

## Resonance propagation in heavy-ion scattering

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MS received 28 September 1999; revised 21 December 2000

**Abstract.** The formalism developed earlier by us for the propagation of a resonance in the nuclear medium in proton–nucleus collisions has been modified to the case of vector boson production in heavy-ion collisions. The formalism includes coherently the contribution to the observed di-lepton production from the decay of a vector boson inside as well as outside the nuclear medium. The medium modification of the boson is incorporated through an energy dependent optical potential. The calculated invariant  $\rho$  mass distributions are presented for the  $\rho$ -meson production using optical potentials estimated within the VDM and the resonance model. The shift in the invariant mass distribution is found to be small. To achieve the mass shift (of about 200 MeV towards lower mass) as indicated in the high energy heavy-ion collision experiments, an unusually strong optical potential of about  $-120$  MeV is required. We also observe that, for not so heavy nuclear systems and/or for fast moving resonances, the shape, magnitude and peak position of the invariant mass distribution is substantially different if the contributions from the resonance decay inside and outside are summed-up at the amplitude level (coherently) or at the cross section level (incoherently).

**Keywords.** Heavy-ions;  $\rho$ -meson; medium modification.

**PACS Nos** 25.75.+r; 24.30.-v; 24.50.+g; 24.10.-i

### 1. Introduction

In recent times there has been much interest in the medium modification of hadron masses. Brown and Rho [1] suggested that the masses of vector bosons, like  $\rho, \omega, \dots$  (and also baryons) scale as

$$\frac{m_V^*(\varrho)}{m_V} \sim \frac{f_\pi^*(\varrho)}{f_\pi}, \quad (1)$$

where the star denotes the in-medium quantities.  $f_\pi$  is the pion weak decay constant. If the decay constant  $f_\pi^*$  decreases, as it does at higher nuclear densities/temperatures due to restoration of the chiral symmetry, the vector boson masses decrease. At the nuclear saturation density ( $\varrho_0$ ) the decrease in mass for  $\rho(770)$  and  $\omega(782)$  mesons is thought to be around 20% or so. At densities/temperatures ( $\sim 3\varrho_0/150$  MeV) achievable in relativistic heavy-ion collisions, the decrease is estimated to be around a factor of 2. This decrease

in vector meson mass, linked to the modification of quark condensates in the medium, is also corroborated by the effect obtained by using QCD sum rule techniques in the medium [2]. Decrease in hadron masses are also seen in the relativistic mean field calculation by Walecka *et al* [3]. A model calculation by Herrmann *et al* [4] of the density dependent two pion self energy in nuclear matter suggests a strongly increased decay width, but, unlike others, only a negligible change of the in-medium  $\rho$  mass. At high densities, ( $\rho_N = 2 - 3\rho_0$ ), they also find another peak in the  $\rho$  meson spectral distribution function at the invariant mass  $\approx 3m_\pi$ . This branch corresponds to the decay of the  $\rho$  meson into a pion and a  $\Delta$ -hole state. Similar conclusions have been achieved by the vector dominance model calculation of Asakawa and Ko [5], which include the effect of collision broadening due to the  $\pi - N - \Delta - \rho$  dynamics. More recently, using the quark-meson coupling model a detailed calculation on light nuclei has been reported by Saito *et al* [6]. They find that the average mass of a  $\rho$ -meson gets reduced by about 50 MeV in  $^{12}\text{C}$  nucleus.

Experimentally, the indication of the medium effect on  $\rho$ -meson is believed to have been seen in the enhanced dilepton yield in CERES and HELIOS relativistic heavy-ion reaction data taken at CERN-SPS [7]. The indicated reduction in the  $\rho$ -meson mass, however, is drastic ( $m_\rho^* \approx 350$  MeV). Besides, reports also exist that this enhancement could also be due to some other scenarios [8].

Thus we see that medium modification of hadron masses is an issue of much interest currently, and is under much debate. Another issue which also has relevance to the above is the manner in which the di-lepton data are analysed. Experimentally the masses of unstable hadrons are explored by producing them in nuclear reactions and then measuring the invariant masses of their leptonic decay products [9]. The medium modification of the unstable hadron from these data is, normally, inferred by adding *incoherently* the decay of the hadron inside and outside the nuclear medium [10]. In a correct method these contributions should be added coherently, i.e. at the amplitude level. This will lead to interference effects in the measured cross sections, and may modify the invariant mass distribution as coming from the incoherent calculations. For the proton–nucleus collisions such studies have been done recently by us [11] and Boreskov *et al* [12] and have been found to have substantial effect. These formalisms incorporate the interaction of the resonance with the nuclear medium through an optical potential. The formalism of Jain and Kundu [11] also include the medium effect on the decay products of the resonance if the latter are hadrons. In the present paper we modify our earlier formalism to heavy-ion collisions, and apply it to the propagation and decay of  $\rho$ -meson. We investigate the difference introduced in the mass distribution if the contribution from inside and outside decay of the resonance is added coherently instead of incoherently. We then also present a calculation to exhibit the sensitivity of our calculation to the magnitude of the medium mass modification of the  $\rho$ -meson.

In our formalism we assume that the resonance is produced when two nucleons, one in the target  $B$  and another in the projectile  $A$  collide. From there on the resonance moves suffering interaction with the medium. We do not worry about the state of the  $(A + B)$  system. The resonance coordinate is with respect to the c.m of the  $(A + B)$  system. The propagating resonance, as said earlier, decays either inside or outside the  $(A + B)$  system and both these contributions add coherently. The interaction of the resonance with the nuclear medium is described by an optical potential. The formalism developed here could be applied in general to any propagating resonance produced in nucleus–nucleus collisions. However, we study, specifically, the invariant mass spectra of a  $\rho$  resonance produced

in Pb+Pb and S+Au collisions traversing at different speeds. We also give the decay probability of the  $\rho$  resonance inside the nuclear medium as a function of its speed of propagation.

The general findings of the paper are that the probability of the  $\rho$  resonance decaying inside the  $A + B$  system is large. Still, the summed cross-section (i.e. its magnitude, shape and the peak position), especially at higher speeds where the contribution from the outside increases, and for lighter systems, like S+Au, is determined equally by the contributions from inside as well as outside. We also find that, for such situations, the coherently calculated cross sections differ a lot from those calculated incoherently. We also find that the calculated invariant mass distribution is sensitive to the medium modification of the  $\rho$ -meson masses. However, the shift in the peak position of the distribution within the vector dominance model (VDM) and the resonance model estimated  $\rho$ -meson potential is small compared to that indicated in the high energy heavy-ion collision data. Therefore, the large  $\rho$ -mass shift seen in the high energy heavy-ion data does not seem to be the final state interaction effect. To achieve such a large effect one would need an abnormally large (unphysical) optical potential. The observed mass reduction seems to be an indicator of the QCD effect at the time of the formation of the  $\rho$ -meson.

## 2. Formalism

Let us suppose that two heavy nuclei, one the projectile  $A$  and another the target  $B$ , collide at high energies. We assume that a resonance  $R$  is produced by the collision of one nucleon in the projectile and one in the target collide at the collision point. This resonance then moves and decays at some subsequent point. That is, we study the reaction,



with the resonance  $R$  decaying into say  $x$  and  $y$ ,



Denoting by  $\mathbf{r}$  the relative coordinate between the target and the projectile, and by  $(\mathbf{r}_B, \mathbf{r}_A)$  the intrinsic coordinates of the target and projectile nucleons, respectively (see figure 1) the resonance coordinate is written as

$$\mathbf{r}_R = \mathbf{r}_A + \frac{B}{A+B} \mathbf{r}. \quad (4)$$

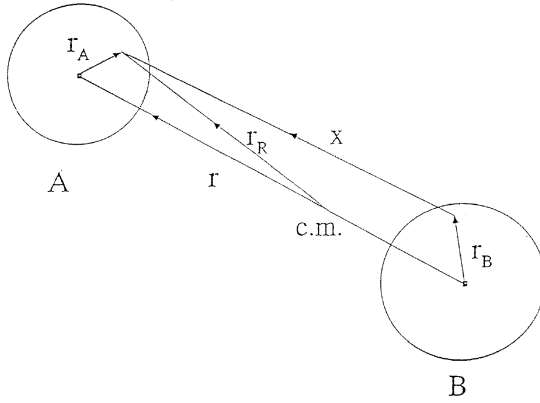
The cross-section for the process shown in eq. (2) is given by

$$d\sigma = [PS] \langle |T_{fi}|^2 \rangle, \quad (5)$$

where

$$T_{fi} = \langle \chi_x^- \chi_y^- \Phi_f(\xi, \mathbf{r}) | \Gamma_{Rxy} G_R(\mathbf{r}'_R, \mathbf{r}_R) \Gamma_{NNR} | \Phi_A(\xi_A) \Phi_B(\xi_B) \chi_{\mathbf{k}_i}^+ \rangle. \quad (6)$$

Here  $\chi$ s and  $\Phi$ s denote the wave functions of the continuum particles and bound systems respectively.  $G_R$  is the in-medium resonance propagator.  $\Gamma_{NNR}$  and  $\Gamma_{Rxy}$  are the amplitudes for resonance production in  $NN$  collision and its decay into  $x$  and  $y$  respectively.



**Figure 1.** Vector diagram for the reaction  $A + B \rightarrow R + X$ .

Since in the final state, states of the final nuclear system are not observed, we sum over them. Then,

$$\sum_f |T_{fi}|^2 = \int d\mathbf{r} d\xi_A d\xi_B |\langle \chi_x \chi_y | \Gamma_{Rxy} G_R \Gamma_{NNR} | \Phi_A \Phi_B \rangle|^2, \quad (7)$$

where we have used the closure for the final states and plane waves for  $\chi_{k_i}$ . If we do not use PW for  $\chi_{k_i}$ , we will get an attenuation factor outside.

Since the resonance production in  $NN$  collision is a high momentum transfer event, it happens at very close separations of the two interacting nucleons. We therefore approximate various coordinates in eq. (7) taking this separation equal to zero (i.e.  $x = 0$  in figure 1) and factor out  $\Gamma$ 's from the above integral. We further write

$$\Gamma_{Rxy} \Gamma_{NNR} = \Gamma_{NN \rightarrow xy}, \quad (8)$$

which gives a measure of the strength of  $xy$  production in  $NN$  collision (except for the free space resonance propagator). With this we get

$$\sum_f |T_{fi}|^2 = |\Gamma_{NN \rightarrow xy}|^2 \int d\mathbf{r} d\xi_A \rho(\mathbf{r}_A) \rho(\mathbf{r} + \mathbf{r}_A) |G(\mathbf{r}_R; \mathbf{k}_R, \mu)|^2, \quad (9)$$

where, for plane waves for  $x$  and  $y$  [11],

$$G(\mathbf{r}_R; \mathbf{k}_R, \mu) = \int d\mathbf{r}'_R \exp(-\mathbf{k}_R \cdot \mathbf{r}'_R) G(\mathbf{r}'_R, \mathbf{r}_R). \quad (10)$$

This function physically gives the probability amplitude for finding particles  $x$  and  $y$  in the detector with the total momentum

$$\mathbf{k}_R = \mathbf{k}_x + \mathbf{k}_y, \quad (11)$$

and the invariant mass  $\mu$

$$\mu^2 = (E_x + E_y)^2 - (\mathbf{k}_x + \mathbf{k}_y)^2, \quad (12)$$

if the resonance  $R$  is produced at a point  $r_R$  in the nucleus (for details see refs [11–13]). The function  $G(\mathbf{r}_R; \mathbf{k}_R, \mu)$  depends upon  $\mu$  through  $G(\mathbf{r}'_R, \mathbf{r}_R)$ . If the resonance  $R$  does not experience any nuclear interaction, function  $G(\mathbf{r}_R; \mathbf{k}_R, \mu)$  reduces to a free space propagator,  $(\mu^2 - m_R^2 + i\Gamma_R m_R)^{-1}$ , and eq. (9) becomes

$$\sum_f |T_{fi}|^2 = [AB] |\Gamma_{NN \rightarrow xy} (\mu^2 - m_R^2 + i\Gamma_R m_R)^{-1}|^2, \quad (13)$$

which can be written as

$$\sum_f |T_{fi}|^2 = [AB] |t_{NN \rightarrow xy}|^2, \quad (14)$$

where  $t_{NN \rightarrow xy}$  represents the quantity under the modulus in eq. (13) and is the  $t$ -matrix for the  $xy$  production through a resonance in a nucleon–nucleon collision.

From the above, the ratio of the cross sections for the resonance production in  $AB$  and  $NN$  collisions, for an inclusive situation where the state of the  $(A + B)$  system is not identified, can, thus, be written as

$$\frac{d\sigma_R^{AB}}{[KF]d\sigma_R^{NN}} = \int d\mathbf{b} \int dz \int d\mathbf{r}_A \rho_A(\mathbf{r}_A) \rho_B(\mathbf{r} + \mathbf{r}_A) |G(\mathbf{r}_R; \mathbf{k}_R, \mu)|^2, \quad (15)$$

where  $\rho_x$  are the nuclear densities. [KF] is the relevant kinematic factor.

The function  $G(\mathbf{r}'_R, \mathbf{r}_R)$  in eq. (10) satisfies

$$[\nabla^2 + E^2 - m_R^2 + i\Gamma_R m_R - \Pi_R]G(\mathbf{r}'_R, \mathbf{r}_R) = \delta(\mathbf{r}'_R - \mathbf{r}_R). \quad (16)$$

Here  $\Pi_R$  is the self energy of the resonance in the medium and  $\Gamma_R$  is its free space decay width.

In the eikonal approximation the function  $G(\mathbf{r}_R; \mathbf{k}_R, \mu)$  appearing in eq. (15) can be written as

$$G(\mathbf{r}_R; \mathbf{k}_R, \mu) = \exp(-i\mathbf{k}_R \cdot \mathbf{r}_R) \phi_R(\mathbf{r}_R; \mathbf{k}_R, \mu), \quad (17)$$

where  $\phi_R$  is a slowly varying modulating function. With this, and using the eqs (10) and (16),  $\phi_R$  approximately works out to

$$\begin{aligned} \phi(\mathbf{r}_R; \mathbf{k}_R, \mu) &= \frac{1}{2ik_R} \int dz'_R \exp \left[ \frac{1}{2ik_R} (\mu^2 - m_R^2 + i\Gamma_R m_R) (z_R - z'_R) \right] \\ &\times \exp \left[ \frac{E_R}{ik_R} \int_{z_R}^{z'_R} V_R(b_R, z''_R) dz''_R \right], \end{aligned} \quad (18)$$

where we have written the self energy  $\Pi$  in terms of the corresponding optical potential  $V_R$  as

$$\Pi_R = 2E_R V_R. \quad (19)$$

$V_R$ , in general, is complex and energy dependent. Its real part, as we shall see later, is related to the mass shift of the resonance and the imaginary part gives the collision broadening of the resonance in the medium.

For a nucleus with a sharp surface, function  $\phi(\mathbf{r}_R; \mathbf{k}_R, \mu)$  splits into a sum of two terms, one corresponding to the decay of the resonance inside the nucleus and another to the decay outside the nucleus, i.e.

$$\phi(\mathbf{r}_R; \mathbf{k}_R, \mu) = \phi_{\text{in}}(\mathbf{r}_R) + \phi_{\text{out}}(\mathbf{r}_R), \quad (20)$$

where

$$\phi_{\text{in}}(\mathbf{r}_R) = \frac{1}{2ik_R} \int_{z_R}^{\sqrt{(R^2 - b_R^2)}} dz'_R \phi_R(\mathbf{b}_R; z_R, z'_R), \quad (21)$$

and

$$\phi_{\text{out}}(\mathbf{r}_R) = \frac{1}{2ik_R} \int_{\sqrt{(R^2 - b_R^2)}}^{\infty} dz'_R \phi_R(\mathbf{b}_R; z_R, z'_R), \quad (22)$$

with

$$\begin{aligned} \phi_R(\mathbf{b}_R; z_R, z'_R) = & \exp \left[ \frac{1}{2ik_R} (\mu^2 - m_R^2 + i\Gamma_R m_R) (z_R - z'_R) \right] \\ & \times \exp \left[ \frac{E_R}{ik_R} \int_{z_R}^{z'_R} V_R(b_R, z''_R) dz''_R \right]. \end{aligned} \quad (23)$$

Since in the cross section (eq. (15)) the square of  $\phi(\mathbf{r}_R; \mathbf{k}_R, \mu)$  appears, the interference between the contributions from the decay inside and outside the nucleus is automatically included in our formalism.

After a little bit of manipulation, the final expressions for  $\phi_{\text{in}}$  and  $\phi_{\text{out}}$  work out to

$$\phi_{\text{in}}(\mathbf{r}_R; k_R, \mu) = \frac{G_0^*}{2m_R} \left[ 1 - \exp \left( \frac{i}{v_R G_0^*} [L(b_R) - z_R] \right) \right], \quad (24)$$

$$\phi_{\text{out}}(\mathbf{r}_R; k_R, \mu) = \frac{G_0}{2m_R} \left[ \exp \left( \frac{i}{v_R G_0} [L(b_R) - z_R] \right) \right], \quad (25)$$

where  $v_R (= \frac{k_R}{E_R})$  is the speed of the resonance and  $L (= \sqrt{(R^2 - b_R^2)})$  is the length from the production point to the surface of the nucleus.  $G_0$  and  $G_0^*$  in eqs (24) and (25) are the free and in-medium resonance propagators, written as

$$G_0 = \frac{2m_R}{\mu^2 - m_R^2 + i\Gamma_R m_R}, \quad (26)$$

$$G_0^* = \frac{2m_R}{\mu^2 - m_R^{*2} + i\Gamma_R^* m_R} \quad (27)$$

with

$$E_R^2 = k_R^2 + \mu^2, \quad (28)$$

$$\begin{aligned} m_R^{*2} &= m_R^2 + \Pi_R \\ &= m_R^2 + 2E_R U_R. \end{aligned} \quad (29)$$

or

$$m_R^* \approx m_R + \frac{E_R}{m_R} U_R, \quad (30)$$

$$\begin{aligned} \Gamma_R^* &= \Gamma_R - 2 \frac{E_R}{m_R} W_R \\ &= \Gamma_R + \frac{E_R}{m_R} |2W_R| \\ &= \Gamma_R + \frac{E_R}{m_R} \Gamma_{\text{collision}}. \end{aligned} \quad (31)$$

In the above we have written

$$V_R = U_R + iW_R. \quad (32)$$

These potentials, as given in eqs (30), (31), give a measure of the mass and width modification of the resonance in the nuclear medium. Their values are an open question and a subject of much research internationally. In one approach they can be treated as completely unknown quantities and data on appropriate experiments can be used to extract their values. This exercise would be of use if the theoretical formalism describes the reaction dynamics correctly and the data do not have much uncertainty. Alternatively, they can be estimated in a particular model and the ensuing values can be used to make an estimate of the cross section for the  $\rho$  production. In the literature, various efforts [14,15] have been made to estimate  $V_R$  using the high energy ansatz, i.e.

$$U_R = -\alpha \left[ \frac{1}{2} v_R \sigma_T^{RN} \rho_0 \right] \quad (33)$$

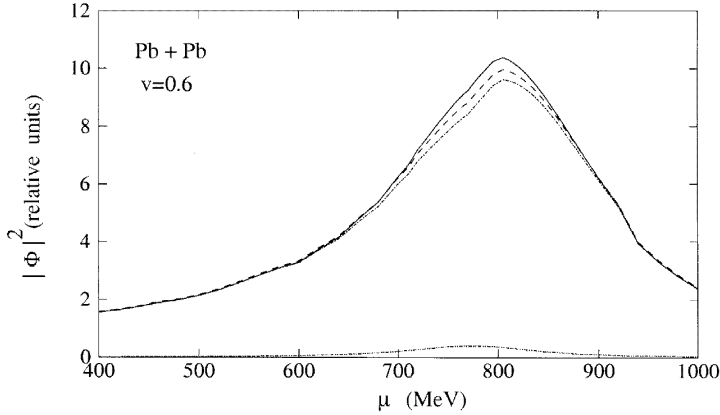
and

$$W_R = - \left[ \frac{1}{2} v_R \sigma_T^{RN} \rho_0 \right], \quad (34)$$

where  $\alpha$  is the ratio of the real to the imaginary part of the elementary  $RN$  scattering amplitude and  $\sigma_T^{RN}$  is the total cross section for it.  $\rho_0$  is the typical nuclear density. A detailed calculation for the  $\rho$ -meson has been done on these lines by Kondrayuk *et al* [15] which gives these potentials as a function of momentum. They use VDM at high energies and resonance model at low energies to generate the  $\rho N$  scattering parameters. We have used these values for our calculations in the present paper. Some representative values of the optical potentials required in our calculations are given in table 1.

**Table 1.**  $\rho$ -meson optical potentials following ref. [15] for  $\mu = 770$  MeV.

$v/c$	$U(\text{MeV})$	$W(\text{MeV})$
0.04	-20.4	-40.8
0.6	37.9	-50.6
0.9	25.8	-54.7

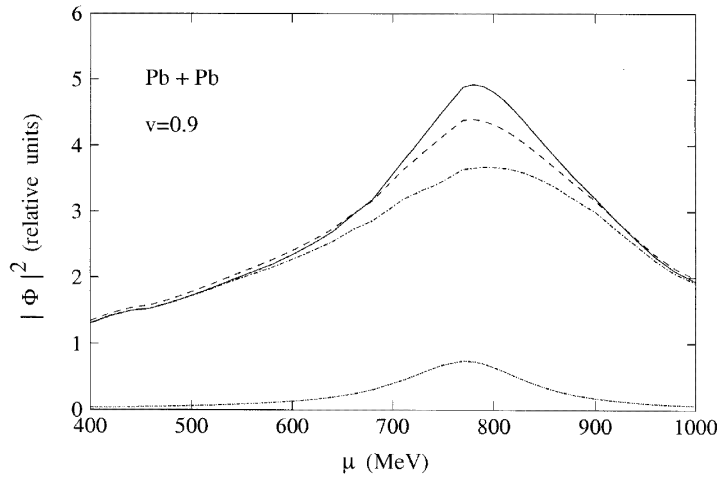


**Figure 2.** The invariant mass spectra of the  $\rho$ -meson produced in Pb+Pb collisions. The solid curve is obtained after adding the inside and the outside decay coherently, while the dashed curve is obtained by adding the same incoherently. The dash-dotted curve is the inside decay contribution and the dash-dot-dot curve is the outside decay contribution separately. The resonance speed is  $0.6c$ .

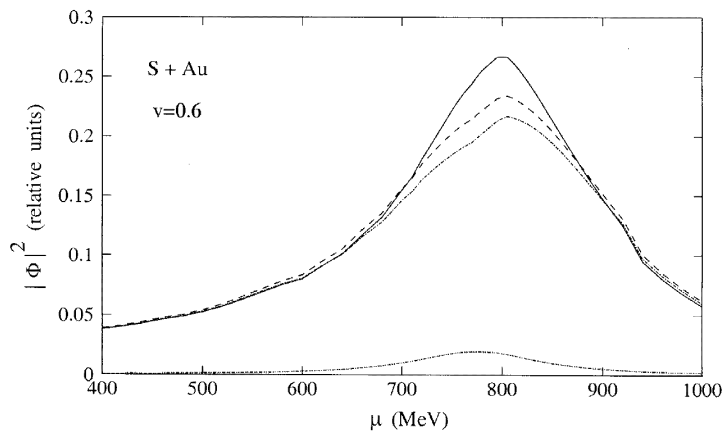
### 3. Results and discussion

Examining eqs (18), (21)–(23) for  $\phi(\mathbf{r}_R; \mathbf{k}_R, \mu)$  in the above we find that the cross sections for the decay of the resonance in the nucleus depends upon the length of the nuclear medium, the speed ( $v_R$ ), the free decay width and the self energy of the resonance. To represent the effect of all these quantities, we present results for the decay of the  $\rho$ -meson for different values of  $v_R$  and for two sets of nuclear systems, viz. Pb+Pb and S+Au. The free width of the  $\rho$ -meson is taken equal to 150 MeV. The optical potentials, which we need at several  $\rho$  momenta, are taken, as mentioned above, from Kondratyuk *et al* [15]. Nuclear densities are taken from ref. [16].

Denoting the ratio  $\frac{1}{[KF]} \frac{d\sigma_R^{AB}}{d\sigma_R^N}$  in eq. (15) as  $|\Phi|^2$ , we plot  $|\Phi|^2$  as a function of the invariant mass,  $\mu$ , of the decay products  $x$  and  $y$  of the  $\rho$  meson. Figures 2–5 show the invariant mass spectra of the  $\rho$  meson in Pb+Pb and S+Au collisions at  $\rho$  velocities of  $0.6c$  and  $0.9c$ . Here  $c$  is the speed of light. The solid curve in all the figures gives the coherently summed cross-section from the decay of  $\rho$ -meson inside and outside the nuclear medium. The dashed curve gives the same added incoherently. The individual



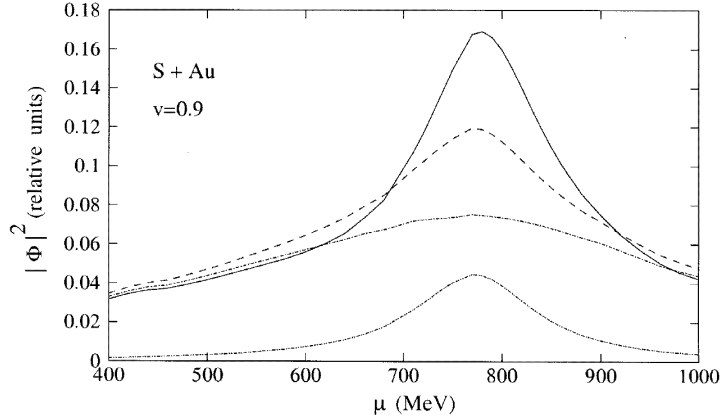
**Figure 3.** All curves have same meaning as in figure 2. The resonance speed is  $0.9c$ .



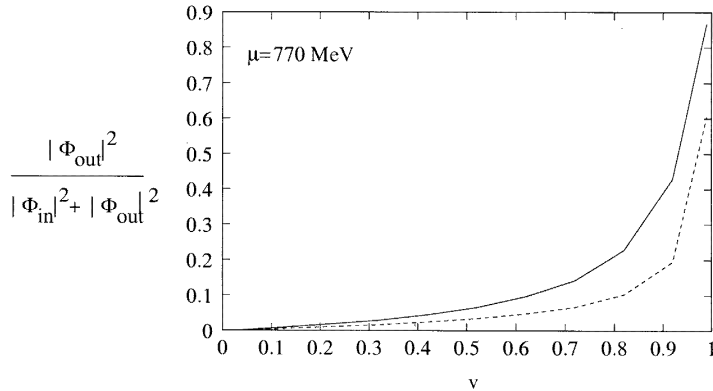
**Figure 4.** All curves have same meaning as in figure 2. The colliding nuclear system is S+Au and the resonance speed is  $0.6c$ .

contributions corresponding to the inside and the outside decay are given by the dash-dotted and dash-dot-dot curves respectively. We observe two things. One, the coherent and the incoherent cross-sections are different, and second, this difference increases with the increase in the  $\rho$ -meson speed. At  $0.6c$  speed, while the coherent and incoherent curves differ only in the peak cross sections, at  $0.9c$  speed their shape and peak cross sections both are different. We also observe that the difference is larger for the smaller system like S on Au.

To get a quantitative idea of the  $\rho$ -meson decay probability outside the nuclear medium, in figure 6 we plot  $\frac{|\Phi_{\text{out}}|^2}{|\Phi_{\text{out}}|^2 + |\Phi_{\text{in}}|^2}$  as a function of the  $\rho$ -meson speed. The results are given for Pb+Pb as well as S+Au system. The invariant mass  $\mu$  is taken equal to 770 MeV.



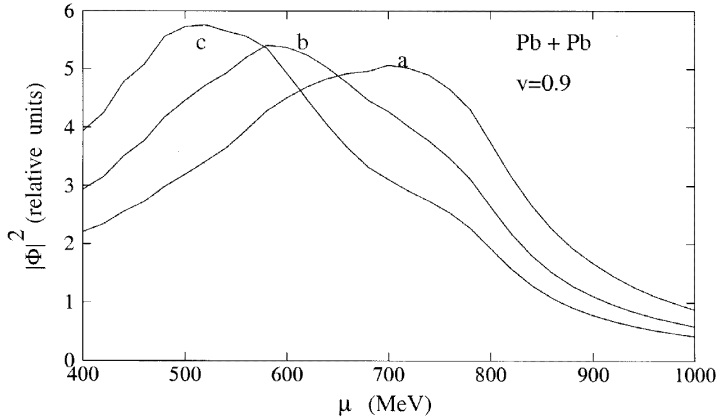
**Figure 5.** Same as in figure 2. The resonance speed is  $0.9c$ .



**Figure 6.** The outside decay probability as a function of the resonance velocity. The solid curve is for S+Au and the dashed curve is for Pb+Pb nuclei.

It is seen that for slow moving  $\rho$ -meson, the decay is mainly inside the nuclear medium. As the speed increases the outside decay contribution increases. For example, for the lighter systems like S+Au the outside decay probability is more than 10% beyond  $0.6c$  speed, and for the heavier systems it becomes more than 10% after  $0.8c$  speed.

If we compare the mass shift seen in our calculations (figures 2–5) with those indicated in the high energy heavy ion collisions, our shifts are small. These are, of course, for the set of optical potentials used by us. We can get an indication of the strength of the optical potentials required to give large mass shifts by studying the sensitivity of our results to the strength of the optical potentials. For this we calculated  $|\Phi|^2$  for Pb+Pb at  $0.9c$  for three arbitrarily chosen values of  $U_R$ , viz.  $-40$ ,  $-80$  and  $-120$  MeV. The imaginary part of the optical potentials is the same as used in figure 2. These results are shown in figure 7. We find that only with  $-120$  MeV value the distribution starts having features resembling



**Figure 7.** Sensitivity of the cross section to the strength of the optical potential. The calculations are presented for Pb+Pb system at  $0.9c$   $\rho$ -meson speed. Curves a, b and c are for  $U_R$  equal to  $-40$ ,  $-80$  and  $-120$  respectively. The imaginary part of the optical potentials is the same as used in figure 2.

those indicated in heavy-ion experiments. But, this value, if we compare with the values coming from the high energy ansatz of Kondratyuk *et al* (see table 1), is abnormally large, and thus appears unphysical. The aim of this observation is not to suggest that the  $\rho$ -meson might have such large unphysical optical potentials. It only indicates that the origin of the observed mass shift, probably, does not lie in the final state interaction. It may have origin in QCD where the  $\rho$ -meson in the nuclear collisions gets formed with a reduced mass only.

#### 4. Conclusions

In summary, we conclude that:

1. The resonance produced in heavy-ion collisions can decay inside the nuclear medium or outside it. The outside decay probability is significant if the nuclear systems involved are not large and/or the resonance speed is large. In such cases the shape of the invariant mass of the decay products of the resonance could be much different if it is calculated using the inside and outside contributions coherently or incoherently. The extracted mass shift of the resonance from the experimental data, thus, should be determined using the coherent calculations only.
2. The mass shift seen in the calculated  $\rho$ -meson invariant mass distribution using the self energy calculated by Kondratyuk *et al* from the high energy ansatz and the inputs within the VDM and resonance model is small.
3. A mass shift towards lower mass by about 200 MeV, as indicated in high energy heavy-ion collisions, can be obtained only by having optical potentials as large as  $-120$  MeV, which is unphysical. Therefore, the large mass shift seen in the RHIC does not seem to arise from the final state interaction. It should have origin in QCD at the birth of the  $\rho$ -meson.

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