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Tilted Bianchi type I dust fluid cosmological model in general relativity

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Abstract. In this paper, we have investigated a tilted Bianchi type I cosmological model filled with dust of perfect fluid in general relativity. To get a determinate solution, we have assumed a condition $A = B^n$ between metric potentials. The physical and geometrical aspects of the model together with singularity in the model are also discussed.

Keywords. Tilted; dust perfect fluid; Bianchi type I.

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1. Introduction

In recent years, there has been a considerable interest in investigating spatially homogeneous and anisotropic universes in which the matter does not move orthogonally to the hyper surface of homogeneity. These are called tilted universes. The general dynamics of tilted universes have been studied in detail by King and Ellis [1], Ellis and King [2], Collins and Ellis [3], tilted Bianchi type I models have been obtained by Dunn and Tupper [4] and Lorentz [5]. Mukherjee [6] has investigated tilted Bianchi type I universe with heat flux in general relativity. To get a determinate model, he has assumed some supplementary conditions on metric potentials. He has shown that the universe assumes a pancake shape. The velocity vector is not geodesic and heat flux is comparable to energy density. Bradley and Sviestine [7] have investigated that heat flow is expected for tilted cosmological models. The cosmological models with heat flow have been studied by a number of authors like Novello and Reboucas [8], Roy and Banerjee [9], Coley and Tupper [10], Mukherjee [11], Banerjee and Santos [12], Coley [13], Roy and Prasad [14].

Bali and Tyagi [15] have investigated magneto Bianchi type I stiff fluid perfect fluid model in general relativity.

In this paper we have investigated tilted Bianchi type I dust fluid model of perfect fluid in general relativity. To get a determinate solution, a supplementary condition $A = B^n$ between metric potentials is used. The behaviour of the singularity in the model with other physical and geometrical aspects of the models are also discussed.

We consider the metric in the form

$$ds^2 = -dt^2 + A^2\,dx^2 + B^2\,dy^2 + C^2\,dz^2, \tag{1.1}$$

where A, B and C are functions of t alone.

The energy momentum tensor for perfect fluid distribution with heat conduction is given by Ellis [16] as

$$T_i^j = (\varepsilon + p)v_i v^j + p g_i^j + q_i v^j + v_i q^j \tag{1.2}$$

together with

$$g_{ij} v^i v^j = -1, \tag{1.3}$$

$$q_i q^i > 0 \tag{1.4}$$

and

$$q_i v^i = 0, \tag{1.5}$$

where p is the pressure, ε the density and q_i the heat conduction vector orthogonal to v_i. The fluid flow vector v^i has the components $(\sinh\lambda/A, 0, 0, \cosh\lambda)$ satisfying eq. (1.3), λ being a tilt angle.

The Einstein's field equation

$$R_i^j - \frac{1}{2}R g_i^j = -8\pi T_i^j \quad (\wedge = 0,\ C = G = 1)$$

for the line element (1.1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi\left[(\varepsilon + p)\sinh^2\lambda + p + \frac{2q_1\sinh\lambda}{A}\right], \tag{1.6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -8\pi p, \tag{1.7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -8\pi p, \tag{1.8}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} = -8\pi\left[-(\varepsilon + p)\cosh^2\lambda + p - \frac{2q_1\sinh\lambda}{A}\right], \tag{1.9}$$

$$(\varepsilon + p)A\sinh\lambda\cosh\lambda + q_1\cosh\lambda + \frac{q_1\sinh^2\lambda}{\cosh\lambda} = 0, \tag{1.10}$$

where suffix '4' indicates ordinary differentiation with respect to t.

2. Solution of field equations

Equations (1.6)–(1.10) are five equations in seven unknowns $A, B, C, \varepsilon, p, \lambda$ and q_1. Thus we require two conditions.

First we assume that the model is filled with dust of perfect fluid which leads to

$$p = 0. \tag{2.1}$$

We also assume that

$$A = B^n. \tag{2.2}$$

Using $p = 0$ in eq. (1.8), we get

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = 0. \tag{2.3}$$

Using eq. (2.2) in eq. (2.3), we have

$$\frac{(n+1)}{n^2}\,\frac{B_{44}}{B_4} + \frac{B_4}{B} = 0 \tag{2.4}$$

which on integration leads to

$$B_4 = \left(\frac{m}{B}\right)^{n^2/(n+1)}$$

which again leads to

$$B = \left[\left(\frac{n^2+n+1}{n+1}\right)\ m^{n^2/(n+1)}\ t + b\right]^{(n+1)/(n^2+n+1)} \tag{2.5}$$

which implies that

$$B = (at+b)^s, \tag{2.6}$$

where

$$a = \left(\frac{n^2+n+1}{n+1}\right) m^{n^2/(n+1)}, \tag{2.7}$$

$$s = \frac{n+1}{n^2+n+1} \tag{2.8}$$

and m and b are constants of integration.

Thus

$$A = (at+b)^{ns}. \tag{2.9}$$

Using the condition $p = 0$ in eq. (1.7) and then putting $A = (at+b)^{ns}$, we have

$$(at+b)^2C_{44}+ans(at+b)C_4=\alpha C, \tag{2.10}$$

where

$$\alpha=a^2(1-ns)ns. \tag{2.11}$$

Substituting $(at+b)(\mathrm{d}C/\mathrm{d}t)=(\mathrm{d}C/\mathrm{d}\tau)$ in (2.10), we have

$$\frac{\mathrm{d}^2C}{\mathrm{d}\tau^2}+(ans-a)\frac{\mathrm{d}C}{\mathrm{d}\tau}=\alpha C$$

which leads to

$$C=R\left[(at+b)^{\beta}-K(at+b)^{-\beta}\right](at+b)^{(1-ns)/2}, \tag{2.12}$$

where R is the constant of integration and $\beta=(M/a)=(1/a)\sqrt{\alpha+\frac{a^2(ns-1)^2}{4}}$ which implies

$$M=\frac{a}{2}\left(\frac{2n+1}{n^2+n+1}\right). \tag{2.13}$$

Therefore, after suitable transformation of coordinates, the metric (1.1) reduces to

$$\mathrm{d}s^2=-\frac{\mathrm{d}T^2}{a^2}+T^{2ns}\mathrm{d}X^2+T^{2s}\mathrm{d}Y^2+R^2\left(T^{M/a}-KT^{-M/a}\right)T^{(1+ns)/2}\mathrm{d}Z^2. \tag{2.14}$$

3. Some physical and geometrical properties

The density of the model (2.14) is given by

$$\varepsilon=\frac{a^2(n+1)(2n+1)}{4\pi(n^2+n+1)^2}\,\frac{1}{T^2}\left[\frac{T^{M/a}}{T^{M/a}-KT^{-M/a}}\right]. \tag{3.1}$$

The reality conditions $\varepsilon+p>0$ and $\varepsilon+3p>0$ given by Ellis [17] lead to

$$T^{2M/a}>0, \tag{3.2a}$$

where

$$\frac{a^2(n+1)(2n+1)}{4\pi(n^2+n+1)^2}>0. \tag{3.2b}$$

The tilt angle λ is given by

$$\cosh\lambda=\sqrt{\frac{n+1}{2n+1}}=N>1 \tag{3.3}$$

and

$$\sinh\lambda = \sqrt{N^2-1} = \sqrt{\frac{-n}{2n+1}}, \quad n<0 \tag{3.4}$$

where

$$n = \frac{N^2-1}{1-2N^2}. \tag{3.5}$$

From eqs (3.3) and (3.4), we find that $n<0$ and $n>-1/2$.

The expression for fluid velocity vector v^i and heat conduction vector q^i for the model (2.14) are given by

$$v^1 = \frac{\sqrt{N^2-1}}{T^{ns}}, \tag{3.6}$$

$$v^4 = N, \tag{3.7}$$

$$q_1 = -\frac{a^2(n+1)^2(2n+1)\sqrt{N^2-1}}{4\pi(n^2+n+1)^2}\ \frac{T^{\left((-2n^2-3)/[2(n^2+n+1)]\right)}}{\left(T^{M/a}-KT^{-M/a}\right)}, \tag{3.8}$$

$$q_4 = \frac{a^2(n+1)^{3/2}(2n+1)^{3/2}(N^2-1)}{4\pi(n^2+n+1)^2\left(T^2-KT^{(2n^2+1)/(n^2+n+1)}\right)}. \tag{3.9}$$

The scalar of expansion θ calculated for the flow vector v^i is given by

$$\theta = \sqrt{\frac{n+1}{2n+1}}\left[\frac{(2n^2+4n+3)a}{2(n^2+n+1)} + \frac{M(1+KT^{-2M/a})}{(1-KT^{-2M/a})}\right]\frac{1}{T}. \tag{3.10}$$

The non-vanishing components of shear tensor (σ_{ij}) and rotation tensor (ω_{ij}) are given by

$$\sigma_{11} = \frac{a(n+1)}{6(n^2+n+1)}\sqrt{\frac{n+1}{2n+1}}\left[\frac{(4n^2+2n-3)}{6(2n+1)} - \frac{1+KT^{-2M/a}}{1-KT^{-2M/a}}\right] \times\left[\frac{(n^2+n-1)}{T^{(n^2+n+1)}}\right], \tag{3.11}$$

$$\sigma_{22} = \frac{a}{6(n^2+n+1)}\sqrt{\frac{n+1}{2n+1}}\left[(-2n^2+2n+3) - \frac{(2n+1)(1+KT^{-2M/a})}{(1-KT^{-2M/a})}\right]\left[\frac{(n^2+n-1)}{T^{(n^2+n+1)}}\right], \tag{3.12}$$

$$\sigma_{33} = \frac{aR^2(T^{M/a} - KT^{-M/a})^2}{3(n^2+n+1)} \left(\frac{-n^2-n}{T^{n^2+n+1}}\right) \sqrt{\frac{n+1}{2n+1}} \times \left[\frac{(2n+1)(1+KT^{-2M/a})}{(1-KT^{-2M/a})} - (n^2+2n)\right], \tag{3.13}$$

$$\sigma_{44} = \frac{a^2(N^2-1)}{6(n^2+n+1)} \sqrt{\frac{n+1}{2n+1}} \left[(4n^2+2n-3) - \frac{(2n+1)(1+KT^{-2M/a})}{(1-KT^{-2M/a})}\right], \tag{3.14}$$

$$\sigma_{14} = \frac{a(n+1)(N^2-1)}{6(2n+1)(n^2+n+1)} \left[(-16n^2-8n+3) + \frac{(2n+1)(1+KT^{-2M/a})}{(1-KT^{-2M/a})}\right] \times \left(\frac{-1}{T^{n^2+n+1}}\right), \tag{3.15}$$

$$\omega_{14} = \frac{an(n+1)^2}{(2n+1)(n^2+n+1)} \sqrt{(N^2-1)}T^{-1/(n^2+n+1)}. \tag{3.16}$$

The rate of expansion (H_i) is given by

$$H_1 = \frac{an(n+1)}{(n^2+n+1)} \frac{1}{T}, \tag{3.17}$$

$$H_2 = \frac{a(n+1)}{(n^2+n+1)} \frac{1}{T}, \tag{3.18}$$

$$H_3 = \frac{M(T^{M/a} + KT^{-M/a})}{T(T^{M/a} - KT^{-M/a})} + \frac{a}{2(n^2+n+1)} \frac{1}{T}. \tag{3.19}$$

4. Discussion

The model starts with a big-bang at $T = 0$ and the expansion in the model decreases as T increases and the expansion in the model stops at $T = \infty$. The model has cigar type singularity at $T = 0$ [18] when $ns < 1$ and $n, s < 0$ while Mukherjee [6] has investigated that his universe assumes a pancake shape. When $T \to 0$, energy density $\varepsilon \to \infty$ if $M < a$. When $T \to \infty$, $\varepsilon \to 0$ if $M < a$.

Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large values of T. The model in general represents shearing and rotating universe.

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