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Magnetic fluid based squeeze film between porous annular curved plates with the effect of rotational inertia

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Abstract. The squeeze film behaviour between rotating annular plates was analysed theoretically when the curved upper plate with a uniform porous facing approached the impermeable and flat lower plate, considering a magnetic fluid lubricant in the presence of an external magnetic field oblique to the plates. Expressions were obtained for pressure and load capacity; and response time is given by a differential equation. The increases in pressure and load capacity depended only on the magnetization. However, the increase in response time depended on magnetization, fluid inertia and speed of rotation of the plates.

Keywords. Magnetic fluid; lubrication; annular curved plates.

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1. Introduction

Gupta *et al* [1] analysed the squeeze film between porous annular curved plates and found that the load capacity increased considerably with the curvature in the case of concave plates. Gupta *et al* [2] extended the above analysis by including the effect of rotation of both the plates. The load capacity decreased when the speed of rotation of the upper plate increased up to a certain value of the curvature parameter.

Recently Bhat and Deheri [3] investigated the effects of magnetic fluid lubricant on the action of the squeeze film between porous annular plates. Shah and Bhat [4,5] investigated the squeeze film based on magnetic fluid between curved porous rotating circular plates and between two curved annular plates respectively. The performance of the bearing with magnetic fluid lubricant was found to be better than the corresponding one with conventional lubricant. It may be noted that in MHD lubrication the effective pressure gets contribution from the magnetic field while in magnetic fluid lubrication it is from the magnetization of the particles in the fluid.

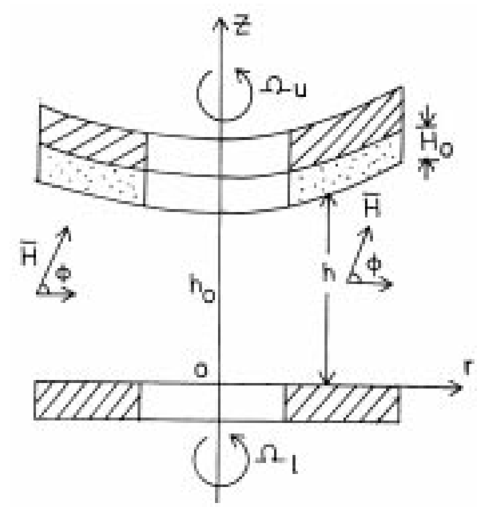

Figure 1. Configuration of the problem.

In the present investigation we extend analysis [3] by including the effects of curvature of the upper plate and rotation of both the plates.

2. Analysis

The bearing shown in figure 1 consists of two annular plates, each of inside radius a and outside radius b. The upper plate is curved and has a porous facing of thickness H_0 which is backed by a solid wall. The film thickness h is taken as

$$h = h_0 \exp(-Br^2); \quad a \leq r \leq b \tag{1}$$

where r is the radial coordinate, h_0 is the central film thickness and B is the curvature of the upper plate. The impermeable and flat lower plate is normally approached by the upper plate with a uniform velocity $\mathrm{d}h_0/\mathrm{d}t = \dot{h}_0$. The upper and lower plates rotate with angular velocities Ω_u and Ω_1 respectively. Assuming axially symmetric flow of the magnetic fluid between the plates under an external magnetic field $\underline{H} = (H(r)\cos\phi, 0, H(r)\sin\phi)$ whose magnitude H vanishes at $r = a, b$ and which is inclined at an angle ϕ to the lower plate, the modified Reynolds equation governing the film pressure p is [2,3]

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[(h^3 + 12\bar{k}H_0)r\frac{\mathrm{d}}{\mathrm{d}r}(p - 0.5\mu_0\bar{\mu}H^2)\right] = 12\mu\dot{h}_0 + 24\rho\bar{k}H_0\Omega_u^2 + \rho(0.3\Omega_r^2 + \Omega_r\Omega_1 + \Omega_1^2)\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r^2h^3), \tag{2}$$

where $\bar{k}$ is the permeability of the porous region, μ_0 is the permeability of the free space, $\bar{\mu}$ is the magnetic susceptibility, μ is the fluid viscosity, $\Omega_r = \Omega_u - \Omega_1$ and ρ is the fluid density.

Taking, for example,

$$H^2 = K(r-a)(b-r), \tag{3}$$

where K is a constant to be chosen according to the field strength and assuming the external magnetic field to arise from a potential function, the angle $\phi = \phi(r,z)$ satisfies the equation

$$\cot\phi \frac{\partial\phi}{\partial r} + \frac{\partial\phi}{\partial z} = \frac{2r-a-b}{2(r-a)(b-r)}$$

whose solution is given by

$$c^2 \operatorname{cosec}^2\phi = (r-a)(b-r) \quad \text{and} \quad \sin\frac{z}{c} = \frac{2r-a-b}{[(b-a)^2-4c^2]^{1/2}},$$

c being an arbitrary constant.

We introduce the following dimensionless quantities

$$\left.\begin{array}{l} P = \dfrac{-h_0^3 p}{\mu a^2 \dot{h}_0},\ R = \dfrac{r}{a},\ k = \dfrac{b}{a},\ \psi = \dfrac{\bar{k}H_0}{h_0^3},\ S = -\dfrac{\rho\Omega_u^2 h_0^3}{\mu\dot{h}_0} \\ \Omega_f = \dfrac{\Omega_l}{\Omega_u},\ \bar{B} = Ba^2,\ \mu^* = -\dfrac{\mu_0\bar{\mu}h_0^3 K}{\mu\dot{h}_0} \end{array}\right\}. \tag{4}$$

From eqs (1)–(4) we obtain

$$\begin{aligned} &\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}R}\left[\left\{\exp(-3\bar{B}R^2) + 12\psi\right\} R\frac{\mathrm{d}}{\mathrm{d}R}\left\{P - 0.5\mu^*(R-1)(k-R)\right\}\right] \\ &= -12 + 24\psi S + \frac{S}{10}(3\Omega_f^2 + 4\Omega_f + 3)\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}R}\left\{R^2\exp(-3\bar{B}R^2)\right\}. \end{aligned} \tag{5}$$

3. Solution

Solving eq. (5) under the boundary conditions

$$P(1) = P(k) = 0, \tag{6}$$

we obtain the dimensionless pressure P as

$$\begin{aligned} P = {}& 0.5\mu^*(R-1)(k-R) + \frac{I(R)}{I(k)}\left\{\frac{-1+2\psi S}{12\psi B}L_1(k) + \frac{S}{60B}(3+4\Omega_f+3\Omega_f^2)L_2(k)\right\} \\ &+ \frac{1-2\psi S}{12\psi B}L_1(R) - \frac{S}{60B}(3+4\Omega_f+3\Omega_f^2)L_2(R) \end{aligned} \tag{7}$$

and the load capacity w of the bearing is given in dimensionless form as

$$\begin{aligned} W &= \frac{-h_0^3 w}{2\pi\mu a^4\dot{h}_0} = \int_1^k PR\,\mathrm{d}R \\ &= \frac{\mu^*}{24}(k+1)(k-1)^3 + 3(1-2\psi S)I_1 - \frac{S}{20}(3+4\Omega_f+3\Omega_f^2)I_2 \\ &\quad + \frac{L_1(k)}{144\psi B^2 I(k)}\left[\frac{-1+2\psi S}{12\psi}L_1(k) + \frac{S}{60}(3+4\Omega_f+3\Omega_f^2)L_2(k)\right], \end{aligned} \tag{8}$$

where

$$L_1(R) = \ln \frac{1+12\psi \exp(3\bar{B})}{1+12\psi \exp(3\bar{B}R^2)}, \tag{9}$$

$$L_2(R) = \ln \frac{\exp(-3\bar{B}R^2)+12\psi}{\exp(-3\bar{B})+12\psi}, \tag{10}$$

$$I(R) = \int_1^R \frac{\mathrm{d}x}{x\{\exp(-3\bar{B}x^2)+12\psi\}}, \tag{11}$$

$$I_1 = \int_1^k \frac{R^3 \exp(3\bar{B}R^2)}{1+12\psi \exp(3\bar{B}R^2)}\mathrm{d}R, \tag{12}$$

$$I_2 = \int_1^k \frac{R^3}{1+12\psi \exp(3\bar{B}R^2)}\mathrm{d}R. \tag{13}$$

The time t taken by the upper plate to reach a central film thickness h_0 starting from an initial film thickness h_1 can be determined in dimensional form from the equation

$$\begin{aligned} \frac{\mathrm{d}\bar{t}}{\mathrm{d}\bar{h}} = {} & \left[3I_1^* - L_1^{*2}(k)/\{1728\bar{B}^2\psi_1^2 I^*(k)\}\right] \div \Bigg[-\frac{1}{2\pi} + \mu_1^*(k+1)(k-1)^3/24 \\ & -6\psi_1 S_1 I_1^* - \frac{S_1}{20}(3+4\Omega_f+3\Omega_f^2)\bar{h}^3 I_2^* + L_1^{*2}(k)S_1/\{864\bar{B}^2\psi_1 I^*(k)\} \\ & +L_1^*(k)L_2^*(k)(3+4\Omega_f+3\Omega_f^2)S_1/\{8640\bar{B}^2\psi_1 I^*(k)\}\Bigg], \end{aligned} \tag{14}$$

where

$$\bar{h} = \frac{h_0}{h_1}, \quad \bar{t} = \frac{w h_1^2 t}{\mu a^4}, \quad \mu_1^* = \frac{\mu_0 \bar{\mu} a^4 K}{w}, \quad S_1 = \frac{\rho \Omega_u^2 a^4}{w}, \quad \psi_1 = \frac{\bar{k} H_0}{h_1^3}, \tag{15}$$

$$L_1^*(k) = \ln\left[\frac{\bar{h}^3 + 12\psi_1 \exp(3\bar{B})}{\bar{h}^3 + 12\psi_1 \exp(3\bar{B}k^2)}\right],\ L_2^*(k) = \ln\left[\frac{\bar{h}^3 \exp(-3\bar{B}k^2) + 12\psi_1}{\bar{h}^3 \exp(-3\bar{B}) + 12\psi_1}\right], \tag{16}$$

$$\left.\begin{aligned} I^*(k) &= \int_1^k \frac{\bar{h}^3}{R\{\bar{h}^3 \exp(-3\bar{B}R^2) + 12\psi_1\}}\ \mathrm{d}R, \\ I_1^* &= \int_1^k \frac{R^3 \exp(3\bar{B}R^2)}{\bar{h}^3 + 12\psi_1 \exp(3\bar{B}R^2)}\mathrm{d}R, \\ I_2^* &= \int_1^k \frac{R^3}{\bar{h}^3 + 12\psi_1 \exp(3\bar{B}R^2)}\mathrm{d}R \end{aligned}\right\}. \tag{17}$$

4. Results and discussion

Dimensionless pressure P and load capacity W are given by eqs (7) and (8) which show that they increase due to magnetization by quantities $0.5\mu^*(R-1)(k-R)$ and $\mu^*(k+1)(k-1)^3/24$ respectively. Thus the bearing performance is improved by the application of magnetic fluid.

Setting $\bar{B} = \Omega_u = \Omega_1 = 0$ we obtain the results for parallel annular plates [3] while setting $\mu^* = \Omega_u = \Omega_1 = 0$ we obtain the results for curved annular plates [1].

Setting $\mu^* = 0$ we obtain the results for the nonmagnetic case [2] which discusses various special cases and the effects on load capacity due to variations in the speed parameter Ω_f, rotational inertial parameter S and curvature parameter $\bar{B}$. Since these parameters are independent of μ^*, their effects remain the same.

While the increases in P and W purely depend on the magnetization parameter μ^*, the increase in the response time $\bar{t}$ depends on all the three parameters μ_1^*, S_1 and Ω_f unlike in the corresponding non rotating case. Solving the differential eq. (14) by Simpson's rule the computed values of $\bar{t}$ are displayed in tables 1 and 2.

Table 1 shows that $\bar{t}$ increases when the magnetic field strength increases and decreases when the angular velocity of the upper plate increases. Table 2 shows that $\bar{t}$ attains a minimum for $\bar{B} = \bar{B}_0$, where $0.0625 < \bar{B}_0 < 0.25$. $\bar{t}$ takes an optimum value when both plates rotate with nearly the same angular velocities but in opposite directions.

Table 1. Values of $\bar{t}$ for various values of $H^* = \max H$ in Am^{-1} and Ω_u in rad s^{-1}.

	H^*			
Ω_u	0	0.625×10^5	10^5	1.875×10^5
50	0.4133	0.4480	0.5270	0.6305
100	0.1512	0.1525	0.1564	0.1634
150	0.0706	0.0709	0.0717	0.0732
200	0.0404	0.0405	0.0408	0.0412

$\Omega_f = 2, \bar{B} = 0.125$.

Table 2. Values of $\bar{t}$ for various values of $\bar{B}$ and Ω_f.

	$\bar{B}$				
Ω_f	−0.125	−0.0625	0.0625	0.125	0.25
−3	0.0477	0.0724	0.2250	0.2058	0.5158
−2	0.1316	0.1926	0.5597	0.5045	1.3145
−1	0.5726	0.7601	1.7870	1.4315	4.4283
1	0.0891	0.1326	0.3979	0.3615	0.9242
2	0.0369	0.0563	0.1770	0.1634	0.4042
3	0.0198	0.0306	0.0977	0.0898	0.2218

$H^* = 1.875 \times 10^5\ \mathrm{Am}^{-1}$, $\Omega_u = 100$ rad s^{-1}.

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