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Metal–insulator crossover in high T_c cuprates: A gauge field approach

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Abstract. A metal–insulator crossover appears in the experimental data for in-plane resistivity of underdoped cuprates and a range of superconducting cuprates in the presence of a strong magnetic field suppressing superconductivity. We propose an explanation for this phenomenon based on a gauge field theory approach to the t-J model. In this approach, based on a formal spin-charge separation, the low energy effective action describes gapful spinons (with a theoretically derived doping dependence of the gap $m_s^2 \sim \delta|\ln\delta|$) and holons with finite Fermi surface ($\varepsilon_F \sim t\delta$) interacting via a gauge field whose basic effect on the spinons is to bind them into overdamped spin waves, shifting their gap by a damping term linear in T, which causes the metal–insulator crossover. The presence of a magnetic field perpendicular to the plane acts by increasing the damping, in turn producing a big positive transverse in-plane magnetoresistance at low T, as experimentally observed.

Keywords. Resistivity; crossover; t-J.

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1. Introduction

We start by summarizing the relevant experimental features which we are aiming to explain theoretically.

1. The in-plane resistivity of strongly underdoped LSCO ($0.04 \lesssim \delta \lesssim 0.08$) and YBCO exhibits a metal–insulator (MI) crossover as T decreases [1,2], with a big positive transverse magnetoresistance at low T [3,4].
2. Superconducting samples of a variety of cuprates (LSCO, BSLCO, PCCO, ...) [5], in strong magnetic field (up to 60 Tesla) suppressing superconductivity also show a metal–insulator crossover for in-plane resistivity.

A basic difference between the two sets of data is given by the estimated value of $k_F\ell$ at the metal–insulator crossover, where ℓ is the mean free path, with $k_F\ell$ of $O(1)$ for the first set and of $O(10)$ for the second, apparently ruling out explanations based on localization.

Our main claims are the following:

1. The two phenomena have the same origin.

2. They can be qualitatively explained within the t-J model for cuprates using:
   a) A formal spin-charge separation of the electron field into a spinon and a holon field coupled by gauge fluctuations.
   b) Ioffe–Larkin rule: the physical resistivity is the sum of the resistivity due to spinons and the resistivity due to holons [6].

3. The metal–insulator crossover is due to a competition between short-range antiferromagnetic order (SRAFO), induced by spinons, and a linear in T dissipation due to gauge fluctuations, induced by holons.

4. The magnetic field decreases the damping, via the cyclotron effect, thus causing an increase of transverse magnetoresistance at low T.

2. Effective action

The theoretical treatment of the t-J model from which we derived the above claims is based on the following theorem [7,8]:

If we couple the fermions of the t-J model to a $U(1)$ gauge field, B_μ, gauging the global charge symmetry and to an $SU(2)$ gauge field, V_μ, gauging the global spin symmetry of the model, and we assume that the dynamics of the gauge fields is described by the Chern–Simons actions:

$$S_{\text{c.s.}}(B) = -\frac{1}{2\pi}\int \mathrm{d}^3x \varepsilon^{\mu\nu\rho} B_\mu \partial_\nu B_\rho \tag{1}$$

$$S_{\text{c.s.}}(V) = \frac{1}{4\pi}\int \mathrm{d}^3x Tr \varepsilon^{\mu\nu\rho}\left[V_\mu \partial_\nu V_\rho + \frac{2}{3} V_\mu V_\nu V_\rho\right], \tag{2}$$

then the gauged model so obtained is exactly equivalent to the original t-J model. Let us give an idea of the proof of the above theorem for the partition function. We expand the partition function of the gauged model in the first-quantized formalism in terms of worldlines of the fermions. After integrating out the gauge fields, the effect of the coupling to $B_\mu(V_\mu)$ is only to give a factor $\mathrm{e}^{-\mathrm{i}\pi/2}(\mathrm{e}^{\mathrm{i}\pi/2})$ for each exchange of the fermion particles, so that the two effects cancel each other exactly.

To the fermion field of the gauged model, χ_α (α spin index), we apply the spin-charge separation: $\chi_\alpha \sim H z_\alpha$, where H denotes a spinless fermion (holon) field and z_α a spin $\frac{1}{2}$ hard-core boson (spinon) field satisfying the constraint $\bar{z}_\alpha z_\alpha = 1$ which implements the 'no-double' occupation of the t-J model.

The above spin-charge decomposition, introduces a further $U(1)$ gauge symmetry:

$$z_\alpha(x) \to \mathrm{e}^{\mathrm{i}\Lambda(x)} z_\alpha(x), \quad H(x) \to H(x)\mathrm{e}^{-\mathrm{i}\Lambda(x)}, \tag{3}$$

with Λ a real gauge parameter, to which is associated a self-generated gauge field A_μ, analogous to the one appearing in the slave boson and slave fermion approaches [6,9].

In a 'mean-field-approximation' [8] to the gauged $U(1) \times SU(2)$ t-J model, in a region of parameters which should be compared with the 'pseudogap phase' of high T_c cuprates, the role of the three gauge fields is the following:

– B_{MFA} carries a flux π per plaquette, converting via Hofstadter mechanism the spinless holon H into a Dirac fermion with a 'pseudospin structure' related to the two Néel sublattices and exhibiting a ('small') Fermi surface with $\varepsilon_{\rm F} \sim t\delta$, δ being the doping concentration.

– $V_{\rm MFA}$ dresses the holons by spin vortices of opposite chirality in the two Néel sublattices. The spinons in the presence of this gas of 'slowly moving' dressed holons acquire a mass $m_{\rm s} \sim \sqrt{\delta|\ln\delta|}$ yielding SRAFO (in formula this is due to a coupling at large scales of the form $(V_{\rm MFA}^2 \bar{z}_\alpha z_\alpha)$). Self-consistency of this treatment relies on the inequality $\varepsilon_{\rm F} \sim t\delta \ll \varepsilon_{\rm s} \sim J\sqrt{\delta|\ln\delta|}$ for small δ.

– The self-generated 'photon' field A_μ couples the Fermi liquid of holons to the gapped spinons, described by a massive (CP^1) NLσ model.

The low-energy effective action for A is dominated by the contribution due to the gapless holons and the propagator of the transverse component A^T of the gauge field turns out to be, for ω, $|\vec{q}|$, $\omega/|\vec{q}| \sim 0$, of the form

$$\langle A^T A^T\rangle(\omega, [\vec{q}]) \sim \left(-\chi|\vec{q}|^2 + {\rm i}\kappa\frac{\omega}{|\vec{q}|}\right)^{-1} \tag{4}$$

where χ is the diamagnetic susceptibility and κ the Landau damping.

3. Resistivity and magnetoresistance

In a regime where one can neglect the 'photon' drag (self-consistent for not too low T) the resistivity is given by the sum of the resistivity, $\rho_{\rm s}$, due to the spinon-'photon' subsystem and the resistivity, $\rho_{\rm h}$, due to the holon–'photon' subsystem (Ioffe–Larkin rule). Each one of them can be evaluated through the Kubo formula

$$\rho_\sharp = \lim_{\omega\to}\left(\frac{I_m\Pi_\sharp(\vec{q}=0,\omega)}{\omega}\right)^{-1} \tag{5}$$

where $\Pi_\sharp$ denotes the corresponding current polarization bubble.

$\rho_{\rm h}$ is the resistivity of a Fermi liquid in the presence of gauge fluctuations and it was evaluated, taking into account also impurities, as [9]

$$\rho_h \sim \delta\left[\varepsilon_{\rm F}\tau_{\rm imp} + \left(\frac{T}{\varepsilon_{\rm F}}\right)^{4/3}\right], \tag{6}$$

where $\tau_{\rm imp}$ is the transport relaxation time due to impurities.

We evaluated $\rho_{\rm s}$ using an eikonal approximation for spinons, partially justified by their mass gap. The Fourier transform of the spinon-current bubble is dominated at large scales by a complex saddle point. Evaluating the contribution of small scales by scaling renormalization one obtains [10] that for small ω, $\vec{q} = \vec{0}$ the gauge fluctuations induce a binding of spinons in to a massive overdamped spin waves with a polar structure of the form $(\omega - 2\sqrt{m_{\rm s}^2 - {\rm i}cT/\chi})^{-1}$ where c is a positive constant $O(1)$. The T-damping appearing in this formula is due to the structure (4) of the transverse 'photon' propagator: for $|\vec{q}| < q_0 = (\kappa T/\chi)^{1/3}$, a momentum scale related to anomalous skin effect, the contribution of gauge fluctuations is dominated by the thermal factor $\sim T$ [9]. The complex saddle point quoted above is actually effective in the range $m_{\rm s}^2 \gtrsim T/\chi \gtrsim q_0 m_{\rm s}$, which for physical values of the parameters correspond to a region of temperature between few tens and few hundreds K. The 'photon' interaction induces also a wave-function renormalization yielding

$$\frac{\mathrm{Im}\,\Pi_{\mathrm{s}}(\vec{q}=\vec{0},\omega)}{\omega} \sim_{\omega\to 0} \mathrm{Im}\,\frac{\sqrt{\delta}}{(m_{\mathrm{s}}^2 - \mathrm{i}cT/\chi)^{1/4}}. \tag{7}$$

Combining the above results we obtain for the in-plane resistivity $\rho = \rho_{\mathrm{h}}$ (eq. (6)) $+\rho_{\mathrm{s}}$ with

$$\rho_{\mathrm{s}} = \frac{1}{\sqrt{\delta}} \frac{(m_{\mathrm{s}}^4 + (cT/\chi)^2)^{1/8}}{\sin\left[\frac{1}{4}\ \arctan(cT/\chi m_{\mathrm{s}}^2)\right]} \tag{8}$$

and for low T, $\rho_{\mathrm{s}} \gg \rho_h$.

The basic feature of the resistivity formula [eqs (6) and (8)] can be summarized as follows: for low T the effect of the spinon gap is dominating, leading to an insulating behavior $\rho \sim 1/T$; at higher temperatures we find a metallic behavior $\rho \sim T^{1/4}, T^{4/3}$, due to the dissipation induced by gauge fluctuations becoming the dominant effect. Therefore a MI crossover is recovered decreasing the temperature (see figure 1, top). This crossover is determined by the interplay between the AF correlation length $\xi \sim (\delta|\ln\delta|)^{-1/2}$ and the thermal de Broglie wavelength $\lambda \sim (\chi/T)^{1/2}$. When $\lambda \lesssim \xi$ the 'peculiar' localization effect due to SRAFO is not 'felt' and a metallic behavior is observed. In the limit $\lambda \gg \xi$ we found the insulating behavior. The minimum of ρ as a function of T, $T_{\mathrm{MF}}(\delta)$, is weakly decreasing in δ, due to a delicate cancellation of doping dependence in the ratio λ/ξ [since $\chi \sim t/\delta$, $\lambda/\xi \sim (T/(t|\ln\delta|))^{1/2}$], in qualitative agreement with experiments. The resistivity curves derived from (6) and (8) exhibit an inflection point (around 200–300 K), also found in the experimental data. One might try to identify this temperature as the pseudogap temperature, corresponding to a crossover to new 'phase' where our approximations are no more valid.

A magnetic field H perpendicular to the plane modifies the contribution of the saddle point dominating the behavior of ρ_{s}, via the cyclotron effect, by reducing the damping cT/χ in (7) to $(cT/\chi(H) - \varepsilon^2 H^2/3q_0^2)$, where $0 < \varepsilon < 1$ is the effective 'charge' of spinons and $\chi(H)$ is the diamagnetic susceptibility in the presence of H, given by $\chi + (\sigma_{\mathrm{h}}^2(H)/4\pi^2\gamma)$, with γ the gap of A^0 and $\sigma_{\mathrm{h}}(H)$ the holon Hall conductivity [11]. Similar changes occur in the spinon resistivity, $\rho_{\mathrm{s}}(H)$, which exhibits a low T behavior

$$\rho_{\mathrm{s}}(H) \sim \left(\frac{cT}{\chi(H)m_{\mathrm{s}}^2} - \frac{\varepsilon^2 H^2}{3q_0^2 m_{\mathrm{s}}^2}\right)^{-1}. \tag{9}$$

The physical resistivity $\rho(H)$ still exhibit a MI crossover. The region of divergence of $\rho_{\mathrm{s}}(H)$ is outside the range of validity of the saddle point used in the above calculation; however, the shift in the MI crossover to higher temperature, due to the shift of the divergent temperature in (9), induces a big positive transverse in-plane magnetoresistance $(\rho(H) - \rho)/\rho[\equiv \Delta R(H)/R(0)]$, at low T (scaling quadratically in H), and comparison with experimental data in LSCO [4], shows a semi-quantitative agreement [11] (see figure 1, bottom). This effect was absent in previous gauge-field treatments of resistivity in magnetic field, where the leading effect (still present in our approach but subdominant) was attributed to the modification of the diamagnetic susceptibility ($\chi \to \chi(H)$), which in the region where dissipation dominates leads to a negative magnetoresistance at low T [12].

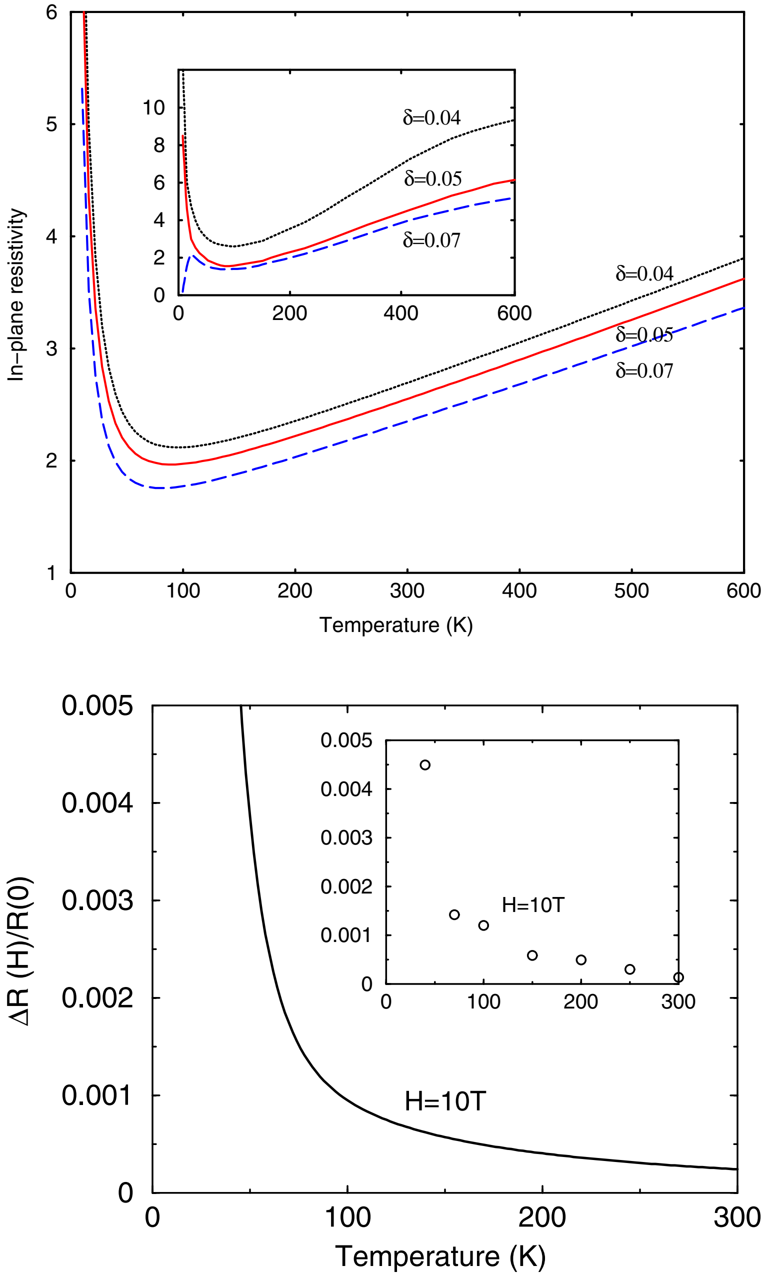

Figure 1. Top: The calculated temperature dependence of in-plane resistivity (sum of (6) and (8)) for various dopings δ in comparison with the corresponding experimental data (inset) on $La_{2-\delta}Sr_{\delta}CuO_4$ in units of mΩ cm, taken from [1]. **Bottom:** The calculated temperature dependence of the magnetoresistance for doping $\delta = 0.075$, in comparison with experimental data on $La_{1.925}Sr_{0.075}CuO_{4+\varepsilon}$ (inset), taken from [4].

References

[1] H Takagi *et al*, *Phys. Rev. Lett.* **69**, 2975 (1992)
B Wuyts *et al*, *Phys. Rev.* **B53**, 9418 (1996)
[2] B Keimer *et al*, *Phys. Rev.* **B46**, 14034 (1992)
B Wuyts *et al*, *Phys. Rev.* **B53**, 9418 (1996)
[3] T Kimura *et al*, *Phys. Rev.* **B53**, 8733 (1996)
Y Abe *et al*, *Phys. Rev.* **B59**, 14753 (1999)
[4] A Lacerda *et al*, *Phys. Rev.* **B49**, 9097 (1994)
[5] P Fournier *et al*, *Phys. Rev. Lett.* **81**, 4720 (1998)
S Ono *et al*, *Phys. Rev. Lett.* **85**, 638 (2000)
K Segasawa and Y Ando, *Phys. Rev.* **B59**, R3948 (1999)
Y Ando *et al*, *Phys. Rev. Lett.* **75**, 4662 (1995)
G S Boebinger *et al*, *Phys. Rev. Lett.* **77**, 5417 (1996)
Y Ando *et al*, *J. Low Temp. Phys.* **105**, 867 (1996)
[6] L Ioffe and A Larkin, *Phys. Rev.* **B39**, 8988 (1989)
[7] J Fröhlich *et al*, *Nucl. Phys.* **B374**, 511 (1992)
J Fröhlich and P A Marchetti, *Phys. Rev.* **B46**, 6535 (1992)
[8] P A Marchetti, Z B Su and L Yu, *Phys. Rev.* **B58**, 5808 (1998)
[9] P A Lee and N Nagaosa, *Phys. Rev. Lett.* **65**, 2450 (1990); *Phys. Rev.* **B46**, 5621 (1992)
L B Ioffe and P B Wiegman, *Phys. Rev. Lett.* **65**, 653 (1990)
[10] P A Marchetti *et al*, *J. Phys. Condens. Matter* **12**, L329 (2000)
[11] P A Marchetti, Z B Su, L Yu, *Phys. Rev. Lett.* **86**, 3831 (2001)
[12] L Ioffe and G Kotliar, *Phys. Rev.* **B42**, 10348 (1990)
L Ioffe and P Wiegmann, *Phys. Rev.* **B45**, 519 (1992)