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Pump induced normal mode splittings in phase conjugation in a Kerr nonlinear waveguide

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Abstract. Phase conjugation in a Kerr nonlinear waveguide is studied with counter-propagating normally incident pumps and a probe beam at an arbitrary angle of incidence. Detailed numerical results for the specular and phase conjugated reflectivities are obtained with full account of pump depletion. For sufficient strengths of the pump a normal mode splitting is demonstrated in both the specular and the phase conjugated reflectivities of the probe wave. The splitting is explained in terms of a simple model under undepleted pump approximation.

Keywords. Phase conjugation; nonlinear waveguide; normal mode splitting.

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1. Introduction

In the recent past there has been a great deal of interest in nonlinear phenomena in layered geometries [1]. Layered structures, in particular, guided wave structures offer resonances with high quality factors and large local field enhancements, which can be exploited for low threshold nonlinear optical phenomena. Recently it was shown that guided or surface wave structures in the close vicinity of a phase conjugate mirror (PCM) [2] can exhibit several interesting features like enhanced back scattering and bistability. Investigations were carried out for both linear as well as Kerr nonlinear waveguides [3,4]. A slowly varying envelope approximation was used to deal with the nonlinear structure [4]. The validity of Agarwal's theorem [5] pertaining to the ability of the PCM to correct for static aberrations [6] was demonstrated by explicit calculations for both linear and nonlinear structures. In the regime of angle of incidence where guided waves are excited, enhanced back scattering was predicted [3]. In the context of the nonlinear layered media bistable response was demonstrated for both specular and phase conjugated reflectivities [4]. In these studies the effect of grating induced by the counter propagating waves in the nonlinear layer was neglected treating the propagation in the nonlinear layer by means of nonlinear characteristics method [7]. Very recently an exact method to deal with such structures was proposed by Laine and Friberg [8]. However, all the above investigations consider the phase conjugate mirror to be given with specified phase conjugate reflectivity, whereas, the structure of the

PCM itself with a nonlinear medium can be complicated [2]. In this paper we address a very general problem of degenerate four wave mixing in a Kerr nonlinear waveguide retaining the effects of the grating induced by the interacting waves to the first order. The treatment is exact in the sense that full Maxwell boundary conditions are used and no approximations as regards the mutual strengths of the pump and signal are made. In other words, approximations like the slowly varying envelope and undepleted pump, leading to significant simplifications, are not used. We calculate the specular and phase conjugated reflectivities from the structure. We show that there is an enhancement in the reflectivities, whenever the guided modes are excited. Moreover, for larger pump strengths, we demonstrate a splitting of the guided mode resonances. The origin of the splittings is explained in terms of a simple model of coupled waves derived from the exact equations under the undepleted pump approximation. The coupling strength is shown to be proportional to the pump intensity. Besides, the possibility of resolving the split resonances is shown to be critically dependent on the quality factors associated with the guided modes.

2. Mathematical formulation

Consider the structure shown in figure 1 consisting of a nonlinear guiding layer of width d_2 on a linear substrate with dielectric constant ε_1. The guiding layer is loaded on top by a high index prism with dielectric constant ε_4 after a spacer layer with width d_3 and dielectric constant ε_3. The nonlinearity is given by the intensity dependent dielectric function ε_2 as

$$\varepsilon_2 = \varepsilon_{20}[1 + \alpha|E|^2], \tag{1}$$

where ε_{20} is the dielectric constant at vanishing power levels and α is the nonlinearity parameter. Let the structure be illuminated from top and bottom by pump waves with complex amplitudes A_{4+} and A_{1-}, respectively. Let also a probe wave with amplitude B_{4+} be incident on the structure from top at an angle θ. We assume all the waves to be

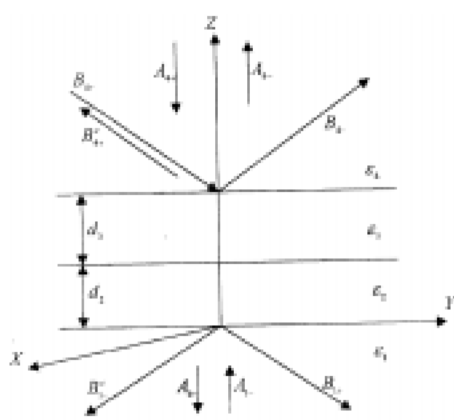


Figure 1. Schematic view of the nonlinear waveguide. The parameters are as follows: ε_1=3.1329, ε_{20}=12.9599+0.045i, ε_3=1.0, ε_4=6.145, d_2=5.5 μm, d_3=0.12 μm, λ=0.82 μm.

s-polarized plane waves with frequency ω. For the only non-vanishing x-component of the field E_2 in the nonlinear layer the wave equation reads as follows:

$$\frac{\partial^2 E_2}{\partial y^2} + \frac{\partial^2 E_2}{\partial z^2} + \varepsilon_{20}[1 + |E_2|^2]E_2 = 0. \tag{2}$$

Equation (2) is written in terms of the dimensionless quantities using the following transformations

$$y \to k_0 y, \quad z \to k_0 z, \quad E \to \sqrt{\alpha} E, \tag{3}$$

where, $k_0 = \omega/c$ is the vacuum wave vector. There is no x dependence in eq. (2) since the structure is assumed to be infinite along the x direction. The most important step is to split the field in three parts corresponding to the pump, probe (signal) and the phase conjugated waves

$$E_2 = E_p + E_s \mathrm{e}^{ip_y y} + E_c \mathrm{e}^{-ip_y y}, \tag{4}$$

where

$$p_y = \sqrt{\varepsilon_4} \sin\theta \tag{5}$$

is the surface component of the wave vector (normalized to k_0) continuous across the interfaces. In writing eq. (4) we ignored the higher order scatterings containing terms like $\mathrm{e}^{\pm 2ip_y y}$. Substituting eq. (4) in eq. (2) and collecting terms proportional to e^0, $\mathrm{e}^{\pm ip_y y}$, one can write down the coupled nonlinear equations for E_c, E_p and E_s as follows:

$$\frac{\mathrm{d}^2 E_p}{\mathrm{d}z^2} = -\varepsilon_{20}\left[E_p(1 + |E_p|^2 + 2|E_s|^2 + 2|E_c|^2) + 2E_s E_c E_p^*\right], \tag{6}$$

$$\frac{\mathrm{d}^2 E_s}{\mathrm{d}z^2} = -p_z^2 E_s - \varepsilon_{20}\left[E_s(2|E_p|^2 + |E_s|^2 + 2|E_c|^2) + E_p^2 E_c^*\right], \tag{7}$$

$$\frac{\mathrm{d}^2 E_c}{\mathrm{d}z^2} = -p_z^2 E_c - \varepsilon_{20}\left[E_c(2|E_p|^2 + 2|E_s|^2 + |E_c|^2) + E_p^2 E_s^*\right], \tag{8}$$

where $p_z = \sqrt{\varepsilon_{20} - p_y^2}$ is the normalized z-component of the wave vector in the limit of vanishing intensities in the nonlinear layer. It is clear from eqs (6)–(8) that an analytical solution of this system poses a formidable problem. However, for undepleted pump, simple solutions can be worked out. Under this approximation, when weak signal and phase conjugated waves do not affect the pump, eq. (6) can be solved in terms of the forward (along negative z direction) and backward waves. This leads to the nonlinear characteristics matrix approach which has been used extensively in the context of various layered media [7,9]. The information about the pump distribution in the nonlinear layer can then be used to solve eqs (7) and (8). In this paper we do not resort to such approximate method. We solve eqs (6)–(8) numerically with appropriate boundary conditions. Note that the propagation of ordinary (along $+y$ direction) and the phase conjugated waves (along $-y$ direction) in the linear layers can be handled by means of standard characteristic matrices [10].

In what follows, we present the outline of the boundary conditions and the numerical scheme to obtain the ordinary and phase conjugate reflectivities. Writing the solutions in the linear media as in refs [3] and [4], the fields and their derivatives at $z = 0$ can be expressed by the following set of equations

$$\begin{pmatrix} E_p \\ E'_p \end{pmatrix}_{z=0} = \begin{pmatrix} 1 & 1 \\ -ip_1 & ip_1 \end{pmatrix} \begin{pmatrix} A_{1+} \\ A_{1-} \end{pmatrix}, \tag{9}$$

$$\begin{pmatrix} E_s \\ E'_s \end{pmatrix}_{z=0} = \begin{pmatrix} 1 \\ -ip_{z1} \end{pmatrix} B_{1+}, \tag{10}$$

$$\begin{pmatrix} E_c \\ E'_c \end{pmatrix}_{z=0} = \begin{pmatrix} 1 \\ ip_{z1} \end{pmatrix} B^c_{1-}. \tag{11}$$

Analogous expressions for the fields and derivatives at $z = d_2$ are given by

$$\begin{pmatrix} E_p \\ E'_p \end{pmatrix}_{z=d_2} = M_p^{-1} \begin{pmatrix} 1 & 1 \\ -ip_4 & ip_4 \end{pmatrix} \begin{pmatrix} A_{4+} \\ A_{4-} \end{pmatrix}, \tag{12}$$

$$\begin{pmatrix} E_s \\ E'_s \end{pmatrix}_{z=d_2} = M^{-1} \begin{pmatrix} 1 & 1 \\ -ip_{z4} & ip_{z4} \end{pmatrix} \begin{pmatrix} B_{4+} \\ B_{4-} \end{pmatrix}, \tag{13}$$

$$\begin{pmatrix} E_c \\ E'_c \end{pmatrix}_{z=d_2} = M_c^{-1} \begin{pmatrix} 1 \\ -ip_{z4} \end{pmatrix} \begin{pmatrix} B_{4+} \\ B_{4-} \end{pmatrix}. \tag{14}$$

In eqs (9)–(14) primes denote the derivatives with respect to z and we have used the following notations

$$M = \begin{pmatrix} \cos(p_{z3}d_3) & -\frac{1}{p_{z3}}\sin(p_{z3}d_3) \\ -p_{z3}\sin(p_{z3}d_3) & -\cos(p_{z3}d_3) \end{pmatrix}, \tag{15}$$

$$\begin{aligned} M_c &= M(p_{z3} \to -p_{z3}), \\ M_p &= M(p_{z3} \to p_3), \\ p_{zj} &= \sqrt{\varepsilon_j - p_y^2}, \quad p_j = \sqrt{\varepsilon_j}, \quad j = 1, 3, 4. \end{aligned} \tag{16}$$

Note that the characteristic matrix given by eq. (15) is written in a slightly different form since instead of the tangential-to-surface components of the magnetic field we used the derivatives of E.

In our numerical scheme we treat the amplitudes A_{1+}, B_{1+} and B^c_{1-} as free complex parameters. Since A_{1-} (being the pump amplitude incident from below) is given, one knows the total field in medium 1. Making use of eqs (9)–(11) one can evaluate the fields and their derivatives at $z = 0$. With these initial conditions eqs (6)–(8) are solved and the respective quantities are found at $z = d_2$. The remaining conditions given by eqs (12)–(14) can then be used to calculate all the amplitudes in medium 4. Since A_{4+}, B_{4+} are also given, this leads to two equations

$$A_{4+}(A_{1+}, B_{1+}, B^c_{1-}) = A_{4+}, \tag{17}$$

$$B_{4+}(A_{1+}, B_{1+}, B^c_{1-}) = B_{4+}. \tag{18}$$

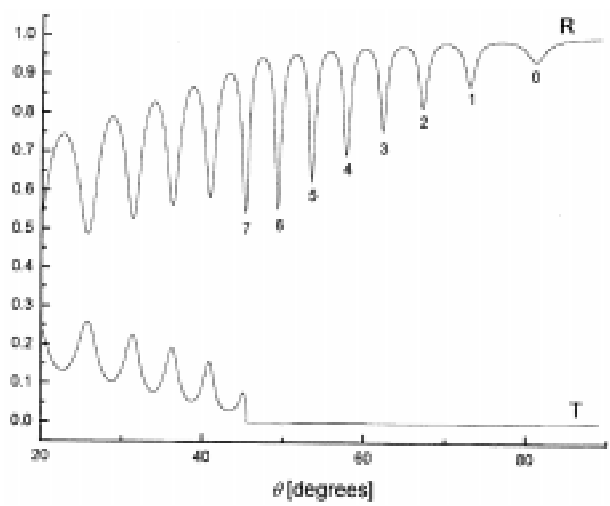
The third equation (the second row of eq. (14)) arises from the continuity of the phase conjugated field at the prism-spacer layer interface. One thus has a closed system of three complex equations for three complex unknowns A_{1+}, B_{1+} and B^c_{1-}. These equations are solved numerically. The knowledge of A_{1+}, B_{1+} and B^c_{1-} finally leads to the evaluation of the ordinary and phase conjugated reflected wave amplitudes B_{4-} and B^c_{4+}, respectively. The calculation of the corresponding reflection coefficients (R and R_c, respectively) is then straightforward

$$R = \left|\frac{V_{4-}}{V_{4+}}\right|, \quad R_c = \left|\frac{V^c_{4+}}{V_{4+}}\right|, \tag{19}$$

where V's denote the dimensionless intensities, e.g. $V_{4+} = |B_{4+}|^2$. Analogous intensities for the pump will be denoted by U's.

3. Numerical results

For numerical calculations the system parameters (corresponding to a GaAs waveguide on a Al_2O_3 substrate) were chosen as follows: $\varepsilon = 3.1329, \varepsilon_{20} = 12.9599 + 0.045i, \varepsilon_3 = 1.0$, $\varepsilon_4 = 6.145$, $d_2 = 5.5\ \mu$m, $d_3 = 0.12\ \mu$m, $\lambda = 0.82\ \mu$m. Linear results for the reflection and transmission of the signal wave in absence of the pump are shown in figure 2a. It is clear from figure 2a that for angles of incidence larger than about 46°, there is almost null transmission through the structure. This is the regime where the guided modes can be excited. Excitation of eight such modes leading to dips in the reflection coefficient



(a)

Figure 2a.

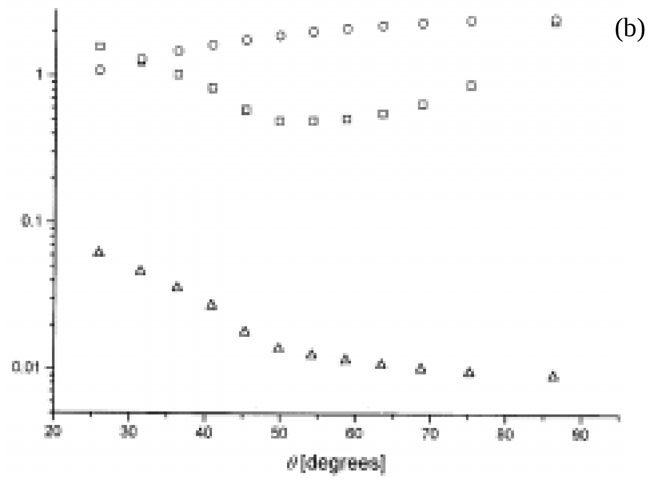


Figure 2. (**a**) Intensity reflection R and transmission T from the linear structure. Dips in R labeled by integers 0 to 7 correspond to the guided mode resonances; (**b**) roots of the dispersion equation: Im(θ) (squares), Re(p_y) (circles) and Im(p_y) (triangles) plotted against Re(θ). Other parameters are as in figure 1.

R are shown in figure 2a. Various modes are labeled by their corresponding mode numbers. For angles below 46°, the oscillations in reflection and transmission correspond to Fabry–Perot (FP) type of resonances. The angular width of the resonances can be quite misleading in inferring about the losses associated with these modes. For example, the fundamental guided mode (labeled by 0) is known to possess the lowest damping, it exhibits the largest angular width. Information about the losses associated with the modes can be obtained by solving the dispersion equation, treating the angle of incidence as a complex parameter. The real(imaginary) part of θ gives the approximate location(width) of the resonances. Knowledge of θ can then be used to calculate the real and imaginary parts of p_y. The propagation constant Re(p_y) gives the effective index of the mode, while Im(p_y) determines the losses of the mode. The results for Re(θ), Im(θ), Re(p_y) and Im(p_y) are shown in figure 2b, where the latter three quantities (denoted by squares, circles and triangles, respectively) are plotted against the first. It is clear from figure 2b that the losses of the guided modes increase as the mode number increases. Note also that the FP modes are characterized by much larger decay rates compared to the guided modes.

We next consider the case when the pump is on. We adopt the procedure outlined in the previous section to calculate the ordinary and phase conjugated reflectivities. The results for R_c and R for $V_{4+} = 1.0 \times 10^{-8}$, $U_{4+} = 0.001$ and for two different values of U_{1-}, namely, 0.001 (dashed line) and 0.005 (solid line) are shown in figures 3a and 3b, respectively. It is clear from figure 3a that there is an enhancement of the phase conjugated reflectivity R_c, whenever a mode (be it the FP or the guided mode) is excited. The enhancement is significant for the guided modes. The most important feature that can be noted from figures 3a and 3b is that for larger pump strengths there is a splitting of some of the resonances (see solid curves), while the FP modes and the higher order guided modes with larger damping do not exhibit the splitting.

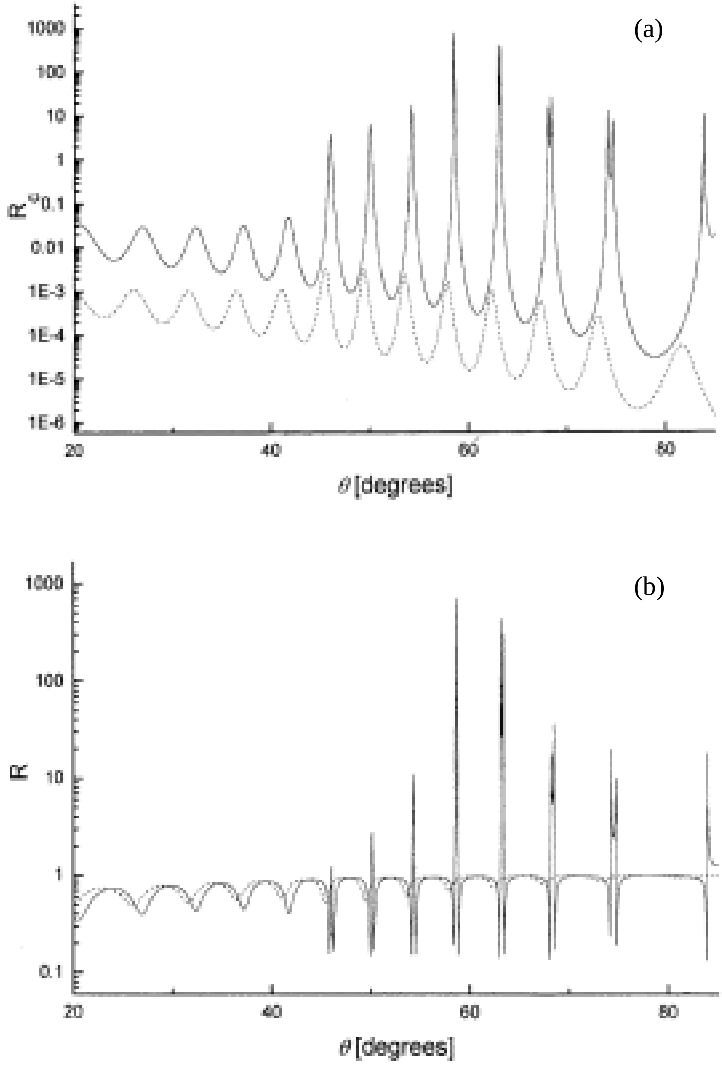


Figure 3. (**a**) Phase conjugated and (**b**) ordinary reflection coefficients R_c and R, respectively, for the nonlinear waveguide for incident probe intensity $V_{4+} = 1.0 \times 10^{-8}$, pump intensity from top $U_{4+} = 0.001$ and for two different values of pump intensity from below U_{1-}, namely, 0.001 (dashed line) and 0.005 (solid line). Other parameters are as in figure 1.

We now show that a simple model based on the undepleted pump approximation can explain the phenomenon of splitting and its major features. Under undepleted pump and slowly varying envelope approximation, i.e. writing

$$E_{s,c}(z) = A_{s,c}(z) \exp(\pm i p_z z), \tag{20}$$

and treating the A's as slowly varying, eqs (6)–(8) can be reduced to the following set of equations

$$\frac{\mathrm{d}A_s}{\mathrm{d}z} = -i\gamma A_s - i\delta A_c^*, \tag{21}$$

$$\frac{\mathrm{d}A_c^*}{\mathrm{d}z} = -i\gamma A_c^* - i\delta^* A_s. \tag{22}$$

In eqs (20), (21) the constant γ is real while δ is complex for real dielectric constant ε_{20}. It is important to note that the modulus of δ is proportional to the pump intensity. The eigenvalues $\lambda_{1,2}$ of the system (21), (22) can be written as

$$\lambda_{1,2} = i(\gamma \pm |\delta|). \tag{23}$$

It is clear from eq. (23) that there is a splitting, the magnitude of which is determined by the absolute value of δ which increases as a function of the pump intensity. Note that damping was ignored in deriving eqs (21)–(23). In presence of damping the split resonances will be resolved if $|\delta|$ exceeds the losses so that the splitting is not masked by the loss induced broadening. It is now clear why the modes with higher quality factors (consequently lower losses) exhibit the splitting and why the splittings show up only at larger power levels of the pump.

4. Conclusions

In conclusion, we studied a nonlinear waveguide in a degenerate four wave mixing configuration and calculated the reflectivities of the signal wave in both specular and backscattering directions. We have shown that guided mode resonances can lead to a significant enhancement of the phase conjugated reflectivity. We also reported a novel pump induced splitting in the guided mode resonances. The phenomenon of splitting was explained in terms of a simple model based on the undepleted pump and slowly varying envelope approximations. It is well known that nonlinear guided wave structures can exhibit bistability and nonreciprocity [1] leading to potential practical applications. Studies on these issues in the degenerate four wave mixing configuration are under way and will be reported elsewhere.

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