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Role of magnetic shear on the electrostatic current driven ion-cyclotron instability in the presence of parallel electric field

HARSHA JALORI and A K GWAL
Space Science Laboratory, Institute of Physics and Electronics, Barkatullah University, Bhopal 462 026, India

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Abstract. Recent observation and theoretical investigations have led to the significance of electrostatic ion cyclotron (EIC) waves in the electrodynamics of acceleration process. The instability is one of the fundamental of a current carrying magnetized plasma. The EIC instability has the lowest threshold current among the current driven instabilities. On the basis of local analysis where inhomogeneities like the magnetic shear and the finite width current channel, have been ignored which is prevalent in the magnetospheric environment. On the basis of non-local analysis interesting modification has been incorporated by the inclusion of magnetic shear. In this paper we provide an analytical approach for the non-local treatment of current driven electrostatic waves in presence of parallel electric field. The growth rate is significantly influenced by the field aligned electron drift. The presence of electric field enhances the growth of EIC waves while magnetic shear stabilizes the system.

Keywords. Magnetic shear; electrostatic ion cyclotron waves; magnetosphere.

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1. Introduction

The EIC waves are characterized by frequencies close to the harmonics of the ion cyclotron frequency and perpendicular wave numbers $k_\perp \leq n/r_i$, where n is the harmonic number and r_i is the ion Larmour radius. Also these waves are characterized by parallel wave number $k_\parallel \ll k_\perp$. In most of the studies, the field aligned currents (current parallel to the magnetic field) or ion beams have served as a driving force for such instability [1]. EIC instability is generated due to the coupling of ion-acoustic mode and Larmour ions. Drummond and Rosenbluth [1] were the first to examine this instability analytically, while Kindel and Kennel [2] studied it in the context of space plasmas in the Earth's magnetosphere. It should be noted however that the analysis of both [1] and [2] is a local analysis.

It is widely recognized that the electric field plays an important role in the dynamics of ionosphere and magnetosphere plasmas. In the auroral regions, charged particles are accelerated to very high energies by electric fields parallel to the magnetic field. They consider a uniform zero order magnetic field (i.e., $B = B_{0z}$), thereby neglecting the self-consistent magnetic field generated by the field aligned currents. This magnetic field (usually small) will give rise to a shear in the zero order field, space dependent i.e., $B(x) = B_y(x) \cdot \hat{y} + B_z \cdot \hat{z}$. Ganguli and Bakshi [3], Bakshi *et al* [4], Ganguli and Bakshi [5], Ganguli *et al* [6] have generalized and improved the previous model of Drummond and Rosenbluth [1] by using a non-local analysis. Note the interesting modifications when the model is made more realistic by the inclusion of inhomogeneities. Ion cyclotron waves can also be excited by the field aligned ion beams. In general magnetic shear is a damping agent and can significantly alter the local mode structure. Ganguli and Bakshi [3] have concluded that even a small shear can significantly reduce the growth rate.

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2. Theoretical considerations

In the present paper, we have considered magnetospheric plasma to contain an uniform static magnetic field B_0 and the electric field E_0 along the z axis of the co-ordinate system. The electron distribution function is assumed to be of the Boltzmann–Vlasov type. In the presence of electric field parallel to B_0, the equilibrium distribution function is slightly modified in terms of the drift velocity due to electric field.

The effective collision frequency ν is independent of the particle velocity. Since it is of very small order in magnetospheric plasma, it is neglected in our case.

This electric field is weak enough so that the induced drift velocity of electron is much smaller then the phase velocity of the propagating wave. In the present problem the presence of such a weak electric field does not even lead to runway electrons.

The dispersion relation for low frequency electrostatic wave in magnetized plasma consisting of beam ions, target ions, and drifting bulk electrons is given by Singh *et al* [7]

$$\begin{aligned} D(\omega,k) = 1 &+ \frac{1}{k^2\lambda_e^2}\left\{1 + \frac{\omega + k_\parallel v_{de}}{k_\parallel v_e} Z\left(\frac{\omega + k_\parallel v_{de}}{k_\parallel v_e}\right)\right\} \\ &+ \frac{1}{k^2\lambda_t^2}\left\{1 + \sum_n \Gamma_n \frac{\omega}{k_\parallel v_t} Z\left(\frac{\omega - n\Omega_i}{k_\parallel v_t}\right)\right\} \\ &+ \left\{1 + \sum_n \Gamma_n \frac{\omega - k_\parallel U_b - nA_b\Omega_i}{k_\parallel v_{b\parallel}}\right\} \\ &\times Z\left(\frac{\omega - k_\parallel U_b - n\Omega_i}{k_\parallel V_{b\parallel}}\right), \end{aligned} \qquad (1)$$

where $k^2 = k_\parallel^2 + k_\perp^2$; $k_\perp$ and $k_\parallel$ are parallel and perpendicular wave numbers with respect to magnetic field B_0. V_{de} is the electron drift velocity which is opposite in direction to the beam velocity U_b. Z is the plasma dispersion function [8].

The thermal velocities of electrons and target ions are

$$v_e = \sqrt{\frac{2k_bT_e}{m_e}}, \qquad v_t = \sqrt{\frac{2k_bT_t}{M}},$$

m_e and M are their masses. We also define

$$\lambda_e = \frac{v_e}{\sqrt{2\omega_{pe}}}, \quad \lambda_t = \frac{v_t}{\sqrt{2\omega_t}}, \quad \lambda_{b\|} = \frac{v_{b\|}}{\sqrt{2\omega_b}},$$

where $\omega_{p\alpha}$ $(\alpha = e, t, b)$ denotes the plasma frequencies of the various plasma components.

The function $\lceil n$ is defined by $\lceil n(\mu_i) = \exp(-\mu_i)I_n(\mu_i)$, where

$$\mu_i = \frac{k_\perp^2 v_{\perp i}^2}{2\Omega_i^2} = \frac{1}{2}k_\perp^2\rho_i^2.$$

The Boltzmann–Poisson system of equation is solved analytically through Fourier transformation which is equivalent of replacing the temperature $T_\|$ by complex temperature

$$T_{cj} = T_j[1 - iq_jE\cdot k/Kk^2T_j]$$

for the electrons and ions (j labeling the species) in the well-known dispersion relation for electrostatic plasma waves. This procedure is valid only if the distribution function is actually a drifted bi-Maxwellian [9].

So the effect of parallel electric field E_0 with respect to magnetic field B_0 has been incorporated through the modification of particles thermal velocity. Under this effect temperature $T_\|$ modifies to a complex temperature $T_{\|c}$ in the direction of the magnetic field [9–11] as given by

$$T_{\|c} = T_\| \left(1 - \frac{jeE_0}{kKT_\|}\right), \tag{2}$$

where e is the charge of particles and K is the Boltzmann constant, while j is an imaginary term and $eE_0/kKT_\| \ll 1$.

Using the basic definition of growth rate γ as

$$\gamma = \frac{-\mathrm{Im}D(\omega,k)}{\frac{\partial}{\partial\omega}\{\mathrm{Re}D(\omega,k)\}} \tag{3}$$

assuming that real part of frequency $\omega_r \gg \gamma$.

The expression of the growth rate for fundamental EIC mode $(n = 1)$ in presence of parallel electric field is given by

$$\frac{\gamma}{\Omega_i} = \frac{\left(\frac{\lambda_{b\|}}{\lambda_e}\right)^2 \times \sqrt{\pi}\sum_n \Gamma_n \frac{(x+X)}{y}\exp\frac{-(x+X)^2}{y} + \frac{\lambda_{b\|}^2}{\lambda_t^2} \times \sqrt{\pi}\cdot\exp\frac{-(x-1)^2}{z} \times \frac{x}{z} + \sqrt{\pi}\sum_n \Gamma_n \exp\frac{-(x-\tilde{y}-1)^2}{z}\cdot\frac{(x-\tilde{y}-1)}{z} - \frac{T_{b\|}}{T_{b\perp}} \times \tilde{k} \times \frac{1}{(x-\tilde{y}-1)}}{\left(\frac{\lambda_{b\|}^2}{\lambda_t}\right)^2\left(\frac{1}{(x-1)^2}\right) + \frac{2T_{b\|}}{T_{b\perp}} \times \sqrt{\pi} \times \tilde{k} \times \exp\frac{-(x-\tilde{y}-1)^2}{z} \times \frac{1}{xz} \times \sum_n \Gamma_n + 2\sum_n \Gamma_n \times \frac{T_{b\|}}{T_{b\perp}} \times \sqrt{\pi} \times \frac{(x-\tilde{y}-1)}{z} \times \exp\frac{-(x-\tilde{y}-1)^2}{z} \times \frac{1}{z^2} \times \tilde{k}}, \tag{4}$$

where

$$X = \frac{k_{\|} v_{de}}{\Omega_e}; \quad x = \frac{\omega}{\Omega_e}; \quad \tilde{k} = \frac{qE_0}{Mk\alpha_{\|b}^2},$$

$$Y = \frac{k_{\|} v_e}{\Omega_e}; \quad \tilde{y} = \frac{k_{\|} U_b}{\Omega_i},$$

$$Z = \frac{k_{\|} v_t}{\Omega_e}; \quad z = \frac{k_{\|} v_{b\|}}{\Omega_i},$$

$$\alpha_{\|} = \frac{\omega}{k_{\|} v_t}; \quad \alpha_{\|b} = \frac{\omega}{k_{\|} v_{b\|}}.$$

Using magnetosphere plasma parameters, the growth are computed from eq. (4).

Effects of magnetic shear

Presence of magnetic shear introduces a non-local behaviour that can alter the local results quite significantly. Generally, the effect of shear is introduced (i) locally (i.e., $k_{\|} \to k_{\|}(x)$) and (ii) globally (i.e., $ikx \to \partial/\partial x$) as given by Ganguli and Bakshi [3].

The current profile is found to be

$$j(x) = n_0 e v_d^0 g(\xi), \tag{5}$$

where

$$g(\xi) = \mathrm{e}^{-\xi^2} \quad \text{and} \quad \xi = x/L_{\mathrm{c}}.$$

L_{c} is a size of finite current channel and such a current profile generates a self consistent shear in the magnetic field, given by

$$\frac{B_y(x)}{B_z} = \frac{L_{\mathrm{c}}}{L_{\mathrm{s}}} \int_0^{\xi} g(\xi) \mathrm{d}\xi, \tag{6}$$

where the shear length is defined by

$$\frac{1}{S} = L_{\mathrm{s}} = \frac{cB_z}{4\pi n_0 e v_d^0}. \tag{7}$$

The corresponding variation in the parallel wave number is

$$k_{\|}(x) = k_{\|}^0 + k_y B_y(x)/B_z \tag{8}$$

which leads to the variation in the angle of propagation

$$u(x) = \frac{k_{\|}(x)}{k_y} = u_0 + \frac{L_{\mathrm{c}}}{L_{\mathrm{s}}} \int_0^{\xi} g(\xi) \mathrm{d}\xi. \tag{9}$$

So the non-local effects are described by $k_{\|} \to k_{\|}(x)$ as given by (8) and $v_d \to v_d(x)$ as implied by eq. (7) which will modify the growth rate given by eq. (4). We introduce shear by replacing $k_{\|}$ by $k_{\|}^{\circ} + sk_y \cdot x$ where $s = 1/L_{\mathrm{s}}$.

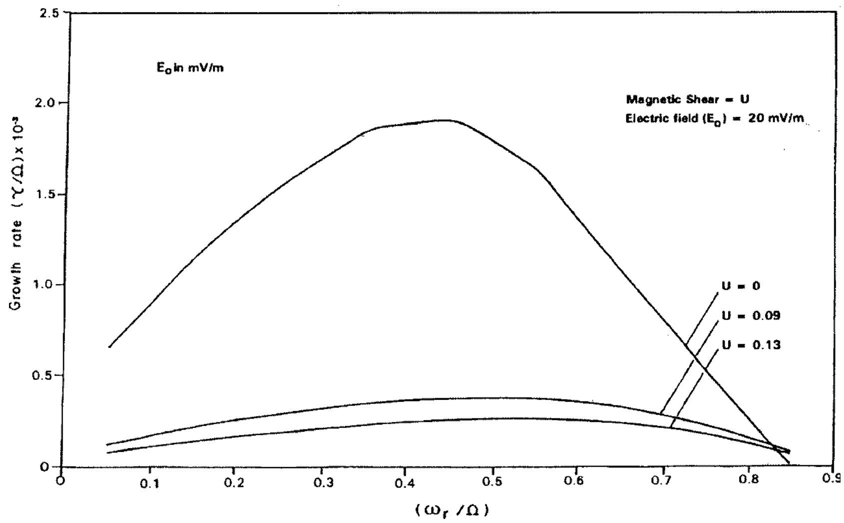

Figure 1. Variation of growth rate (γ/Ω) of electrostatic ion cyclotron wave with ion cyclotron frequencies $(\omega_{\mathrm{r}}/\Omega)$ for different values of shear in the presence of electric field (20 mV/m).

3. Results and discussion

We have chosen magnetospheric plasma parameters to compute the growth rate from eq. (4). The effect of electric field is being discussed with reference to temperature anisotropy of the beam and electron drift. We provide an analytical approach for the non-local treatment of current driven EIC waves in a sheared magnetic field. We study the effects of magnetic shear by replacing $k_{\|}$ in eq. (4) by $k_y \cdot u$ where $u = sx(= k_{\|}^{(x)}/k_y)$ where $1/\mathrm{s} = L_{\mathrm{s}} = cB_z/(4\pi n_0 e v_d^0)$ and the transverse wave number given by $b = 1/2(k_y \cdot \rho_i^2)$. The independent variable u is the physical angle $u = x/L_{\mathrm{s}}$, the origin of x is at the position where the field line is perpendicular to the given k $(\mathbf{k}\cdot\mathbf{B} = 0)$. $|\phi|$ is significant between 0.9 and 0.12 with a peak around $u_{mo} = 0.09$. We have demonstrated the importance of non local effects due to a magnetic shear (produced by the field aligned current). We find that shear significantly reduces the local growth rate.

In the present analysis we have chosen magnetospheric plasma parameter as used by Singh *et al* [7] and calculate the growth rate of EIC waves for the case of with and without shear in the presence of electric field as shown in figure 1. It is observed that the effects of magnetic shear are to decrease the growth rate. Anyhow the increase in the electric field can give rise to the growth rate of electrostatic ion-cyclotron waves.

4. Conclusions

(1) The magnetic shear decreases the growth rate of EIC waves.
(2) The growth rate depends on the strength of parallel electric field.

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References

[1] W E Drummond and M N Rosenbluth, *Phys. Fluids* **5**, 1502 (1962)
[2] J M Kindel and C F Kennel, *J. Geophys. Res.* **76**, 3055 (1971)
[3] G Ganguli and P Bakshi, *Phys. Fluids* **23**, 1830 (1982)
[4] P Bakshi, G Ganguli and P Palmadesso, *Phys. Fluids* **26**, 1808 (1983)
[5] G Ganguli and P Bakshi, *J. Geophys. Res.* **89**, 945 (1984a)
[6] G Ganguli, P Bakshi and P Palmadesso, *J. Phys. Fluids* **27**, 2039 (1984b)
[7] Nagendra Singh, J R Conard and R W Schunk, *J. Geophys. Res.* **90**, 12, 219 (1985)
[8] B D Fried and S D Conte, *The plasma dispersion function* (Academic, New York, 1961)
[9] D Pines and J R Schrieffer, *Phys. Rev.* **124**, 1387 (1961)
[10] A Bers and S R G Brueck, *Quantum Prog. Rep.* **89**, 159 (1968)
[11] B Juhl and R A Treumann, *J. Plasma. Phys.* **34**, 47 (1985)