(E^0_{\mathbf{k}}) - n(E^0_{\mathbf{k+Q}})] \times \left(v^2_{\mathbf{k+q}} u^2_{\mathbf{k}} + \frac{\Delta_{\mathbf{k}}\Delta_{\mathbf{k+Q}}}{4E^0_{\mathbf{k}}E^0_{\mathbf{k+Q}}}\right)\delta(\Omega + E^0_{\mathbf{k}} + E^0_{\mathbf{k+Q}})\,. \tag{2}$$

These terms are neglected in the s-wave case as the low ultrasound frequency means that the delta function condition cannot be satisfied. In d-wave superconductors, the nodes in the gap imply that there is a finite phase space available over which these terms will contribute. We have explicitly calculated this contribution to the UAR (the details will be reported elsewhere [6]). We find that in the regime $k_{\mathrm{B}}T \ll Qv_{\mathrm{F}} \ll \Delta_0$ the UAR is linearly dependent on T. But the magnitude of this term is larger than that obtained in [1] approximately by a factor of 10 along the antinodal direction for parameters appropriate to the cuprates. This is understandable as the Fermi functions in eqs (1) and (2) are very small at low temperatures thus making the thermal factor in the terms considered by us $(1 - n(E^0_{\mathbf{k+Q}}) - n(E^0_{\mathbf{k}})$ larger than that $(n(E^0_{\mathbf{k+Q}}) - n(E^0_{\mathbf{k}}))$ occurring in the terms considered in [1]. In the other regime $Qv_{\mathrm{F}} \ll k_{\mathrm{B}}T \ll \Delta_0$ we find a contribution to the UAR that is proportional to $1/T$. As this term is additive with respect to the linear in T contribution obtained in [1], the ratio of the coefficients of T and $1/T$ in the measured attenuation rate α_S for two different choices of $\mathbf{Q}$ would enable us to extract the ratio $v_{\mathrm{F}}/v_{\Delta}$.

3. Longitudinal ultrasonic attenuation in the mixed phase

In this section we present our preliminary results for the quasiparticle contribution to the attenuation rate in the presence of a magnetic field $H_{\mathrm{c1}} \leq H \ll H_{\mathrm{c2}}$. The ultrasonic attenuation in the mixed state couples the lattice vibrations to the vortex motion. However, the dragging of the flux lines due to ionic displacements will be weak in the clean limit considered by us as pinning potentials are very weak and so these effects have been ignored in our work. In contrast to conventional s-wave superconductors, where the UAR comes primarily from the core bound states, here it will come mainly from nodal quasiparticles which are part of the scattering state spectrum.

To describe the nodal quasiparticles we make use of a semi-classical approximation first introduced in the context of the cuprates by Volovik [7]. Within this approximation, the currents associated with a vortex are treated as constant in the first instance. The attenuation rate is then determined to first order in the current. The position dependence of the current is now restored and the expression for α_S is then averaged over one unit cell of the vortex lattice to obtain the field dependent attenuation rate $\alpha_S(H, T)$. To first order in the current, the quasiparticle energy becomes

$$E_{\mathbf{k}} = E^0_{\mathbf{k}} + \mathbf{q} \cdot \nabla_{\mathbf{k}} \xi_{\mathbf{k}} \tag{3}$$

where $E^0_{\mathbf{k}}$ is the quasiparticle energy in the Meissner phase and $\xi_{\mathbf{k}}$ is the electronic band energy. The superconducting order parameter is assumed to be $\Delta(\mathbf{r}, \mathbf{r}') = \Delta_0(\mathbf{r} - \mathbf{r}') \exp \mathrm{i}\mathbf{q} \cdot (\mathbf{r} + \mathbf{r}')$ where Δ_0 is the order parameter in the abscence of $\mathbf{q}$. Then solving the associated Bogulibov equations the quasiparticle energies are found, to first order in $\mathbf{q}$, to be those given by eq. (3) and to the same order the coherence factors $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ and the gap $\Delta_{\mathbf{k}}$ are found to be unaffected. Making use of these results we calculate $\chi''(\mathbf{Q}, \omega)$ to obtain the UAR. Our partial results obtained so far, which only include processes corresponding to the creation of a quasiparticle and quasihole pair, indicate the existence of a temperature independent contribution to the UAR which is proportional to $\sqrt{H}$ whose origin is probably related to the 'Volovik effect' [7] which is the presence of a finite density-of-states at the Fermi energy whose magnitude is proportional to $\sqrt{H}$.

4. Conclusion

We have calculated the longitudinal ultrasonic attenuation rate in d-wave superconductors in the Meissner phase as well as the mixed phase. In the Meissner phase, we find that previously ignored terms are extremely important. In the mixed phase, our results indicate that there is a temperature independent contribution to the attenuation rate which goes as $\sqrt{H}$.

The shortcomings of our calculation are: (a) Non-inclusion of disorder effects which limits the applicability only to the cleanest samples. (b) We have ignored the incoherent part of the spectral function whose inclusion would require a microscopic theory incorporating strong electronic correlations.

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References

[1] I Vekhter, E J Nicol and J P Carbotte, *Phys. Rev.* **B59**, 7123 (1999)
[2] T Gupta and D M Gaitonde, *Mod. Phys. Lett.* **B15**, 269 (20001)
[3] J Moreno and P Coleman, *Phys. Rev.* **B53**, R2995 (1996)
[4] M Randeria *et al*, *Phys. Rev. Lett.* **74**, 4951 (1995)
[5] M R Norman *et al*, *Phys. Rev.* **B52**, 615 (1993)
[6] T Gupta and D M Gaitonde (to be published)
[7] G E Volovik, *JETP Lett.* **58**, 469 (1993)