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Propagation of waves in a multicomponent plasma having charged dust particles

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Abstract. Propagation of both low and high frequency waves in a plasma consisting of electrons, ions, positrons and charged dust particles have been theoretically studied. The characteristics of dust acoustic wave propagating through the plasma has been analysed and the dispersion relation deduced is a generalization of that obtained by previous authors. It is found that nonlinear localization of high frequency electromagnetic field in such a plasma generates magnetic field. This magnetic field is seen to depend on the temperatures of electrons and positrons and also on their equilibrium density ratio. It is suggested that the present model would be applicable to find the magnetic field generation in space plasma.

Keywords. Dusty plasma; acoustic waves; high and low frequency; magnetic field generation.

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1. Introduction

Propagation of waves in a multicomponent plasma having charged dust particles has been investigated by various authors in recent times as the presence of charged dust grains give rise to a new kind of modes called dust modes and it has wide applications in magnetosphere and space plasma [1–3]. In fact, Rao *et al* [4] initiated to study theoretically the propagation of dust acoustic mode in a plasma. Subsequently, Shukla [5] and other authors [6,7] investigated different aspects of wave propagation in dusty plasma from which it has been observed that charge fluctuation of dust particles has significant role on the formation of solitons and shocks as well as instability of the wave [8–10]. The positrons are also found to exist in space plasma [11]. Several authors [12,13] considered the presence of positrons together with the electrons and ions for their studies on the propagation of waves. As we see that both the charged dust particles and positrons exist with the electrons and the ions in space plasma we consider here a model plasma consisting of all the above four species for our present investigation. We have derived the dispersion relation which is more general than that obtained by the previous authors. Moreover, we have obtained the growth rate of the magnetic field which is generated inside the plasma due to weakly nonlinear localization of high frequency field. It is to be mentioned that there are several mechanisms of generation of magnetic field in laboratory and space plasma [14–16]. In hot astrophysical bodies, the dynamo effect is one of the main sources of magnetic field generation [17,18]. However, our present model for the generation of magnetic field may be applicable in space plasma as the strength of magnetic field in space is found to be weak.

2. Formulation

We consider an unmagnetized collisionless multicomponent plasma consisting of electrons, ions, positrons and negatively charged dust particles. The dynamics of such plasmas are assumed to be represented by the following equations

$$\frac{\partial n_s}{\partial t} + \vec{\nabla} \cdot (n_s \vec{v}_s) = 0, \tag{1}$$

$$m_s n_s \left[\frac{\partial}{\partial t} + (\vec{v}_s \cdot \vec{\nabla}) \right] \gamma_s \vec{v}_s = \vec{F}_s, \tag{2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{C} \frac{\partial \vec{B}}{\partial t}, \tag{3}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{C} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{C} \sum_s Z_s q_s n_s v_s, \tag{4}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \tag{5}$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi \sum_s Z_s n_s q_s, \tag{6}$$

$$\vec{E} = -\frac{1}{C} \frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \phi, \tag{7}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}, \tag{8}$$

where,

$$\gamma_s = \left(1 - \frac{v_s^2}{C^2} \right)^{-1/2},$$

$$F_s = q_s n_s \left[-\vec{\nabla} \phi - \frac{1}{C} \frac{\partial \vec{A}}{\partial t} + \frac{1}{C} (\vec{v}_s \times \vec{\nabla} \times \vec{A}) \right] - K_B \nabla (n_s T_s) \tag{9}$$

the subscript s represents the types of plasma species. For electron $s = e$, for ion $s = i$, for positron $s = p$ and dust particles $s = d, m_s, n_s, T_s, v_s, q_s$ are respectively the mass, number density, temperature, velocity and the charge of the plasma species. $\vec{E}$ and $\vec{B}$ are the electric and magnetic field intensity, ϕ and $\vec{A}$ are the scalar and vector potentials. K_{B} is the Boltzmann constant and C is the velocity of light. Z_s is the number of charges, $Z_e = 1, Z_i = 1, Z_p = 1$ and $Z_d = Z$.

Now, assuming all field variables in eqs (1) to (9) have slow and fast components given by

$$f_s = f_{s0} + f_{s1}. \tag{10}$$

In the right hand side of (10), the first term is the slow component and the second term is the fast component.

The time and space variation of the fast component of the field variable are given by

$$f_{s1} = f_{s1} \exp i(Kx - \omega t), \tag{11}$$

where K and ω are the wave number and frequency of the wave.

It is also to be noted that the quasineutrality condition in the multicomponent is given by

$$n_{e0} + Z_d n_{d0} = n_{p0} + n_{i0}. \tag{12}$$

Moreover, we assume that the charge fluctuation of dust particle is governed by

$$\frac{\partial q_{d1}}{\partial t} = I_{e1} + I_{i1} + I_{p1} \tag{13}$$

from which we obtain

$$q_{d1} = |I_{p0}| \left(\frac{n_{p1}}{n_{p0}} - \frac{n_{e1}}{n_{e0}} + \frac{n_{i1}}{n_{i0}} \right) \left(\frac{1}{\eta_1 - i\omega} \right), \tag{14}$$

where

$$\eta_1 = e\,|I_{e0}| \left(\frac{1}{T_e c_1} + \frac{1}{T_p c_1 - e q_{d0}} + \frac{1}{T_i c_1 - e q_{d0}} \right). \tag{15}$$

c_1 is the capacity of dust grains.

3. Dispersion relation

We have assumed, due to large inertia, that $(n_d, \vec{v}_d)$ have no fast component. For the vector potential's, fast component we set

$$\vec{A}_1 = \vec{A}_1(x,z,t) \exp(-i(\omega t - Kx)) = \vec{a} \exp(-i(\omega t - Kx)) = \vec{a} \mathrm{e}^{-i\theta}, \tag{16}$$

where $\vec{a} = a(\vec{y} - i\vec{z})$ the other fast components are written as; $n_{j1} = n_{j1}\mathrm{e}^{-i\theta}; v_{j1} = v_{j1}\mathrm{e}^{-i\theta}$ etc. We now substitute (10) in eqs (1) to (8) and separate the slow and fast components. In this respect it is to be noted that an average over the total cycle is to be taken where only the $\omega = 0$ part of the product of two fast component only contribute. Using eq. (10) we have

$$m_j n_{j0} [\partial t + (\vec{v}_{j0} \cdot \vec{\nabla})] \left(1 + \frac{|v_{j0}|^2}{2C^2} \right) \vec{v}_{j0} = \vec{A}_j + \vec{\Pi}_j, \tag{17}$$

where $j = e, i, p$ and $\vec{A}_j = q_j n_{j0}(\vec{v}_{j0} \times \vec{\nabla} \times (A_0/C) - \vec{\nabla}\phi_0 - (1/C)(\partial \vec{A}_0/\partial t) - K_{\rm B}\vec{\nabla}(n_{j0}T_j)$ for the slow components. Also the corresponding part of the Maxwell's equation read

$$\vec{\nabla} \times \vec{E}_0 = -\frac{1}{C}\frac{\partial \vec{B}_0}{\partial t}, \tag{18}$$

$$\vec{\nabla} \times \vec{B}_0 = \frac{4\pi}{C}\vec{J}_0, \tag{19}$$

$$\vec{\nabla} \cdot \vec{B}_0 = 0, \quad \vec{\nabla} \cdot \vec{E}_0 = 0, \tag{20}$$

$$\vec{B}_0 = \vec{\nabla} \times \vec{A}_0, \quad \vec{E}_0 = -\frac{1}{C}\frac{\partial \vec{A}_0}{\partial t} - \vec{\nabla}\phi_0, \tag{21}$$

where

$$\begin{aligned}\vec{J}_0 = q_e[n_{e0}\vec{v}_{e0} + \langle n_{e1}\vec{v}_{e1}\rangle] + q_i[n_{i0}\vec{v}_{i0} + \langle n_{i1}\vec{v}_{i1}\rangle]\\ +q_p[n_{p0}\vec{v}_{p0} + \langle n_{p1}\vec{v}_{p1}\rangle] - (Z_d n_{d0} q_{d0} \vec{v}_{d0}),\end{aligned} \tag{22}$$

$$\begin{aligned}\vec{\Pi}_j = -m_j n_{j0}\Bigg[\left\langle (\vec{v}_{j1}\cdot\vec{\nabla})\left(\left(1+\frac{|v_{j0}|^2}{2C^2}\right)\vec{v}_{j1}\right)\right\rangle\\ -\frac{q_j}{m_j}\left\langle \vec{v}_{j1} \times \vec{\nabla} \times \frac{\vec{A}_1}{C}\right\rangle\Bigg] + K_{\rm B}\left\langle \frac{n_{j1}}{n_{j0}}\vec{\nabla}(n_{j1}T_j)\right\rangle\\ -K_B\left\langle \frac{n_{j1}^2}{n_{j0}^2}\right\rangle \vec{\nabla}(n_{j0}T_j),\end{aligned} \tag{23}$$

where $\langle\ \rangle$, denotes the averaging over the full cycle. On the the other hand for the fast component we get,

$$\begin{aligned}m_j n_{j0}\Bigg[(\partial t + (\vec{v}_{j0}\cdot\vec{\nabla}))\left(1+\frac{|v_{j0}|^2}{2C^2}\vec{v}_{j1} + (\vec{v}_{j1}\cdot\vec{\nabla})\right)\\ \times\left(\left(1+\frac{|v_{j0}|^2}{2C^2}\right)\vec{v}_{j0}\right)\Bigg] = Q_j,\end{aligned} \tag{24}$$

where

$$\begin{aligned}Q_j = -K_B\left[1+\frac{\langle n_{j1}\rangle^2}{n_{j0}^2}\right]\vec{\nabla}(n_{j1}T_j) - \frac{n_{j1}}{n_{j0}}\vec{\nabla}(n_{j0}T_j)\\ +q_j n_{j0}\left[-\vec{\nabla}\phi_1 - \frac{\partial}{\partial t}\frac{\vec{A}_1}{C} + \vec{v}_{j0}\times\vec{\nabla}\times\frac{\vec{A}_1}{C} + \vec{v}_{j1}\times\vec{\nabla}\times\frac{\vec{A}_0}{C}\right],\end{aligned} \tag{25}$$

whereas

$$\vec{\nabla} \times \vec{E}_1 = -\frac{1}{C}\frac{\partial \vec{B}_1}{\partial t}, \tag{26}$$

$$\vec{\nabla} \times \vec{B}_1 = \frac{4\pi}{C}\vec{J}_1 + \frac{1}{C}\frac{\partial \vec{E}_1}{\partial t}, \tag{27}$$

$$\vec{\nabla} \cdot \vec{E}_1 = 4\pi \left[q_e n_{e1} + q_i n_{i1} + q_p n_{p1} + Z_d n_{d0} q_{d1}\right], \tag{28}$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0, \tag{29}$$

$$\vec{B}_1 = \vec{\nabla} \times \vec{A}_1, \tag{30}$$

$$\vec{E}_1 = -\frac{1}{C}\frac{\partial \vec{A}_1}{\partial t} - \vec{\nabla}\phi_1, \tag{31}$$

$$\begin{aligned}\vec{J}_1 &= q_e[n_{e0}\vec{v}_{e1} + n_{e1}\vec{v}_{e0}] + q_i[n_{i0}\vec{v}_{i1} + n_{i1}\vec{v}_{i0}] \\ &\quad + q_p[n_{p0}\vec{v}_{p1} + n_{p1}\vec{v}_{p0}] + (Z_d n_{d0} q_{d1}\vec{v}_{d0}).\end{aligned} \tag{32}$$

Actually terms occurring on the right hand side of eq. (24) are representative of ponderomotive forces. Equation (24) can be simplified by taking the following points into consideration.

(a) Any connective term of the form $(\vec{v}_{j0} \cdot \vec{\nabla})\vec{v}_{j1}$ can be ignored compared to $\partial t\vec{v}_{j1}$, because $\frac{|(\vec{v}_{j0}\cdot\vec{\nabla})\vec{v}_{j1}|}{|\partial t(\vec{v}_{j1})|} \approx \frac{v_{j0}Kv_{j1}}{\omega v_{j1}} \approx \frac{v_{j0}}{v_{\mathrm{ph}}} \ll 1$, where v_{ph} stands for the phase velocity $= \omega/K$.

(b) The other connective terms are even smaller. For example $(\vec{v}_{j1} \cdot \vec{\nabla})\vec{v}_{j0}$.

(c) The term $cn_{j0}(\vec{v}_{j1} \times \frac{\vec{B}_0}{C})$ is also small compared to $mn_{j0}\partial t(\vec{v}_{j1})$, because $\frac{e|\vec{v}_{j1}\times\frac{\vec{B}_0}{C}|}{m|\partial t(\vec{v}_{j1})|} \approx \frac{\omega_{j0}}{\omega} \ll 1$, ω_{j0} being the cyclotron frequency, for B_0 in the range of a few mega gauss. So eq. (24) can be written as

$$m_j n_{j0}\partial t\left[\left(1 + \frac{|v_{j0}|^2}{2C^2}\right)\vec{v}_{j1}\right] = -K_B\vec{\nabla}(n_{j1}T_j) + q_j n_{j0}\left(-\vec{\nabla}\phi_1 - \frac{\partial}{\partial t}\frac{\vec{A}_1}{C}\right). \tag{33}$$

Using eq. (33) along with eqs (28) and (16) we solve for the real and imaginary parts of $\vec{v}_j$ which turn out to be

$$\begin{aligned}\vec{v}_{eI} = x_1\Big[&\frac{e}{m\omega}x_4\lambda_D^2\vec{D}_R - \frac{e}{m\eta_1}(1-x_4)\lambda_D^2\vec{D}_I \\ &-\frac{e}{m\omega}\vec{\Phi}_R - \frac{e}{mC\omega}(r_1\vec{y} - r_2\vec{z})\Big],\end{aligned} \tag{34}$$

$$\begin{aligned}\vec{v}_{eR} = x_1\Big[&-\frac{e}{m\omega}x_4\lambda_D^2\vec{D}_I - \frac{e}{m\eta_1}(1-x_4)\lambda_D^2\vec{D}_R \\ &+\frac{e}{m\omega}\vec{\Phi}_I + \frac{e}{mC\omega}(r_2\vec{y} + r_1\vec{z})\Big],\end{aligned} \tag{35}$$

$$\vec{v}_{pI} = x_2\Bigg[-\frac{e}{m\omega}x_5\lambda_D^2\vec{D}_R + \frac{e}{m\eta_1}\left(\frac{T_pn_{e0}}{T_en_{p0}} - x_5\right)\lambda_D^2\vec{D}_I + \frac{e}{m\omega}\vec{\Phi}_R + \frac{e}{mC\omega}(r_1\vec{y} - r_2\vec{z})\Bigg], \quad (36)$$

$$\vec{v}_{pR} = x_2\Bigg[\frac{e}{m\omega}x_5\lambda_D^2\vec{D}_I + \frac{e}{m\eta_1}\left(\frac{T_pn_{e0}}{T_en_{p0}} - x_5\right)\lambda_D^2\vec{D}_R - \frac{e}{m\omega}\vec{\Phi}_I - \frac{e}{mC\omega}(r_2\vec{y} + r_1\vec{z})\Bigg], \quad (37)$$

$$\vec{v}_{iI} = x_3\Bigg[-\frac{e}{m\omega}x_6\lambda_D^2\vec{D}_R + \frac{e}{m\eta_1}\left(\frac{T_in_{e0}}{T_en_{i0}} - x_6\right)\lambda_D^2\vec{D}_I + \frac{e}{m\omega}\vec{\Phi}_R + \frac{e}{mC\omega}(r_1\vec{y} - r_2\vec{z})\Bigg], \quad (38)$$

$$\vec{v}_{iR} = x_3\Bigg[\frac{e}{m\omega}x_6\lambda_D^2\vec{D}_I + \frac{e}{m\eta_1}\left(\frac{T_in_{e0}}{T_en_{i0}} - x_6\right)\lambda_D^2\vec{D}_R - \frac{e}{m\omega}\vec{\Phi}_I - \frac{e}{mC\omega}(r_2\vec{y} + r_1\vec{z})\Bigg], \quad (39)$$

where,

$$x_1 = \left(1 + \frac{|v_{e0}|^2}{2C^2}\right)^{-1}, \quad x_2 = \left(1 + \frac{|v_{p0}|^2}{2C^2}\right)^{-1},$$
$$x_3 = \left(1 + \frac{|v_{i0}|^2}{2C^2}\right)^{-1}, \quad x_4 = \left(1 - \frac{Z_dn_{d0}I_{e0}}{en_{e0}\eta_1}\right),$$
$$x_5 = \frac{T_pn_{e0}}{T_en_{p0}}\left(1 - \frac{Z_dn_{d0}I_{p0}}{en_{p0}\eta_1}\right), \quad x_6 = \frac{T_in_{e0}}{T_en_{i0}}\left(1 - \frac{Z_dn_{d0}I_{i0}}{en_{i0}\eta_1}\right)$$
$$r_1 = (a_t\cos\theta - \omega a\sin\theta), \quad r_2 = (a_t\sin\theta + \omega a\cos\theta)$$
$$\vec{\Phi}_I = \vec{\nabla}(\phi_1\sin\theta), \quad \vec{\Phi}_R = \vec{\nabla}(\phi_1\cos\theta)$$
$$\vec{D}_R = \vec{\nabla}(\vec{\nabla}\cdot\vec{\Phi}_R), \quad \vec{D}_I = \vec{\nabla}(\vec{\nabla}\cdot\vec{\Phi}_I).$$

Now going back to eq. (1) and using the slow, fast decomposition we get

$$n_{e1} = \frac{n_{e0}}{\omega - K_xv_{e0x}}\vec{K}\cdot(\vec{v}_{eR} - i\vec{v}_{eI})$$
$$n_{p1} = \frac{n_{p0}}{\omega - K_xv_{p0x}}\vec{K}\cdot(\vec{v}_{pR} - i\vec{v}_{pI}) \quad (40)$$
$$n_{i1} = \frac{n_{i0}}{\omega - K_xv_{i0x}}\vec{K}\cdot(\vec{v}_{iR} - i\vec{v}_{iI})$$

and the fluctuating dust charge can be represented as

$$q_{d1} = |I_{p0}|\left(\frac{1}{\eta_1 - i\omega}\right)\Bigg[\frac{\vec{K}\cdot(\vec{v}_{pR} - i\vec{v}_{pI})}{(\omega - K_xv_{p0x})} - \frac{\vec{K}\cdot(\vec{v}_{eR} - i\vec{v}_{eI})}{(\omega - K_xv_{e0x})} + \frac{\vec{K}\cdot(\vec{v}_{iR} - i\vec{v}_{iI})}{(\omega - K_xv_{i0x})}\Bigg]. \quad (41)$$

On the other hand we also have

$$
\begin{aligned}
v_{e1x} &= \left[\mathrm{im}_e \left(1 + \frac{|v_{e0}|^2}{2C^2} \right) (\omega - K_x v_{e0x}) - \frac{iT_e K_B K_x^2}{(\omega - K_x v_{e0x})} \right]^{-1} eE_{1x} \\
v_{p1x} &= - \left[\mathrm{im}_p \left(1 + \frac{|v_{p0}|^2}{2C^2} \right) (\omega - K_x v_{p0x}) - \frac{iT_p K_B K_x^2}{(\omega - K_x v_{p0x})} \right]^{-1} eE_{1x} \\
v_{i1x} &= - \left[\mathrm{im}_i \left(1 + \frac{|v_{i0}|^2}{2C^2} \right) (\omega - K_x v_{i0x}) - \frac{iT_i K_B K_x^2}{(\omega - K_x v_{i0x})} \right]^{-1} eE_{1x}.
\end{aligned} \tag{42}
$$

Using these expressions for $n_{e1}, n_{p1}, n_{i1}, v_{e1x}, v_{p1x}, v_{i1x}$ in eqs (6) and (28) we finally obtain the dispersion relation

$$
\begin{aligned}
1 &+ \frac{1}{K^2\lambda_e^2 - a_e\sigma_e^2} \left[1 + \frac{i\beta}{\omega + i\eta_1} \right] + \frac{1}{K^2\lambda_i^2 - a_i\sigma_i^2} \left[1 + \frac{i\beta\delta_1}{\omega + i\eta_1} \right] \\
&+ \frac{1}{K^2\lambda_p^2 - a_p\sigma_p^2} \left[1 + \frac{i\beta\delta_2}{\omega + i\eta_1} \right] = 0
\end{aligned} \tag{43}
$$

where,

$$
\begin{aligned}
&\lambda_e^2 = \frac{K_B T_e}{4\pi n_{e0} e^2}, \quad \lambda_i^2 = \frac{K_B T_i}{4\pi n_{i0} e^2}, \quad \lambda_p^2 = \frac{K_B T_p}{4\pi n_{p0} e^2}, \\
&a_e = \left(1 + \frac{v_{e0}^2}{2C^2} \right), \quad a_i = \left(1 + \frac{v_{i0}^2}{2C^2} \right), \quad a_p = \left(1 + \frac{v_{p0}^2}{2C^2} \right), \\
&\sigma_e^2 = \left(\frac{\omega - K_x v_{e0x}}{\omega_e} \right)^2, \quad \sigma_i^2 = \left(\frac{\omega - K_x v_{i0x}}{\omega_i} \right)^2, \quad \sigma_p^2 = \left(\frac{\omega - K_x v_{p0x}}{\omega_p} \right)^2, \\
&\delta_1 = \frac{n_{e0}}{n_{i0}}, \quad \delta_2 = \frac{n_{e0}}{n_{p0}}, \quad \beta = \frac{|I_{e0}|\, Z_d n_{d0}}{e n_{d0}}, \\
&\omega_e = \frac{4\pi n_{e0} e^2}{m_e}, \quad \omega_i = \frac{4\pi n_{i0} e^2}{m_i}, \quad \omega_p = \frac{4\pi n_{p0} e^2}{m_p}.
\end{aligned}
$$

Equation (16) is the most general form of the dispersion relation than that obtained by the previous authors [8]. Solving (16) one can easily find that both the wave frequency ω and wave number is complex which indicates that the wave will be unstable during its propagation through the plasma. The imaginary part which represents the growth rate of the wave depends on the drift velocity and concentrations of electrons, ions, positrons and charged dust particles. However, if we consider the effect of positron on the instability of the wave we get

$$
\omega = K \left[v_{p0} \mp \left(\frac{\delta_2}{a_p} \right)^{\frac{1}{2}} C_{ds} \right] \pm \frac{i\beta}{2} \left(\frac{\delta_2}{a_p} \right)^{\frac{1}{2}} \cdot \frac{\delta_2 C_{ds}}{\left[v_0 - (\frac{\delta_2}{a_p})^{\frac{1}{2}} C_{ds} \right]}, \tag{44}
$$

where

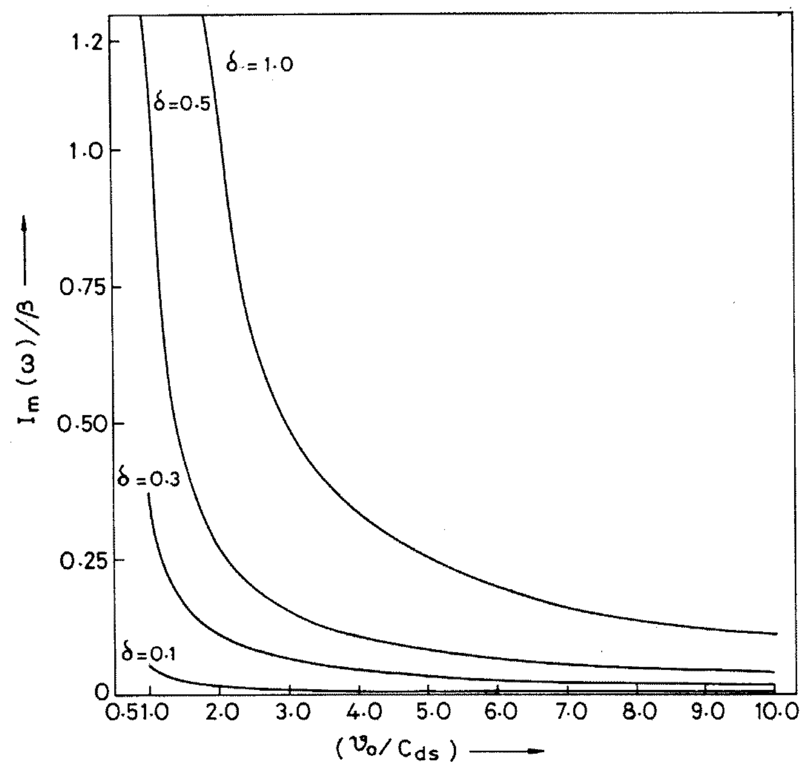

Figure 1. Imaginary part of the unstable root as a function of v_0/C_{ds} for different values of the ratio of background densities of electrons to positrons.

$$v_{p0} > C_{ds} \quad , \qquad C_{ds}^2 = \lambda_e^2 \omega_p^2 .$$

In figure 1, the variation of Im $\frac{\omega}{\beta}$ against $\frac{v_{p0}}{C_{ds}}$ has been shown for various values of the concentrations (δ_2) of electrons and positrons. It is interesting to see that the growth rate significantly depends on the ratio of the number density of positrons and electrons.

4. Magnetic field generation

The magnetic field which is generated inside the plasma due to weakly nonlinear localization of the high frequency electromagnetic field can be found out following Rizzato [16]. From (26), (31) and (41) and using eqs (34)– (39) we can obtain

$$-\beta_1^2 \vec{\nabla}(\nabla^2 \phi_1) = \frac{\omega^2}{C^2} \epsilon_e^1 \left(\partial t \frac{\vec{A}_1}{C} + \vec{\nabla} \phi_1 \right) , \tag{45}$$

where β_1 and ϵ_e^1 stands for

$$\beta_1^2 = \frac{K_B T_e}{mC^2}\left[x_2 \frac{T_p}{T_e}\left(1 - \frac{Z_d n_{d0}\,|I_{e0}|}{e n_{p0}\eta_1} - \frac{i\omega Z_d n_{d0}\,|I_{p0}|}{e n_{p0}\eta_1^2}\right) + x_1\left(1 - \frac{Z_d n_{d0}\,|I_{e0}|}{e n_{e0}\eta_1} - \frac{i\omega Z_d n_{d0}\,|I_{p0}|}{e n_{e0}\eta_1^2}\right)\right],$$

$$\epsilon_e^1 = \left[1 - \frac{\omega_p^2}{\omega^2}\left(x_1 + x_2\frac{n_{p0}}{n_{e0}}\right)\right].$$

Neglecting inertia of electrons and positrons we also get using eqs (23) and (32),

$$2\vec{\nabla}\times\left(\vec{\nabla}\phi_0 + \partial t\frac{\vec{A}_0}{C}\right) = \frac{1}{e}\vec{\nabla}\times\left(\frac{\vec{\Pi}_p}{n_{p0}} - \frac{\vec{\Pi}_e}{n_{e0}}\right). \tag{46}$$

The right hand side of this equation clearly indicates the fact that the generation of magnetic field is due to the presence of the ponderomotive force. Simplifying the above equation, we get

$$2\left(\vec{\nabla}\times\frac{\partial\vec{A}_0}{\partial t}\right) = -\vec{\nabla}\times\bar{\epsilon} \tag{47}$$

with

$$\begin{aligned}\epsilon = \frac{mC}{e}\Big[&\vec{v}_{eR}\times\left(\vec{\nabla}\times\left(a_1\vec{v}_{eR} - \frac{e}{m\omega}\vec{E}_I\right)\right)\\ &+\vec{v}_{eI}\times\vec{\nabla}\times\left(a_1\vec{v}_{eI} + \frac{e}{m\omega}\vec{E}_R\right)\\ &-\vec{v}_{pR}\times\left(\vec{\nabla}\times\left(a_2\vec{v}_{pR} + \frac{e}{m\omega}\vec{E}_I\right)\right)\\ &-\vec{v}_{pI}\times\left(\vec{\nabla}\times\left(a_2\vec{v}_{pI} - \frac{e}{m\omega}\vec{E}_R\right)\right)\Big]\end{aligned} \tag{48}$$

where $a_1 = 1/x_1, a_2 = 1/x_2$.

Now collecting the components of $\bar{\epsilon}$ we get,

$$\bar{\epsilon}_x = 0, \quad \bar{\epsilon}_y = \Psi, \quad \bar{\epsilon}_z = i\Psi, \tag{49}$$

where

$$\Psi = \frac{2ep\phi_1(a_x - iap)}{m}(\Psi_1 - i\Psi_2),$$

$$\Psi_1 = \frac{1}{\omega}\left[x_1\left(1 + \lambda_D^2 p^2\left(1 - \frac{\beta}{\eta_1}\right)\right) - x_2\left(1 + \frac{T_p n_{e0}}{T_e n_{p0}}\left(1 - \frac{\beta\delta}{\eta_1}\right)\lambda_D^2 p^2\right)\right],$$

$$\Psi_2 = \lambda_D^2 p^2\left(\frac{x_1\beta}{\eta_1^2} - x_2\frac{T_p n_{e0}\beta\delta}{T_e n_{p0}\eta_1^2}\right),$$

where $p = \frac{\omega}{\beta_1 C}(\epsilon_e^1)^{\frac{1}{2}}$. So that eq. (47) yields with eqs (18) and (21)

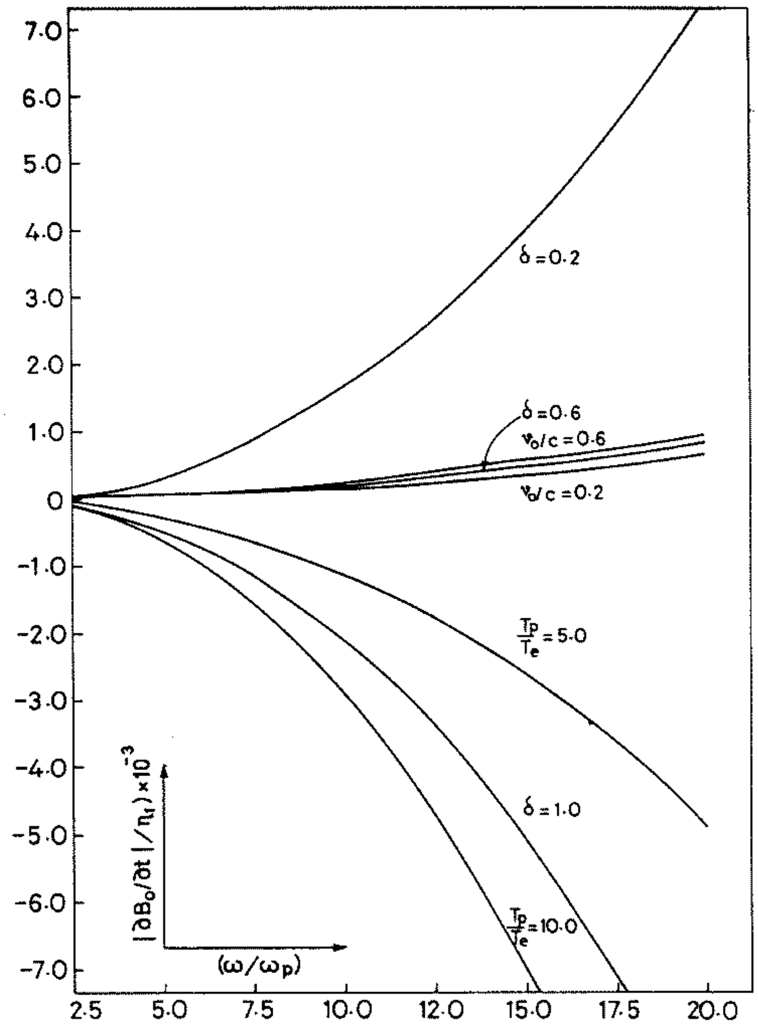

Figure 2. The growth rate of the magnetic field with the variation of frequency ratio (ω/ω_p) at different values of (i) T_p/T_e, (ii) v_0/c (iii) η_{e0}/η_{p0}.

$$\left(\frac{\partial B_0}{\partial t}\right)_x = 0,$$
$$\left(\frac{\partial B_0}{\partial t}\right)_y = -\frac{eapK_1\phi_1(p+K_1)}{m}(\Psi_2 + i\Psi_1),$$
$$\left(\frac{\partial B_0}{\partial t}\right)_z = \frac{eapK_1\phi_1(p+K_1)}{m}(\Psi_1 - i\Psi_2), \tag{50}$$

where we have set $a = a_1 \exp(iK_1 x - i\omega t)$. If η_1 is small compared to ω, we obtain on further simplification

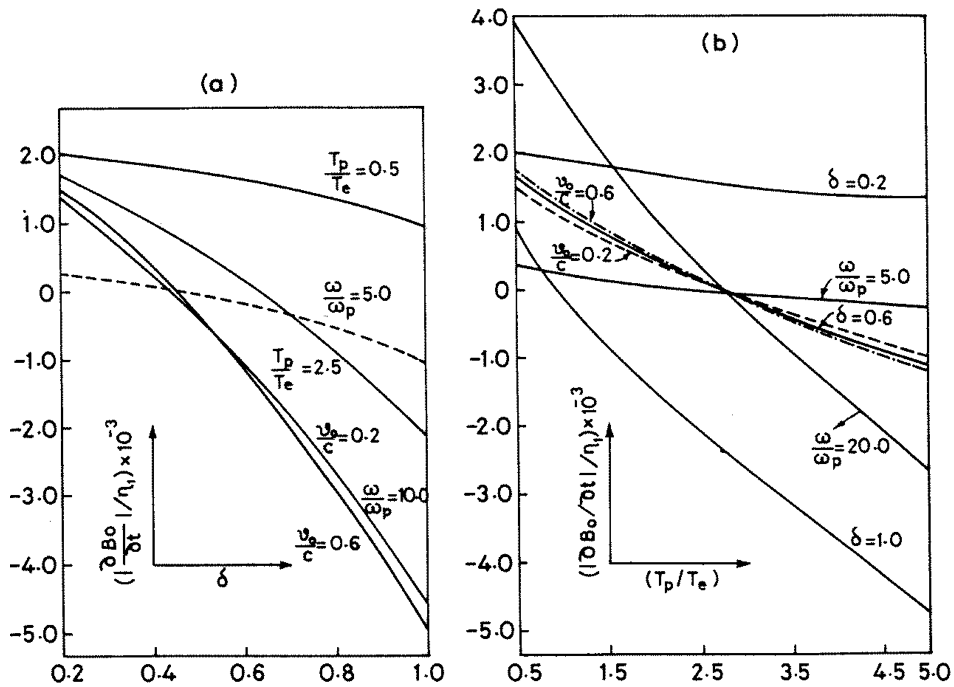

Figure 3. The growth rate of the magnetic field with the variation of (**a**) the number density ratio (η_{e0}/η_{p0}) at different values of (i) T_p/T_e (ii) v_0/c (iii) ω/ω_p and (**b**) temperature ratio (T_p/T_e) at different values of (i) η_{e0}/η_{p0} (ii) v_0/c (iii) n_{e0}/n_{p0}.

$$\frac{(\frac{\partial B_0}{\partial t})}{\eta_1} = \frac{\sqrt{2}\cdot ea_1\omega^{\frac{3}{2}}\phi_1 K_1^2\beta^{\frac{1}{2}}[1-X(1+Y)]^{\frac{3}{2}}(1-\alpha_p\delta_2^2)}{m\beta\lambda_D\omega_p^3(1+\alpha_p\delta_2)^{\frac{3}{2}}}, \tag{51}$$

where

$$X = \frac{\omega_p^2}{\omega^2}, \quad Y = \frac{1}{\delta_2}, \quad \alpha_p = \frac{T_p}{T_e}, \quad \delta_2 = \frac{n_{e0}}{n_{p0}},$$

a_1 and ϕ_1 are the amplitudes of vector and scalar potentials. In figures 2 and 3, the variations of the growth rate of the generated magnetic field are shown for different values of the temperatures and concentrations of electrons and positrons. It is interesting to note from the figures that while the spontaneously generated magnetic field depends on δ_2 and α_p, but relativistic correction is not so important. The observations here are important and actually effects the plasma in space, where electron-positron configuration plays important roles.

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